A New Analytical Model of Gain in Highly-Saturated SOAs

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***Abstract*—This contribution presents a new analytical model in good agreement with experimental data for gain in highly-saturated SOAs. This model, based on Lambert W function, only needs two parameters and offers fast-computing simulations.**

***Index Terms*—Semiconductor Optical Amplifier, Gain, Lambert W function.**

I. Introduction

Modeling the nonlinear behavior of Semiconductor Optical Amplifier (SOA) behavior is a challenging task [1]. Multisection models have been used to obtain a suitable behavior [2], [3]. Here, we introduce a monosection-analytical model, so we can limit the number of parameters involved. This not only makes our model computing efficient, but also allows to render the SOA-nonlinear-gain behavior over a large range of optical-input-power values. Indeed, understanding nonlinearities permits to better adapt the predistortion strategy in order to enhance the optical transmission performance [4]. The proposed model is validated using the experimental characterization of the SOA from [5] (see Fig. 3.13).

II. Mathematical Model

The saturated gain G of an SOA can be expressed in a general way as a function of the SOA total input power P_{in}, linear gain G_0, and saturation power P_{sat} as [6]:

$$G = G_0 e^{-(G-1)\frac{P_{in}}{P_{Sat}}} \tag{1}$$

One should note the term G in both sides of (1), as in this case the gain is necessarily saturated as the optical-input power is non-zero. A simple way to handle this is to compute the 1st order nesting expression as:

$$G = G_0 e^{-(G_0-1)\frac{P_{in}}{P_{Sat}}} \tag{2}$$

Willing to have a better saturation behavior, we can use (1) as a nested relationship of higher orders, setting $G = G(G(...(G(G_0))))$. Fig. 1 presents with markers the reference experimental gain as a function of the SOA-optical-input power. The solid line presents the gain using (2), while dashed and dotted lines present the gain using nested relationships of respectively 8th and 9th orders as an example. The following parameters are used for all analytical results: $G_0 = 16.2$ dB and $P_{sat} = 7$ dBm. These parameters were obtained by fixing the analytical small-signal gain G_0 as the experimental one, and optimizing P_{sat} for better reproducing the experimental data. At high input powers, working with even nesting orders (8th order in Fig. 1), the obtained gain increases with optical-input power, which is not a physical behavior. Working with higher odd nesting orders permits to better reproduce the experimental data up to a given optical-input power after which the behavior moves away from the reference curve, still not being fully satisfactory.

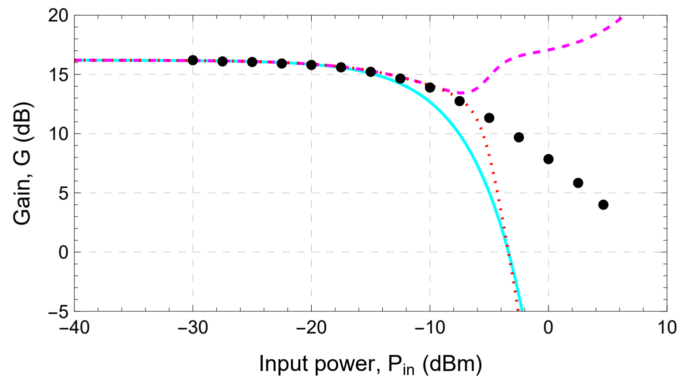

Fig. 1. Gain as a function of the optical-input power. Markers: experimental data. Solid, dashed and dotted lines: analytical gains using respectively 1st, 8th and 9th nesting orders with $G_0 = 16.2$ dB and $P_{sat} = 7$ dBm.

In order to analytically solve (1), we rework it to obtain the following expression:

$$G\frac{P_{in}}{P_{Sat}} e^{G\frac{P_{in}}{P_{Sat}}} = G_0 \frac{P_{in}}{P_{Sat}} e^{\frac{P_{in}}{P_{Sat}}} \tag{3}$$

The formal solution of (3), in which we recognize an expression $xe^x = y$, is $x = W(y)$, where $W(y)$ is the Lambert W function [7], giving finally:

$$G = \left(\frac{P_{in}}{P_{Sat}}\right)^{-1} W\left(G_0 \frac{P_{in}}{P_{Sat}} e^{\frac{P_{in}}{P_{Sat}}}\right) \tag{4}$$

Fig. 2 presents again the previous experimental data that we aim to reproduce. The solid line presents now the use of (4) with the same parameters as before ($G_0 = 16.2$ dB and $P_{sat} = 7$ dBm). We show in dashed and dotted lines the previously obtained gains using nested relationships of respectively 1st and 9th orders. The comparison shows a behavior closer to experimental data and physically realistic for the gain of saturated SOAs in favor of using Lambert W function.

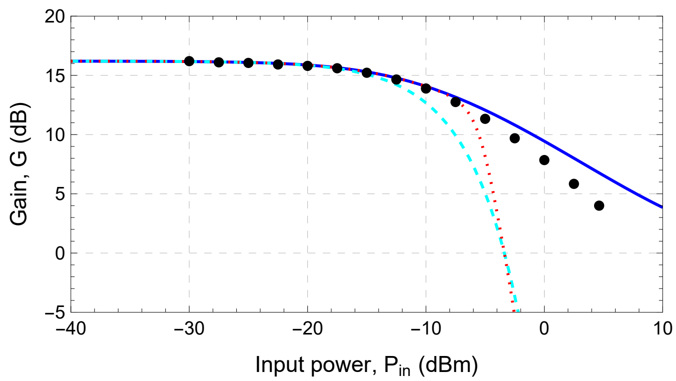

Fig. 2. Gain as a function of the optical-input power. Markers: experimental data. Solid, dashed and dotted lines: analytical gains using respectively Lambert W function, 1st and 9th nesting orders with $G_0 = 16.2$ dB and $P_{sat} = 7$ dBm.

