1D vectorial simulations of anisotropic VCSELs

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***Abstract*—Recent work has shown that elliptically and circularly polarized vertical-cavity surface-emitting lasers (VCSELs) can be realized by leveraging the interaction between tilted anisotropic layers. To harness this effect for the design of custom-polarized emitters, we present a mathematical framework that enables fast and efficient one-dimensional vectorial simulations of anisotropic VCSELs.**

***Index Terms*—VCSELs, polarization, optical anisotropies**

I. Introduction

Optical anisotropies play a key role in determining the polarization of vertical-cavity surface-emitting lasers (VCSELs), whether they arise from electro- and elasto-optic effects [1], [2] or subwavelength gratings (SWG) [3]. Anisotropies cause dichroism and birefringence, and, when tilted to each other, the supported VCSEL polarizations become elliptical due to a resonator chirality [3], [4]. In this work, we develop a mathematical framework to analyze the polarization of single-mode anisotropic VCSELs, enabling the emission of any custom polarization state.

II. Theory and Results

Consider a single-mode VCSEL composed of a stack of anisotropic layers as depicted in Fig. 1. Focusing on a one-dimensional analysis, transverse variations along the (x, y) plane can be neglected, allowing us to focus solely on the longitudinal stack along z. This corresponds to an electric field phasor $\mathbf{E}$ without a z-component. According to [5], $\mathbf{E}$ can be expanded in terms of the modes $\{\mathbf{e}_{\boldsymbol{\mu}}\}$ supported by a uniform medium with a real scalar dielectric constant ϵ_{ref} (and refractive index r), where $\boldsymbol{\mu}$ represents a multi-dimensional label. In this case, $\boldsymbol{\mu} = [i, \alpha]^{\mathrm{T}}$, where $i \in \{x, y\}$ represents the linear polarization either along x or y, while $\alpha \in \{\text{forward (f), backward (b)}\}$ indicates the propagation direction. We define $\mathbf{e}_{\boldsymbol{\mu}} = e_0 \left(\hat{x}\delta_{ix} + \hat{y}\delta_{iy}\right)$, where δ_{ij} is the Kröneker delta and e_0 is arbitrary. Two polarization modes are labeled by $\boldsymbol{\mu} = [i, \alpha]$ and $\boldsymbol{\nu} = [i', \alpha']$, and must be normalized so that:

$$\int_{\mathbb{R}^2} \mathbf{e}_{\boldsymbol{\mu}} \cdot \mathbf{e}_{\boldsymbol{\nu}} \, \mathrm{d}x\mathrm{d}y = \frac{Z_\perp C_{\boldsymbol{\mu}}}{2 s_{\boldsymbol{\mu}}} \delta_{ii'}, \tag{1}$$

where $C_{\boldsymbol{\mu}}$ is the modal power normalization constant, $s_{\boldsymbol{\mu}} = \pm 1$ according to α and $Z_\perp = Z_0 / r$, Z_0 being the vacuum impedance. The field phasor within the VCSEL can be written as $\mathbf{E}(z) = \sum_{\boldsymbol{\mu}} a_{\boldsymbol{\mu}}(z)\mathbf{e}_{\boldsymbol{\mu}}$, shifting the unknown to the 4 z-dependent expansion coefficients $\{a_{\boldsymbol{\mu}}\}$.

Each VCSEL layer is described by its thickness t and by its anisotropic dielectric constants ϵ_{XX} and ϵ_{YY} along their principal axes (X, Y), which can be tilted to our reference system (x, y) by an angle ϕ (top inset of Fig. 1). It is convenient to define the isotropic and anisotropic dielectric constants ϵ_{iso} and ϵ_{ani} as $(\epsilon_{XX} \pm \epsilon_{YY})/2$, respectively. An isotropic refractive index can be associated to the anisotropic layer as $n_{\text{iso}} = \sqrt{\epsilon_{\text{iso}}/\epsilon_0}$, ϵ_0 being the vacuum dielectric constant.

