Numerical modeling of photorefractive crystals for self-adapting external cavity laser mirrors

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***Abstract*— The process of refractive index grating formation in a photorefractive crystal, used as the self-adapting spectral filter in an external cavity laser, is studied with a self-consistent model. The model is based on the Finite Difference solution of the Kukhtarev equations. The results obtained are compared with approximate analytical models.**

I. Introduction

Photorefractive crystals have many applications in optics and optoelectronics. In particular, they are used in holography [1] and in self-adapting external cavity lasers [2]. In the holographic applications, the grating is typically written by obliquely propagating monochromatic waves. However, when a photorefractive crystal is used in external cavity lasers, the grating is written along the direction of the wave propagation by the counter-propagating waves having different frequencies [2]. The former problem has received considerable attention in the literature [1, 3, 4, 5]. In this contribution the authors consider the latter problem, i.e. the one encountered when analyzing the spectral response of the self-adapting external cavity laser mirror [2].

In a self-adapting external cavity laser [6], the counter-propagating waves produce a standing wave pattern in the photorefractive crystal. This standing wave pattern writes a refractive index grating in the crystal via the photorefractive effect. The process of grating formation in the photorefractive crystal is described by the Kukhtarev equations [3]:

$$\frac{\partial\left(N_D^+ - N\right)}{\partial t} = -\frac{1}{e}\nabla\cdot J \tag{1}$$

$$\frac{\partial N_D^+}{\partial t} = \left(N_D - N_D^+\right)\left(sI+\beta\right) - \gamma_R\cdot N_D^+ N \tag{2}$$

$$J = eD_S\nabla N + e\mu N E_{SC} \tag{3}$$

$$\nabla\cdot\varepsilon_S E_{SC} = e\left(N_D^+ - N_A - N\right) \tag{4}$$

where N is the free-electron density; N_D^+ is the ionized donor concentration; N_D is the total donor concentration; J is the current; E_{SC} is the electrostatic space-charge field; I is the intensity distribution; e is the electronic charge; s is the ionization cross section; β is the thermal excitation rate (proportional to the dark current); γ_R is the recombination rate; ε_S is the dielectric constant tensor, $\varepsilon_s = \frac{\mathrm{K}\cdot\varepsilon\,\mathrm{K}}{\mathrm{K}^2}$; μ is the effective static mobility, $\mu = \frac{\mathrm{K}\cdot\mu\,\mathrm{K}}{\mathrm{K}^2}$; D_S is the diffusion constant, $D_S = \frac{k_B\cdot T\cdot\mu}{e}$; N_A is the acceptor concentration and K is the grating vector.

In order to solve (1) – (4) subject to a given illumination pattern, we use the Finite Difference method. For this purpose we first combine the Kukhtarev equations into one nonlinear differential equation for the 1D case:

$$o_1\frac{\partial^2 N(z)}{\partial z^2} + o_2\left(\frac{\partial N(z)}{\partial z}\right)^2 + o_3 N^2(z) + o_4 N^3(z) + o_5 N^4(z) = 0 \tag{5}$$

$$o_1 = \left[D_S + qd_1\left(\mu - \gamma_R\frac{\varepsilon}{e}\right)\right]\cdot N^2(z) - \left(sI+\beta\right)\frac{\varepsilon}{e}qd_1\cdot N(z) \tag{6a}$$

$$o_2 = \gamma_R q d_1\frac{\varepsilon}{e}\cdot N(z) - \left(sI+\beta\right)\frac{\varepsilon}{e}qd_1 \tag{6b}$$

$$o_3 = \left(sI+\beta\right)\left(N_D - N_A\right) \tag{6c}$$

$$o_4 = -\left(sI+\beta+\gamma_R N_A\right) \tag{6d}$$

$$o_5 = -\gamma_R \tag{6e}$$

where $d_1 = \frac{D_S\gamma_{eff}n^3}{2\mu}$. For both the first and second derivatives, we use standard central difference approximations and obtain:

$$o_5 N_i^4 + o_4 N_i^3 + o_3 N_i^2 - \frac{2o_1}{\left(\Delta z\right)^2}N_i + \xi = 0 \tag{7}$$

where ξ is given by the expression:

$$\xi = \frac{o_1\left(N_{i+1}+N_{i-1}\right)}{\left(\Delta z\right)^2} + \frac{o_2}{4\left(\Delta z\right)^2}\left(N_{i+1}-N_{i-1}\right)^2 \tag{8}$$

