Analysis of herladed single photon decoy state protocol using single photon detectors for quantum secure imaging

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***Abstract*— Spontaneous parametric down-conversion heralded single photon pairs (HSPS) are utilized in quantum key distribution (QKD) and quantum imaging (QI) experiments. Decoy state methods enhance security in these fields. HSPS is well-suited for integrating QKD protocols, reducing measurement uncertainty, and ensuring secure quantum imaging. This study examines the performance of the HSPS-decoy state method for quantum imaging under various conditions, focusing on secure key rate and Quantum Bit Error Rate (QBER) for free space applications. We specifically investigate the HSPS within the BB84 decoy state protocol for secure quantum imaging in free space. The dark count rate significantly impacts QBER, affecting the maximum tolerable loss. Low-efficiency, low-noise detectors can outperform high-efficiency, high-noise detectors, highlighting the importance of minimizing dark counts for optimal system performance.**

Keywords*— Quantum secure imaging, Quantum key distribution, BB84 protocol, Decoy state method.

I. Introduction

Ubiquitous quantum properties of nonclassical light sources and their possibility of being integrated into systems for advanced sensing, imaging and secured information processing have been the focus of research and technological revolution in the recent past. Spontaneous parametric down-conversion heralded single photon pairs are utilized in quantum key distribution (QKD) and quantum imaging (QI) experiments. Decoy state methods have also been employed to enhance security in these fields. Due to its superior performance in low-photon-number regimes, the HSPS is well-suited for integrating QKD protocols, which helps to reduce measurement uncertainty and ensure secure quantum imaging[1]. We examine the performance of the proposed HSPS-decoy state method for quantum imaging under various operating conditions, focusing on secure key rate and quantum bit error rate (QBER) as key parameters for free space applications. Recent examinations have evaluated similar systems[2], but this study specifically investigates the HSPS within the BB84 decoy state protocol for secure quantum imaging in free space.

II. Mathematical model

The Decoy state method is helpful in practical situations when the source is not an ideal single-photon source but could also be embedded with multiple photon components in the transmitted information. It is a technique used in QKD to improve the security of information transmitted. The secure key rate, R is given by [3]:

$$R \geq q[Q_1[1 - H_2(e_1)] - Q_\mu\, f(E_\mu)H_2(E_\mu)]$$

Where q is a parameter depending on the QKD protocol, Q_1 is the gain of single photons, H is the binary Shannon entropy, e_1 is the error rate od single photon states, E_μ is the overall quantum bit error rate, and $f(E_\mu)$ is the error correction efficiency. This calculation ensures that a secure key is generated, accounting for the presence of multi-photon states.

The equation that describes the photon number distribution for a SPDC based HSPS(thermal) is given by [4], $p_{k,x}^{HSPS_{per}} = \frac{x^n}{(1+x)^{n+1}} * \frac{(1-(1-\eta_A)^k + d_A)}{P_x^{post}}$ where x is the mean photon number, k is number of photons. η_A represents the detection efficiency at the source end, d_A is the dark count rate for Alice detector, P_x^{post} is the post-selected probability.

Figure 1. illustrate the single-photon contributions from photon source HSPS. The graph given in Figure 1 is a reference for determining optimal mean photon number of the source.

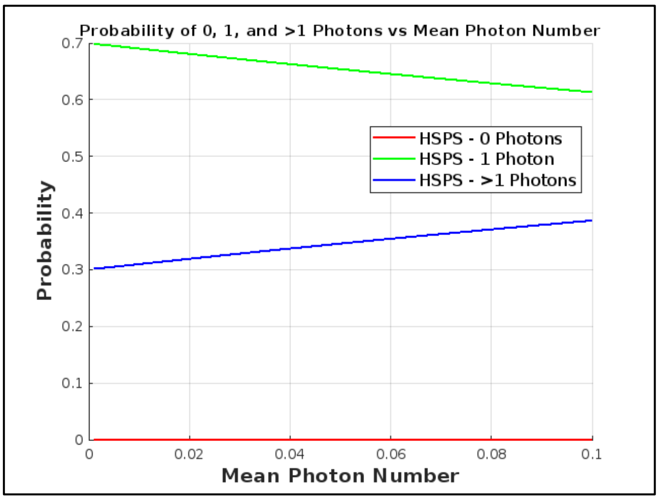

Figure 1. Quantitative Analysis of Single-Photon Emissions from SPDC based Heralded Single Photon Source (HSPS) and Weak Coherent single (WCS) photon source: Implications for QSI. (correlation probability=0.7, $\eta_A = 0.6$ and, $d_A = 10^{-6}$) for mean photon number 0 to 0.1.