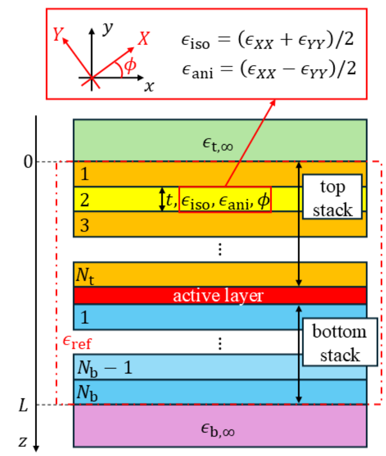

Fig. 1. VCSEL schematic and needed parameters for each layer.

Expressing $\{a_{\boldsymbol{\mu}}\}$ as the vector $\mathbf{a} = [a_{x\text{f}}, a_{x\text{b}}, a_{y\text{f}}, a_{y\text{b}}]^{\mathrm{T}}$, the transmission matrix of one of the VCSEL layers embedded in the reference medium can be obtained solving the coupled mode equations as $\mathbf{T} = \exp\left[(\mathbf{B} + \mathbf{K})\, t\right]$ [5], $\mathbf{B}$ and $\mathbf{K}$ representing the propagation and coupling matrices. $\mathbf{B}$ is diagonal with $B_{11} = B_{13} = -j\beta_\perp$, $B_{22} = B_{44} = j\beta_\perp$, where j is the imaginary unit, $\beta_\perp = 2\pi r/\lambda$ and λ is the optical wavelength. On the other hand,

$$\mathbf{K} = \frac{j\omega(\epsilon_{\text{iso}} - \epsilon_{\text{ref}})Z_\perp}{2}\begin{bmatrix} -1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix} + \tag{2}$$

$$+ \frac{j\omega\epsilon_{\text{ani}} Z_\perp}{2}\begin{bmatrix} -\cos(2\phi) & -\cos(2\phi) & -\sin(2\phi) & -\sin(2\phi) \\ \cos(2\phi) & \cos(2\phi) & \sin(2\phi) & \sin(2\phi) \\ -\sin(2\phi) & -\sin(2\phi) & \cos(2\phi) & \cos(2\phi) \\ \sin(2\phi) & \sin(2\phi) & -\cos(2\phi) & -\cos(2\phi) \end{bmatrix},$$

where ω is the optical pulsation. It is useful to define the matrix $\mathbf{A}$ as the first matrix that appears in (2).

According to the sketch in Fig. 1, defining $\mathbf{T}_{i,\text{t}}$ and the $\mathbf{T}_{k,\text{b}}$ as the transmission matrices of the i-th and k-th layers of the top and bottom stacks, respectively, the transmission matrix of the whole stacks can be obtained as $\mathbf{T}_{\text{t}} = \mathbf{T}_{N_{\text{t}}}\mathbf{T}_{N_{\text{t}}-1} \cdot \ldots \cdot \mathbf{T}_1 = \prod_{i=N_{\text{t}}}^{1} \mathbf{T}_{i,\text{t}}$ and $\mathbf{T}_{\text{b}} = \prod_{k=N_{\text{b}}}^{1} \mathbf{T}_{k,\text{b}}$, where N_{t} and N_{b} are the number of layers of the top and bottom stacks.

Let us now treat the active layer, considered isotropic, with thickness t_{a} and a nominal dielectric constant ϵ_{a}. To support

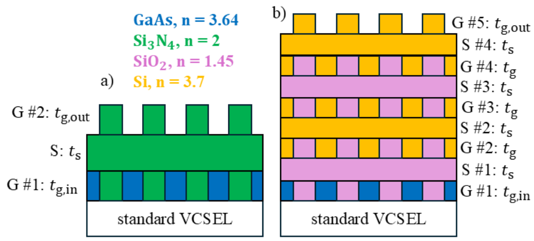

Fig. 2. Structures under investigation. On top of a standard VCSEL epistructure, we consider 2 (a) or 5 (b) additional SWGs, separated by spacers.