If we arrange Eq.(7) as:

$$aN_i^2 + bN_i + c = 0 \tag{9}$$

where a=o$_3$, $b = -\frac{2o_1}{\left(\Delta z\right)^2}$ and $c = o_5 N_i^4 + o_4 N_i^3 + \xi$ we can formally calculate the root:

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$$N_i = \frac{-b-\sqrt{b^2-4ac}}{2a} \qquad (10)$$

Now we can form fixed point iteration by calculating the electron density from (10) subject to known constants a, b and c calculated using an initial guess of the electron density distribution. The newly calculated values of N are then used to update the coefficients and (10) is solved again to obtain the updated electron density distribution. This process continues until convergence is reached. We note that the other root of (9) does not correspond to a physically relevant solution.

II. MODELLING PARAMETERS

As an example, we consider the formation of a refractive index grating in $BaTiO_3$. The calculation parameters are given in Tab.1. [3]

Tab.1 Parameters of $BaTiO_3$

Name	Value	Unit
r_{42}	1640×10^{-12}	m/V
r_{13}	8×10^{-12}	m/V
r_{33}	28×10^{-12}	m/V
n_o	2.32866	
n_e	2.29512	
λ	966.028	nm
γ_R	5×10^{-14}	m^3/s
μ	0.5×10^{-4}	m^2/V-s
N_A	3×10^{22}	m^{-3}
N_D	$200N_A$	m^{-3}
s	1×10^{-5}	m^2/J-s

III. RESULTS

Figure 1 shows the calculated dependence of the refractive index perturbation, which is results from the photorefractive effect, on the longitudinal position. The illumination pattern modulation depth is $m=\frac{2r}{1+r^2}$=0.9986, which is typical for standing wave patterns created by waves counter-propagating within an external laser cavity [6]. This figure shows that an analytical approach is not accurate even if 3 terms are used in the expansion series given in [4]. Figure 2 summarizes the comparison between numerical and analytical methods for various values of the modulation depth. The discrepancy between the numerical and analytical results was quantified using the following error norm:

$$e\_norm_{\Delta n} = \left(\sum_{i=0}^{M+1}\left(\Delta n_{i,numerical}-\Delta n_{i,analytical}\right)^2\right)^{1/2} \qquad (11)$$

As expected for low values of m, the difference between the analytical and numerical results is negligible but the discrepancy grows significantly for large modulation depths.

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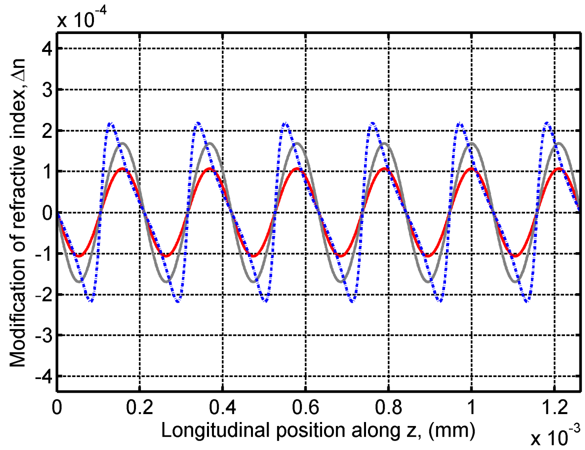

Fig.1 Dependence of the refractive index perturbation on the longitudinal position (m=0.9986)

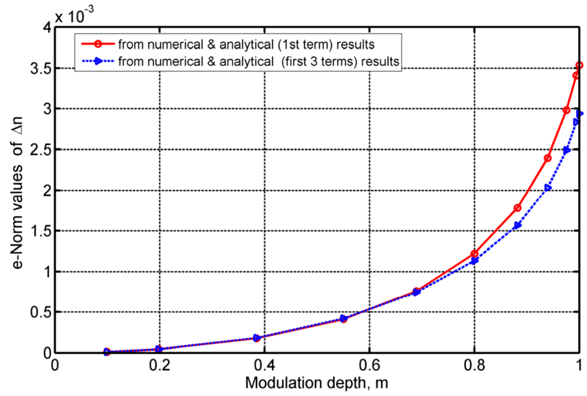

Fig.2 Dependence of the error norm on the modulation depth