# Zak phase of a 1D topological photonic crystal by Finite-Difference Time-Domain simulation


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***Abstract*—Topological properties of one-dimensional periodic systems are characterized by the Zak phase, which is essential for describing protected edge or surface states that are robust against disorder and perturbations. Here, we explicitly calculate the Zak phases of a one-dimensional topological photonic crystal with guided-mode resonance using the Finite-Difference Time-Domain method. The retrieved time-dependent Zak phases are found to be zero for trivial and π for nontrivial photonic crystals respectively, which ensures the bulk-edge correspondence, even in a non-Hermitian condition.**



***Keywords— Zak phases, photonic crystal, FDTD simulation***


## I. INTRODUCTION

Recently, there has been a number of demonstrations of topological photonic crystal lasers [1]. While most of these are implemented in 2D photonic crystal (PhC) lattices, one-dimensional (1D) structures stand out due to their small mode volume and single mode operation [2]. The topological properties of bulk materials are determined by topological invariants such as the Chern number and the Berry phase [3]. In one-dimensional (1D) periodic systems under Hermitian conditions, this is the Zak phase as the 1D variant of the Berry phase, which has quantized values of 0 or π [4].

The commonly employed 1D topological models, though, do not always ensure bulk-edge correspondence under non-Hermitian conditions. Here we focus on a photonic non-Hermitian structure, which has seen a number of demonstrations such as topological photonic lasers. We develop a model that addresses the validity of the bulk-edge correspondence for the case of a 1D semiconductor photonic crystal. This allows us to gain a better understanding of the topological contribution of the lasing of photonic structures under leakage or pulsed pumping.

We choose a 1D nanobeam PhC with guided-mode resonance implementing the Su–Schrieffer–Heeger (SSH) model. We use the finite-difference time-domain (FDTD) method to compute the electromagnetic fields as a function of time explicitly [5]. Thereby, the time-dependent Bloch modes of the magnetic field $H_z$ within the first Brillouin zone (BZ) can be obtained. The resultant Zak phases for both trivial and nontrivial photonic structures are numerically calculated. By considering the temporal evolution of the magnetic fields, we obtain the stationary and dynamic Zak phases, which show a topological feature of the photonic structure under a non-stationary and non-Hermitian condition.

## II. SIMULATION METHOD

The 1D SSH model consists of dimerized holes within the slab structure, as illustrated in Fig. 1 [6]. Two distinct unit cells with an inversion symmetry are designed. The dimerized holes in the unit cell are arranged in two ways, either the normalized distance *d* is less than 0.5 (unit cell A) and greater than 0.5 (unit cell B). As a first step, we calculated the photonic band structure of the PhC slab using the Ansys-Lumerical FDTD solver. The parameters used in the simulation are as follows; The lattice constant $a_x$ = 270 nm, the hole width $w$ = 54 nm, the hole length $y$ = 162 nm, the slab height $h$ = 172.8 nm, the slab index $n$ = 3.4

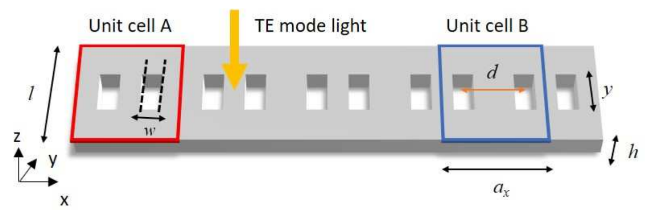


Fig. 1. Schematic of 1D PhC structure

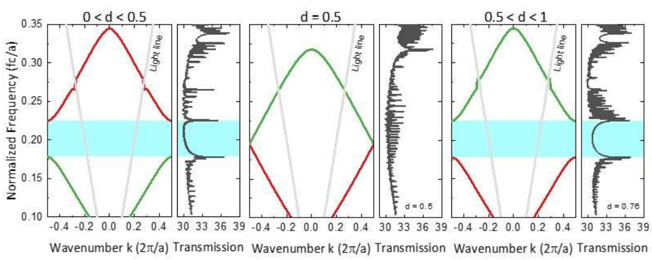


Fig. 2. Photonic bandstructure and transmission spectra for three conditions: 0 < d < 0.5, d = 0.5, and 0.5 < d < 1.

Fig.2 depicts the topological characteristics of photonic band structures and transmission spectra of the top PhC lattice for three different *d* values. The blue region is the photonic bandgap where photons are not allowed. The bandgap at the band edge in the infinite structure(left) is identical to the one from the transmission spectrum of a finite structure consisting of 40 unit cells(right), to which we restrict further simulations.

