# Numerical study of Self-Detection Near Field Optical Microscopy in the Terahertz range


Carlo Silvestri*, Lorenzo Luigi Columbo†, Massimo Brambilla ‡

*School of Information Technology and Electrical Engineering, The University of Queensland, Brisbane, Australia

†Dipartimento di Elettronica e Telecomunicazioni, Politecnico di Torino, Torino, Italy

‡Dipartimento Interateneo di Fisica, Politecnico e Universita' degli Studi di Bari, CNR-IFN, Bari, Italy

Email: c.silvestri@uq.edu.au



***Abstract*—We numerically study the self-detection scattering type near field optical microscopy (SD s-SNOM), a detectorless and high resolution technique exploited for the retrieval of the dielectric properties of a resonant material sample in the terahertz (THz) range. We derive an approximated method for the reconstruction of the dielectric function in the weak feedback regime, where the signal to noise ratio is expected to be higher than in the commonly used very weak feedback regime, reporting reasonable accuracy in the estimation of the phonon resonances for a supposed Cesium Bromide (CsBr) sample.**

***Index Terms*—SNOM, Terahertz, Quantum Cascade Laser**


## I. Introduction

The self-detection scattering-type near field optical microscopy (SD s-SNOM) is a technique that allows to retrieve the dielectric properties of a material sample in the terahertz (THz) region of the electromagnetic spectrum [1]. This configuration provides sub-wavelength resolution, circumventing the diffraction limit [2], and it exploits the light source as a detector, overcoming the lack of efficient detectors affecting the THz range [3]. In this work we numerically study a typical SD s-SNOM experimental layout [1], which is shown in Fig. 1. The beam emitted by a THz quantum cascade laser (QCL), is backscattered by the nanometric tip of an atomic force microscope (AFM) operating in tapping mode, positioned in proximity of the analyzed sample (in our case CsBr), and partially re-injected into the laser cavity. This determines a change in the voltage signal across the terminals of the QCL $\Delta V$, whose demodulation through a lock-in amplifier allows to retrieve the information about the dielectric function of the material sample $\varepsilon_{\rm s}$. A displacement of the piezolectric mirror PZM induces a variation $\Delta L$ of the laser-tip distance $L$. We model the dynamics of the single mode THz QCL in the SD s-SNOM scheme by using the Lang-Kobayashi (LK) equations [4], modified by introducing a complex scattering coefficient $\sigma = Se^{i\phi}$, depending on $\varepsilon_{\rm s}$ [1]. We retrieve approximated reconstruction formulas for $\sigma$ valid in the weak feedback regime (Acket parameter $C < 1$) [5], which in turn allow to determine $\varepsilon_{\rm s}$. The proposed method, applied in this work to numerically simulated self-mixing signals, can be applied to experimental traces.

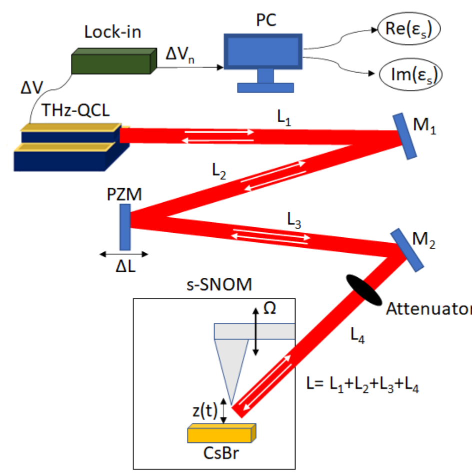


Fig. 1. Experimental layout for the SD s-SNOM.

## II. Theoretical model

Since the AFM tip oscillation frequency is around of 10-100 KHz, while typical time rates for a QCL are in the 10-100 GHz range, we analyze our model using the steady state solutions of the LK equations:

$$\Delta V = VS\cos(\omega_{\rm F}\tau - \phi) \quad (1)$$

$$\omega_{\rm F}\tau = \omega_0\tau - \varepsilon S\sqrt{1+\alpha^2}\sin(\omega_{\rm F}\tau - \phi + \arctan(\alpha)) \quad (2)$$

where $\omega_{\rm F}$ and $\omega_0$ are respectively the angular frequency of the laser in presence of feedback and without feedback, $V = 2\frac{1-R}{\sqrt{R}}a\frac{\tau_p}{\tau_c}$ and $\varepsilon = \frac{\tau}{\tau_c}\left(\frac{1-R}{\sqrt{R}}\right)a$, $\tau_{\rm e}$ and $\tau_{\rm p}$ are respectively the carrier and photon lifetime, $\alpha$ is the linewidth enhancement factor, $\tau = \frac{2L}{c}$ is the roundtrip time in the external cavity $L$, $R$ is the reflectivity of both the QCL facets, $\tau_{\rm c}$ is the roundtrip time in the QCL cavity, $a$ is the attenuation factor. In agreement with the experiments, where the signal is demodulated by a lock-in amplifier and the harmonics are extracted to retrieve the sample properties, we consider $\sigma_{\rm n} = s_{\rm n}e^{i\phi_{\rm n}}$ and $\Delta V_{\rm n}$, respectively the harmonics of $\sigma$ and $\Delta V$, and we derive approximated reconstruction formulas for $s_{\rm n}$ and $\phi_{\rm n}$ based on a first order truncated Taylor expansion of the voltage signal [5]:

$$s_n = \frac{1}{2V}\sqrt{((\Delta V_{\frac{3\pi}{2}})_n - (\Delta V_{\frac{\pi}{2}})_n)^2 + (\Delta V_n - (\Delta V_\pi)_n)^2} \quad (3)$$

$$\phi_n = \omega_0\tau - \arctan\left[\frac{(\Delta V_{\frac{3\pi}{2}})_n - (\Delta V_{\frac{\pi}{2}})_n}{\Delta V_n - (\Delta V_\pi)_n}\right] \quad (4)$$

where $\Delta V_{\frac{\pi}{2}}$, $\Delta V_\pi$, $\Delta V_{\frac{3\pi}{2}}$ are the harmonics of the signals obtained for displacements of the laser-tip distance of $\lambda_0/8$, $\lambda_0/4$, and $3\lambda_0/8$. Once $s_\mathrm{n}$ and $\phi_\mathrm{n}$ are retrieved through Eqs. (3)-(4), they can be exploited to determine the dielectric function of the sample $\varepsilon_\mathrm{s}$ [6].