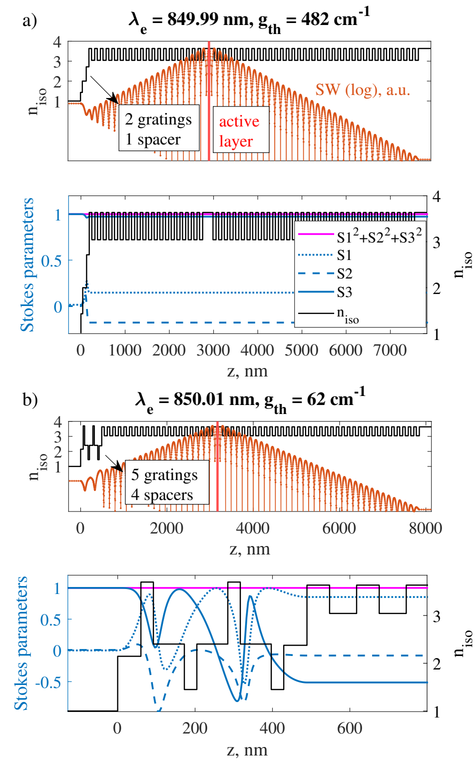

Fig. 3. Refractive index (n_{iso}), emission wavelength, threshold gain, SW and Stokes parameters of the lasing mode supported by structures (a) and (b).

optical modes, its dielectric constant must be modified by a quantity $\Delta\epsilon_{\text{a}}$, whose real part must be zero for cold cavity modes and whose imaginary part is linked to the modal threshold gain and represents an unknown of our problem. The coupling matrix of the active layer can be written as $\mathbf{K}_{\text{a}} = \mathbf{K}_{\text{a},0} + (\Delta\epsilon_{\text{a}}/\epsilon_{\text{ref}})\,\Delta\mathbf{K}_{\text{a}}$, where $\mathbf{K}_{\text{a},0} = j\omega\,(\epsilon_{\text{a}} - \epsilon_{\text{ref}})\,Z_\perp\mathbf{A}/2$ and $\Delta\mathbf{K}_{\text{a}} = j\omega\epsilon_{\text{ref}}Z_\perp\mathbf{A}/2$. Since $|\Delta\epsilon_{\text{a}}| << \epsilon_{\text{a}}$, for thin active layers it is possible to linearize the active transmission matrix as $\mathbf{T}_{\text{a}} = \mathbf{T}_{\text{a},0} + (\Delta\epsilon_{\text{a}}/\epsilon_{\text{ref}})\mathbf{T}_{\text{a},0}\Delta\mathbf{K}_{\text{a}}t_{\text{a}}$, where $\mathbf{T}_{\text{a},0}$ is the transmission matrix of the active layer without any index modification. Using this expression of $\mathbf{T}_{\text{a}}$, the transmission matrix for the whole VCSEL can be written as:

$$\mathbf{T} = \mathbf{T}_{\text{b}}\mathbf{T}_{\text{a}}\mathbf{T}_{\text{t}} = \mathbf{T}^{(1)} + (\Delta\epsilon_{\text{a}}/\epsilon_{\text{ref}})\mathbf{T}^{(2)}. \tag{3}$$