The average distance between analytical and experimental behaviors can be minimized by varying P_{sat} parameter. We show in Fig. 3 the same experimental and analytical behaviors as in Fig. 2 (markers and solid line), to which we add in dotted line the analytical behavior obtained setting $P_{sat} = 4$ dBm.

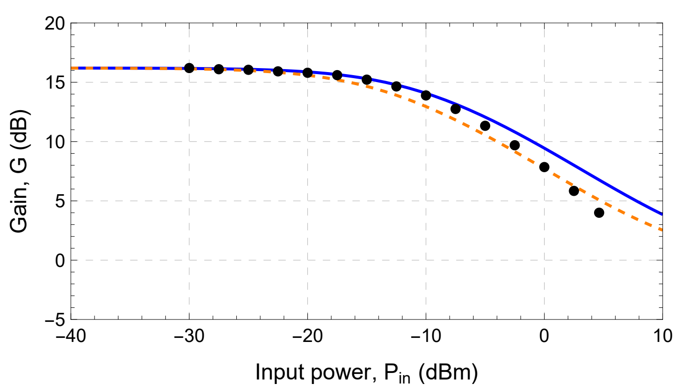

Fig. 3. Gain as a function of the optical-input power. Markers: experimental data. Solid and dashed lines: analytical gains using Lambert W function with $G_0 = 16.2$ dB and respectively $P_{sat} = 7$ dBm and $P_{sat} = 4$ dBm.

We finally show in Fig. 4 the benefit of using our analytical model in term of simulation performance. To do so we use Mathematica, in which we computed as a reference (4) (noted "Lambert W", called "ProductLog" in Mathematica). We see that using nested relationships (noted respectively as "1st nesting order" and "9th nesting order") comes with slightly faster performance but with non fully satisfying the physical behavior. Another solution would be to numerically solve (1), searching for a numerical value for G satisfying this relationship. We also show in Fig. 4 that this solution (noted "NSolve" as calling "NSolve" function in Mathematica) is nearly 70 times slower than the analytical one, while having the same behavior as using (4) (not represented in previous figures).

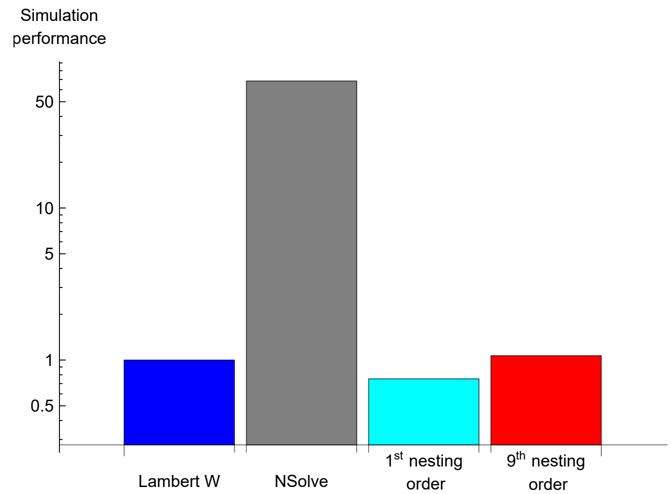

Fig. 4. Relative simulation performance (simulation duration) with reference to using Lambert W function.

III. Conclusion

We have introduced a simple and efficient analytical model to reproduce the gain behavior of highly-saturated SOAs. This model uses only two parameters and provides a behavior close to the experimental one for high optical-input powers.

References

[1] Y. Qi, W. Zhao, J. Dong, S. Wang, S. Ma, Q. Zhang, and L. Zhang, "Enhancing simulation efficiency in semiconductor optical amplifier (soa) models: A comparative study of first and second order approaches," pp. 1686–1690, 2024.

[2] M. J. Connelly, "Wideband Semiconductor Optical Amplifier Steady-State Numerical Model," *IEEE Journal of Quantum Electronics*, vol. 37, no. 3, pp. 439–447, March 2001.

[3] P. Morel and A. Sharaiha, "Wideband Time-Domain Transfer Matrix Model Equivalent Circuit for Short Pulse Propagation in Semiconductor Optical Amplifiers," *IEEE Journal of Quantum Electronics*, vol. 45, no. 2, pp. 103–116, 2 2009.

[4] A. Ghazisaeidi and L. A. Rusch, "On the efficiency of digital back-propagation for mitigating soa-induced nonlinear impairments," *Journal of Lightwave Technology*, vol. 29, no. 21, pp. 3331–3339, 2011.

[5] P. Morel, "Modélisation des amplificateurs optiques à semi-conducteurs : du composant au système," Ph.D. dissertation, Université de Bretagne occidentale, 2006.

[6] G. P. Agrawal and N. A. Olsson, "Self-Phase Modulation and Spectral Broadening of Optical Pulses in Semiconductor Laser Amplifiers," *IEEE Journal of Quantum Electronics*, vol. 25, no. 11, pp. 2297–2306, November 1989.

[7] R. Corless, G. Gonnet, D. Hare, D. J. Jeffrey, and D. E. Knuth, "On the Lambert W function," *Advances in Computational Mathematics*, vol. 5, p. 329–359, December 1996.