Trying to understand semiconductor lasers for 40 years

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***Abstract*—Selected aspects of the mathematical modelling and numerical simulation of semiconductor lasers will be presented.**

***Index Terms*—semiconductor laser, mathematical modeling, numerical simulation**

I. Introduction

The semiconductor laser is a fascinating object for a theoretical physicist. Mathematical modeling of such a device must incorporate several physical disciplines, including the transport of charged carriers and heat, as well as the generation, guidance and propagation of photons. The physics involved extends to both the macroscopic and microscopic levels. Examples include the classical drift-diffusion and wave equations, and the quantum-theoretical based calculation of mobilities and the dielectric susceptibility. Quantum laser theory is required to calculate noise spectra. Another aspect to consider is the range of spatial and temporal scales involved, which vary from nanometers to millimeters and picoseconds to continuous wave. It is nearly impossible to include all disciplines, levels, and scales in a single simulation program. Instead, the entire problem must be broken down into smaller parts. The results can be transferred from one part to another using determined parameters, analytical functions, or look-up tables. Further simplifications are often necessary because numerical implementations must be compatible with the computer resources available when writing the code, such as memory and processing. However, when making such simplifications, it is important to start from first principles and observe basic physical principles.

The three modeling issues I will discuss in my talk are as follows: (i) local charge neutrality; (ii) filamentation; and (iii) orthogonality of cavity modes.

(i) The van Roosbroeck system describes the semi-classical transport of free electrons and holes in a quasi-static electric field using a drift-diffusion approximation [1]. It consists of the Poisson equation for the electric potential and continuity equations for the electron and hole current densities which are driven by the gradients of the quasi-Fermi potentials. The electron and hole densities are related to the electric potential as well as the quasi-Fermi potentials of electrons and holes via so-called state equations [2]. The numerical solution of this highly non-linear system of differential equations is non-trivial [3] and different numerical schemes have been proposed [4]. However, since the Debye length $\lambda_{\mathrm{D}} = \sqrt{\varepsilon_0 \varepsilon_s k_{\mathrm{B}} T / q^2 n}$ is much smaller than the thicknesses of the bulk layers of diode lasers driven above threshold, the solution of the Poisson equation can be avoided by applying the zero-space charge approximation and setting the right-hand side of the Poisson equation to zero [5],

$$n(\xi) - p(\xi) - C = 0 \tag{1}$$

with

$$n = N_{\mathrm{c}} F_{1/2}(\xi) \quad \text{and} \quad p = N_{\mathrm{v}} F_{1/2}\left(\frac{\varphi_{\mathrm{F}} - E_{\mathrm{g}}}{k_{\mathrm{B}} T} - \xi\right) \tag{2}$$

where $F_{1/2}(\cdot)$ is an Fermi integral and $C = N_{\mathrm{D}}^+ - N_{\mathrm{A}}^-$ is the ionized net doping. The solution of the neutrality condition (1) yields the relation between the electron and hole densities n and p, respectively, and the Fermi voltage φ_{F} which depends on the energy gap E_{g}, but not on the conduction and valence band edges separately. In my talk I will compare the results of a full numerical solution of the drift-diffusion system and with a solution based on (1) [6].

(ii) For many years, the multi-peaked lateral field profiles of BA lasers have been often interpreted in terms of filamentation. As first demonstrated by Bespalov and Talanov [7], plane waves propagating in a uniform medium with a focusing Kerr nonlinearity spontaneously break up into small filaments. In diode lasers an indirect Kerr-type nonlinearity can be induced by the dependence of the real part of the susceptibility on the carrier density (often described by Henry's $\alpha-$factor) which in turn depends on the field intensity via the rate of stimulated recombination. There are several reasons why this indirect Kerr-type nonlinearity does not result in the development of filaments. First, the susceptibility in the active region of a laser is complex-valued. In regions with a high intensity the real part of the susceptibility (refractive index) is increased, but the imaginary part (gain) is decreased. This results in both focusing and defocusing phenomena at the same time. Secondly, the medium of an injection laser is always nonuniform because of the formation of a lateral waveguide due to the dependence of the susceptibility on carrier density, local temperature, or on external factors such as etched index-guiding trenches or implanted regions. Third, the excitation of several lateral waveguide modes with different wavelengths leads to mode beating that drives oscillations of the carrier density via the stimulated recombination. Thus, BA lasers exhibit an inherently non-stationary behavior, see Fig. 1. I will show in my talk that the lateral field profile can be surprisingly well understood as the result of the competition and superposition of stationary lateral waveguide modes [8], [9], [10], [11].

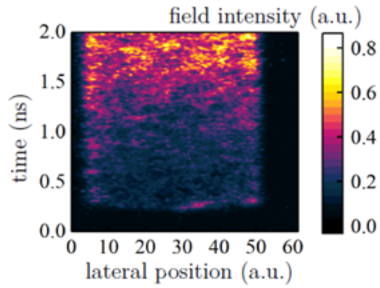

Fig. 1. Near field intensity of a BA laser measured with a streak camera (courtesy of H. Christopher).

(iii) The forward and backward propagating longitudinal modes $\Phi_m^{\pm}$ of a laser cavity fulfill a special orthogonality relationship due to the openness of the cavity and the presence of gain,

$$(\Phi_\mu, \Phi_\nu) := \int_0^L \left[\Phi_\mu^+ \Phi_\nu^- + \Phi_\mu^- \Phi_\nu^+\right] dz = 0 \quad \text{for} \quad \mu \neq \nu, \tag{3}$$

which can be also considered as a consequence of time-reversal symmetry. A similar relation holds also for the transverse modes. The relation (3) has two consequences. Firstly, it is evident that the integral does not define a scalar product, as the Φs are complex-valued. Indeed, it has been demonstrated that $(\Phi_\mu, \Phi_\nu) = 0$ can occur even for $\mu = \nu$ at specific parameter configurations, which is the consequence of a mode degeneracy [12], [13]. The occurrence of such exceptional points is not restricted to lasers but is inherent to non-Hermitian systems [14]. Secondly, the relation (3) results in an enhancement of the intrinsic spectral linewidth compared to the case of power-orthogonal modes. This enhancement is often described in terms of the Petermann factor. In my talk I will demonstrate the occurrence of exceptional points in two-section DFB lasers and I will address the origin of the Petermann factor in the context of calculating the intrinsic spectral linewidth in terms of the cavity mode [15].

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