# Modelling coherent emission in transverse coupled cavity VCSELs

1<sup>st</sup> Martino D'Alessandro Politecnico di Torino Torino, Italy martino.dalessandro@polito.it

2<sup>nd</sup> Valerio Torrelli Politecnico di Torino Torino, Italy

3<sup>th</sup> Pierluigi Debernardi **CNR-IEIIT** Torino, Italy

4<sup>th</sup> Alberto Gullino **CNR-IEIIT** Torino, Italy

5<sup>th</sup> Keyvan Azimi Asrari 6<sup>th</sup> Markus Lindemann 7<sup>th</sup> Thomas de Adelsburg Ettmayer University of Pavia Pavia, Italy

Ruhr-Universität Bochum, Germany

Ruhr-Universität Bochum, Germany

8<sup>th</sup> Guido Giuliani University of Pavia, Julight Srl Pavia, Italy

9<sup>th</sup> Alberto Tibaldi Politecnico di Torino Torino, Italy

Abstract-Transverse coupled cavity (TCC)-VCSELs are promising candidates for the next generation of high-speed direct modulation devices. We present a comprehensive framework for their investigation based on the dynamical solution of the scalar wave equation and compare it with recent experimental results.

Index Terms-Transverse coupled cavity, VCSELs, coherent emission

## I. INTRODUCTION

TCC VCSELs [1] are promising candidates for a number of different applications, such as direct modulation [2], beam steering and terahertz generation [3]. The potential of these devices lies on the presence of two nearly frequencydegenerate (super)-modes [4], which however implies a critical thermal management and impact of technological variations. In this work we present a model based on the dynamical solution of the scalar wave equation. The model naturally handles particular dynamical states for which two modes are coherently phase-locked [2]. Such effects can be observed by adding cross-coupling terms to the standard rate equation model, which are derived from the scalar wave equation [4] and arise from the variations of the refractive index due to gain, self-heating or unwanted variations on the nominal structure.

We consider the structure manufactured in [5], consisting in a standard 850 nm VCSEL with bow-tie oxide aperture, comprising two circular cavities of radius  $R_{R,L}$ , connected by a rectangular region denoted as *bridge*. It is reasonable to assume that, due to the oxidation process, one of the two apertures is slightly bigger than the other  $(R_{\rm R} < R_{\rm L})$  in this case). We assume that the two cavities are electrically isolated, as done in [6], and that we can pump them separately with currents  $I_{R,L}$ , respectively. Fig.1 shows a sketch of the oxide aperture together with a cut of the first two optical modes, computed with our in-house VCSEL modal solver. The two modes are nearly degenerate in frequency and threshold gain.



Fig. 1. Sketch of the oxide aperture under investigation together with the first two optical modes.  $L_{br} = 4\mu m$  in this work.

### **II. THEORY AND RESULTS**

By representing the electric field in the quantum well as a sum of the optical modes (cold-cavity modes), we obtain a set of dynamical equations for the *i*-th modal amplitude  $E_i$ , reading:

$$\partial_t E_i = (i\omega_i - L_i)E_i + \sum_j k_{ij}E_j, \tag{1}$$

where  $L_i$  and  $\omega_i$  are the i-th modal losses and frequency offset with respect to an arbitrary reference  $\omega_0$ , and  $k_{ij}$  is given by the following integral on the active region (AR):

$$k_{ij}(t) = \frac{v_g \Gamma_z}{2} \iint_{AR} \Psi_i^* \Psi_j g(x, y) dx dy,$$
(2)

where  $v_q$  is the group velocity,  $\Gamma_z$  is the optical confinement factor and g(x, y) is a generalized definition of gain accounting for thermal effects, defined by:

$$g = \underbrace{(1 + i\alpha_{\rm h})[G_{\rm d}(N(x,y) - n_{\rm tr})]}_{\rm carriers} - \underbrace{i\frac{\Gamma_{\rm th}}{\Gamma_{\rm z}}\frac{4\pi}{\lambda_0}\frac{dn}{dT}\Delta T(x,y)}_{\rm temperature}$$
(2)

where  $n_{\rm tr}$  is the transparency carrier density,  $G_{\rm d}$  is the differential gain,  $\alpha_h$  is the linewidth enhancement factor,  $\frac{dn}{dT}$  is a