III. Results

The **Figure 2.** depicts the secure key rate versus transmission loss in decibels (dB). In this simulation, several key parameters are used to model the secure key rate[2], [5], [6].

The attenuation coefficient for free space is set to 0.1 dB/km, with an error rate associated with vacuum events e_0= 0.5 and an intrinsic error rate of the detection system e_d = 0.033. The error correction inefficiency factor f (e)=1.22, and the protocol-specific constant for BB84 (q) is 1/2. The receiver and transmitter aperture diameters are 12 mm and 10 mm, respectively, and the divergence angle is 0.025 mrad. The simulation examines various detector types, the first set has an efficiency η_A of 0.6 for Alice and η_B= 8.216*10^{-3} for Bob(or the overall consideration of the channel loss, an additional loss towards the Bob end of the detection is taken as 5 dB, in addition to the detection efficiency of the detectors), with dark count rates d_A and d_B of 10^{-6} and 8*10^{-9}, respectively. The correlated probability P_{cor} = 0.7. The third set further increases Bob's efficiency to 4.898*10^{-2} and dark count rate to 6.8*10^{-5}. The fourth SPAD detector set has Bob's efficiency at 1.106*10^{-1} and dark count rate at 3.5*10^{-8}. For the fifth detector efficiency is 2.686*10^{-2} for Bob, with dark count rate 9.2*10^{-6}.

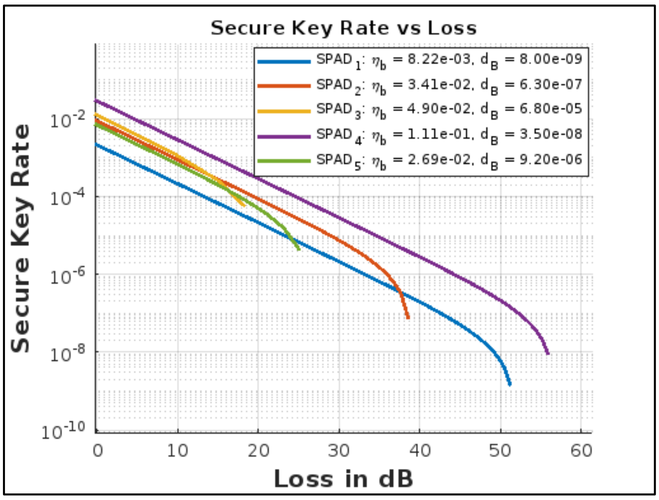

Figure 2. Secure key rate versus transmission loss in decibels (dB). Figure 3 shows the Quantum Bit Error Rate (QBER) versus loss in dB for various detector parameters. The red dashed line at 11% QBER marks the threshold beyond which error rates are too high for practical applications.

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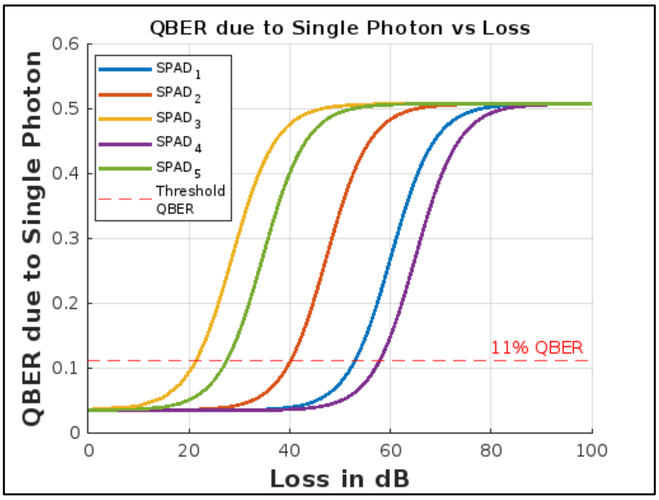

Figure 3: Quantum Bit Error Rate (QBER) versus loss in dB for various detector parameters.

IV. CONCLUSION

Reducing the occurrence of dark counts is essential for maximizing the system's efficiency since the ability to separate the signal from the background noise is more significant than the total amplification of the system. The dark count rate substantially influences the Quantum Bit Error Rate (QBER), which determines the maximum amount of loss the system can handle without exceeding the threshold value. While the total counts are affected by detector efficiency, a detector with low efficiency but low noise may perform better than a detector with high efficiency but high noise. This exemplifies the significance of reducing dark counts to improve the system's performance.

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