Finally, splitting the vector $\mathbf{a} \in \mathbb{C}^4$ into its forward and backward components $\mathbf{a}_{\text{f}}$ and $\mathbf{a}_{\text{b}} \in \mathbb{C}^2$, accounting for the fact that within the interval $(0, L)$ all layers are embedded in the reference medium and evaluating the reflection coefficients from the reference medium to the boundary semi-infinite media at $z = 0$ (Γ_{top}) and $z = L$ (Γ_{bottom}), we end up with the following relationships: $\mathbf{a}_{\text{f}}(L^-) = \mathbf{T}_{\text{ff}}\mathbf{a}_{\text{f}}(0^+) + \mathbf{T}_{\text{fb}}\mathbf{a}_{\text{b}}(0^+)$, $\mathbf{a}_{\text{b}}(L^-) = \mathbf{T}_{\text{bf}}\mathbf{a}_{\text{f}}(0^+) + \mathbf{T}_{\text{bb}}\mathbf{a}_{\text{b}}(0^+)$, $\mathbf{a}_{\text{f}}(0^+) = \Gamma_{\text{top}}\mathbf{a}_{\text{b}}(0^+)$ and $\mathbf{a}_{\text{b}}(L^-) = \Gamma_{\text{bottom}}\mathbf{a}_{\text{f}}(L^-)$. Solving for $\mathbf{a}_{\text{b}}(0^+)$ and splitting all the components of the transmission matrices according to (3), we end up with the following 2×2 generalized eigenvalue problem:

$$\gamma\mathbf{N}^{(1)}\mathbf{a}_{\text{b}}(0+) = \mathbf{N}^{(2)}\mathbf{a}_{\text{b}}(0^+), \tag{4}$$

where $\gamma = -\epsilon_{\text{ref}}/\Delta\epsilon_{\text{a}}$ and $\mathbf{N}^{(1,2)} = \mathbf{T}_{\text{bf}}^{(1,2)}\Gamma_{\text{top}} + \mathbf{T}_{\text{bb}}^{(1,2)} - \Gamma_{\text{bottom}}\mathbf{T}_{\text{ff}}^{(1,2)}\Gamma_{\text{top}} - \Gamma_{\text{bottom}}\mathbf{T}_{\text{fb}}^{(1,2)}$. By determining the wavelengths for which $\Re\{\gamma\} = 0$, one finds the emission wavelength λ_{e} of the two supported modes. At λ_{e}, $\Im\{\gamma\}$ can be used to evaluate the corresponding modal threshold gain g_{th}. The corresponding two eigenvectors $\mathbf{a}_{\text{b}}(0^+)$, defined up to a multiplicative constant, represent the near-field (NF) exiting the device since $\mathbf{a}_{\text{b}}(0^-) \propto \mathbf{a}_{\text{b}}(0^+)$, *i.e.*, $\mathbf{a}_{\text{NF}} = [0, a_{x\text{b}}(0^+), 0, a_{y\text{b}}(0^+)]^{\text{T}} \in \mathbb{C}^4$. The latter can be transmitted from 0^- to 0^+ using the transmission coefficient from the top semi-infinite medium to the reference medium, then it can be propagated using the transmission matrices up to any section z of the VCSEL. This allows the evaluation of the standing wave as $\text{SW}(z) = |\mathcal{E}_x(z)|^2 + |\mathcal{E}_y(z)|^2$, where $\mathcal{E}_x(z) = a_{x\text{f}}(z) + a_{x\text{b}}(z)$ (similarly for $\mathcal{E}_y$), and the Stokes parameters according to [6].

This model is applied to the structures in Fig. 2, optimized for the emission of circularly polarized light. In structure (a), on top of an 850 nm VCSEL structure, we consider two SWGs of thicknesses $t_{\text{g,in}} = 60$ nm and $t_{\text{g,out}} = 41$ nm, tilted by $45°$ and separated by a spacer of thickness $t_{\text{s}} = 87$ nm. In structure (b), 5 gratings and 4 spacers are considered with parameters $t_{\text{s}} = 32$ nm, $t_{\text{g,in}} = t_{\text{g,out}} = 80$ nm, $t_{\text{g}} = 80$ nm. All gratings from the second on are tilted to the first one with an angle increasing in steps of $\Delta\phi = 3.8°$. All gratings are treated as homogeneous anisotropic media using the Born-Wolf formulae with a 50% filling factor [6]. Fig. 3 reports λ_{e}, g_{th}, the SW and the Stokes parameters of the lasing mode of both structures, showcasing how this model can be used for polarization engineering in any section of the VCSEL.

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