Fig. 2. Simulated optical spectrum for varying  $I_r$  for fixed  $I_l = 3mA$ . The vertical lines denote the boundaries of the coherent region. Insets: wavelength resolved near field for (a) 2.8 mA and (b) 3.2 mA. The dashed line is the emission wavelength of the below-theshold supermode in the coherent range, computed by finding the eigenvalues of the matrix  $k_{ij}$ .

phenomenological coefficient describing the variation of the refractive index with respect to the temperature,  $\lambda_0$  is the reference wavelength associated to  $\omega_0$ ,  $\Gamma_{\text{th}}$  is a phenomenological coefficient to describe the overlap of the standing wave with the longitudinal temperature profile, set to 1 in this work,  $\Delta T$  is the self-heating temperature variation profile. In this work, we assume  $\Delta T$  as a superposition of two gaussians centered in the two apertures, whose peak values are proportional to the currents  $I_{\text{R,L}}$ , with a coefficient of 7.5  $\frac{K}{mA}$ . Other material parameters can be found in [4]. The model is closed with the self-consistent solution of the carrier diffusion equation by means of a finite-element discretization, as carried out in [2].

We solve the dynamical equations fixing  $I_L$  and varying  $I_{\rm R}$ , using the first four modes as basis. Fig. 2 shows the resulting optical spectra. At low  $I_{\rm R}$ , each cavity emits at a distinct wavelength, with a large frequency separation due to the temperature difference between the apertures. As  $I_{\rm R}$ increases and approaches  $I_{\rm L}$ , a coherent regime emerges where both cavities emit at the same frequency, which tends toward the lower-wavelength mode. The plot is asymmetric with respect to  $I_{\rm R} = I_{\rm L}$ , reflecting the structural asymmetry of the device. In particular, the coupling occurs when the smaller cavity is hotter. The two insets display the wavelength-resolved near-field, highlighting the considerations made so far. Given a certain gain and temperature profile, we can intepret as pumped-cavity modes the combination of cold-cavity modes that diagonalize the matrix  $k_{ij}$ , where the imaginary part of the correspondent eigenvalues represents the emission frequency. In this way, we can highlight the presence of a below-trheshold supermode in the coherent range, which is responsible for photon-photon-resonance [4], denoted with the dashed line.

The simulated behaviour aligns well with recent experimental results [7] [6]. Evidence of coherent emission has been also recently observed in [2], showing theorethically and experimentally an enhancement of the bandwidth modulation.

#### **III. ACKNOWLEDGEMENT**

Alberto Tibaldi and the members of Ruhr-University and Julight would like to acknowledge the Italian Ministero dell'Università e della Ricerca (MUR) and by the German Bundesministerium für Bildung und Forschung within the EU-ROSTARS project 'COHORT' (E! 6226 ) for having partially funded this research.

#### REFERENCES

- [1] H. Dalir, F. Koyama, Applied Physics Letters 103, 091101 (2013).
- [2] M. Lindemann, et al., Journal of Applied Physics (submitted) (2025).
- [3] Y. Hu, et al., Electronics Letters 61 (2025).
- [4] M. D'Alessandro, et al., IEEE Photonics Journal 16, 1 (2024).
- [5] M. Lindemann, et al., 2023 23rd International Conference on Transparent Optical Networks (ICTON) (2023).
- [6] W. North, et al., Journal of Lightwave Technology 42, 236 (2024).
- [7] S. T. M. Fryslie, et al., IEEE Journal of Selected Topics in Quantum Electronics 23, 1 (2017).