## III. Numerical Results

We simulate the SD s-SNOM setup in the weak feedback regime, where we expect higher signal to noise ratio with respect to the commonly adopted very weak feedback regime ($C < 0.1$), by numerically solving Eqs. 1-2 adopting the algorithm proposed in [7]. We consider $C = 0.13$. The emission wavenumber of the QCL is varied between $50\ \mathrm{cm}^{-1}$ and $140\ \mathrm{cm}^{-1}$, the other simulation parameters are $L = 0.6$ m, $\tau_\mathrm{p} = 32.4$ ps, $\tau_\mathrm{c} = 37.4$ ps and $\alpha = 1.5$. Then, we apply the first order formula Eqs. (3)–(4) to the numerically reproduced self-mixing signals, in order to retrieve the harmonics of $\sigma$. In Figs. 2a-2b we show $s_3$ and $\phi_3$, comparing the first order reconstruction (red curves) with the zero order (black), presented in [1] and rigorously valid in the very weak feedback regime, and with the same quantities calculated by using the finite-dipole (FD) model [6] (blue). The calculated curves are used as reference to quantify the degree of accuracy of the reconstructions. We report a good agreement between the reconstructed and reference lines, noticing however that the first order better approximates the FD curves (see the insets in Figs. 2a-2b). Then, we use the harmonics $\sigma_\mathrm{n}$, with $n = 0, .., 7$, determined with the first order method, for the retrieval of the dielectric function of the CsBr sample. The dependence of $\sigma$ from $\varepsilon_\mathrm{s}$ is modeled with the FD model as in [1]. The reconstructed real part of $\varepsilon_\mathrm{s}$ is compared with fitted experimental data [8] in Fig. 3a, with good agreement reported in most of the considered wavenumber interval. We estimate the transverse optical (TO) and longitudinal optical (LO) phonon resonances, obtained through the condition $\mathrm{Re}(\varepsilon_\mathrm{s}) = 0$, reporting an error of 0.1% for the TO (Fig. 3b) and 0.07% for the LO (Fig. 3c).

## IV. Conclusion

We derived an approximated method for the retrieval of the dielectric function of a resonant material in the THz range, by modeling a typical SD s-SNOM configuration with a modified version of the LK equations. We tested this method on simulated self-mixing signals, finding good accuracy in the retrieval of the phonon resonances of a supposed CsBr sample. The proposed procedure can be applied to any experimental signal.

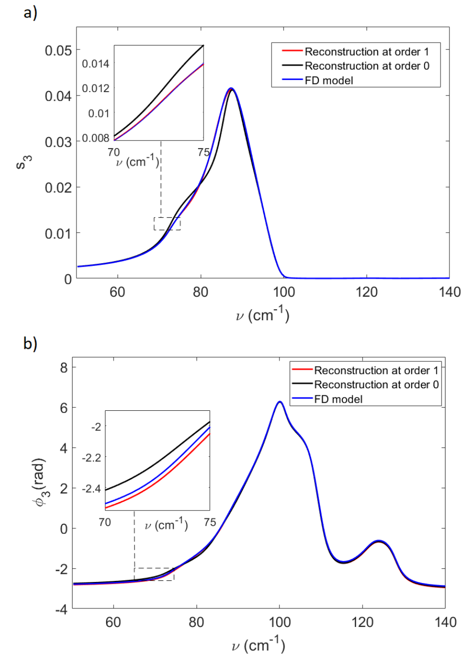


Fig. 2. First order (red) and zero-order (black) reconstructions, and calculated curves by using the FD model (blue) for $s_3$ (a) and $\phi_3$ (b). A zoom in the frequency interval between $70\ \mathrm{cm}^{-1}$ and $75\ \mathrm{cm}^{-1}$ is shown in two insets.

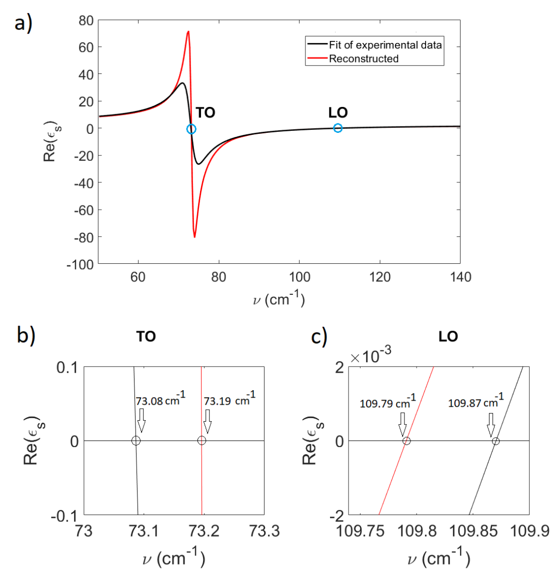


Fig. 3. a) Reconstructed (red) and fitted from experimental data [8] (black) real part of $\varepsilon_\mathrm{s}$ of CsBr; zoom around the TO (b) and LO (c) phonon resonances.

## References

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