## III. Numerical Results

To calculate the Zak phase, we have to obtain Bloch modes in the PhC slab. In this system, Maxwell's equations can be expressed as the following eigenvalue problem [7]:

$$\nabla \times \left[\frac{1}{\varepsilon(\mathbf{r})}\nabla \times H_z(\mathbf{r})\right] = \left(\frac{\omega}{c}\right)^2 H_z(\mathbf{r}) \qquad (1)$$

Owing to the periodicity of the PhC, the solutions of Eq. (1) can be expressed using the Bloch theorem, where $u_k(\mathbf{r})$ is the Bloch function with the lattice constant $\boldsymbol{a}$.

$$H_{z,\boldsymbol{k}}(\mathbf{r}) = e^{i\boldsymbol{k}\cdot\boldsymbol{r}}u_{\boldsymbol{k}}(\mathbf{r}) = e^{i\boldsymbol{k}\cdot\boldsymbol{r}}u_{\boldsymbol{k}}(\mathbf{r}+\mathbf{a}) \qquad (2)$$

The magnetic field ($H_z$) at the band edges for each PhC is depicted in the upper and lower right of Fig. 3. The Zak phase is given as an integral over the BZ:

$$\phi_n = i\oint \langle u_{n,\boldsymbol{k}}|\nabla_{\boldsymbol{k}}|u_{n,\boldsymbol{k}}\rangle \cdot d\boldsymbol{k} \qquad (3)$$

Determining the Zak phase for arbitrary photonic structures necessitates a form of discretization. By dividing the Brillouin zone (BZ) into N equal intervals, the Zak phase for each band can be recast into the following form [8]:

$$\phi = -\mathrm{Im}\left[\ln\left[\langle u_{k_1}(\boldsymbol{r})|u_{k_2}(\boldsymbol{r})\rangle \cdots \langle u_{k_{N-1}}(\boldsymbol{r})|u_{k_N}(\boldsymbol{r})\rangle\right]\right] \qquad (4)$$

This form assumes that the Bloch modes are steady-state. Within the simulation, we can choose an appropriate temporal apodization window to average out the magnetic fields to obtain Bloch modes and the Zak phases.

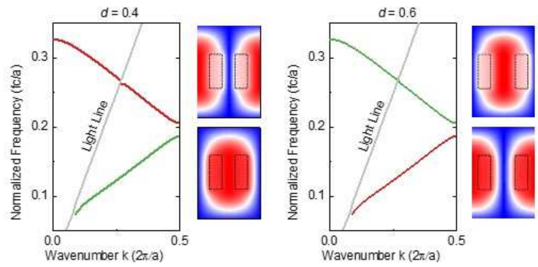


Fig. 3. Photonic band structures and the magnetic field ($H_z$) mode profiles for PhCs composed of unit cells A and B, respectively.

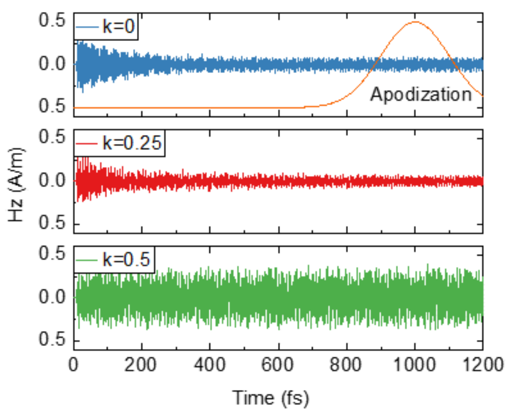


Fig. 4. Time evolution of the magnetic field ($H_z$) $\boldsymbol{k}$ = 0.5 (green), $\boldsymbol{k}$ = 0.25 (red), and $\boldsymbol{k}$ = 0 (blue)

The Gaussian function with a full-width at half-maximum of 250 fs is used as an apodization window, which provides sufficient temporal averaging over rapidly varying magnetic fields. Extracting the Bloch mode under the apodization time is shown in Fig. 4. Then we numerically calculate the Zak phases by substituting the Bloch function obtained from Eq. (2) into Eq. (4), as shown in Fig. 5. As expected, the nontrivial structure with 0.5 < d < 1 results in Zak phases of $\pi$, while it remains zero for the trivial structure.

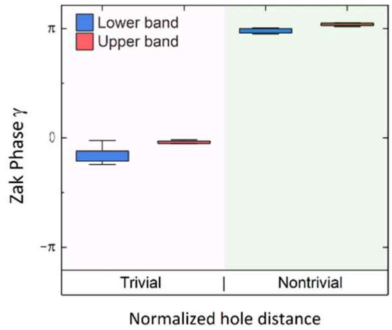


Fig. 5. Calculated Zak phases for Trivial (left) and Nontrivial (right) lattices.

## IV. Conclusion

In summary, we conducted numerical simulations to determine the Zak phases of 1D PhC explicitly, under guided-mode resonance and pulsed excitation as non-Hermitian conditions. This was achieved by discretizing Bloch magnetic fields in the first BZ. Our analysis revealed quantized values of 0 or $\pi$ for trivial and nontrivial PhCs, respectively. Our result shows that non-Hermitian conditions such as pulsed excitation or leakage still maintain the topological nature of the system originating from bulk properties.


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