A Modified Stretching Factor for Perfect Matched Layers in 2D FDTD simulations

Fahmida Ferdous*, Satoshi Ishii*, and Jian Wang

ECE Department, Purdue University, 465 Northwestern Avenue, West Lafayette, Indiana 47907, USA,

fferdous@purdue.edu, *Equal contributors

***Abstract*- A modified definition of the stretching factor for perfectly matched layer is introduced in 2D finite difference time domain method. This modified definition allows us to write equations for only one polarization to calculate both TM_z and TE_z polarized fields by taking the advantage of duality relations. The wave incident at Brewster angle and wave propagation through Fresnel zone plate are simulated by our developed method.**

I. INTRODUCTION

In numerical simulations, truncating the computational domains is very important. It is applicable not only to finite difference time domain (FDTD) method but also for other numerical simulation methods. The boundaries of the computational domains depend on the geometries such as perfect electric/magnetic conductors, scattering boundary conditions, absorbing boundary conditions (ABCs) and periodic boundary conditions (PBCs). ABCs have played a significant role in truncating the computational domains. Perfectly matched layer (PML) is one of the robust ABCs [1]. The main advantage of PML is that it is independent of incident angles, frequencies and polarizations. On the other hand, PBC is an efficient way to simulate infinite structure. Especially, by Bloch theorem, wave propagating in a periodic structure can be simulated in a form of Bloch wave. Therefore, Bloch PBC offers an indispensable method to simulate wave propagation in periodic structures, such as photonic crystals and plasmonic devices.

Often, numerical electromagnetic simulations can be done in 2D. As far as simulating in 2D doesn't change the physical conditions of the problems, it is better to work in 2D to reduce the loads to the computers and to increase the resolution. In 2D, there are two kinds of polarizations when the wave vector is expressed in x-y plane: i) electric field is polarized to z-axis (E_z-polarization or TM_z mode) ii) magnetic field is polarized to z-axis (H_z-polarization or TE_z mode). TM_z waves and TE_z waves interact with the surrounding environment in different ways. Thus, it is important to simulate both TM_z waves and TE_z waves to understand the propagation of electromagnetic waves.

When coding is concerned, it should be concise, simple and easy to understand. To simulate both TM_z mode and TE_z mode for a geometry certainly one way is to write codes for TM_z mode and TE_z mode separately. However, it is possible to write one code and use it for both polarizations by taking the advantage of the duality relations. We present a modified stretching factor for PML to realize an easy and simple way of coding for both TM_z mode and TE_z mode in FDTD method.

II. METHOD

It is possible to express the equations for the solution domain and PML in a single set of equations [2]. Here only equations for z component are shown.

$$\epsilon \frac{\partial E_{sxz}}{\partial t} + \sigma_x E_{sxz} = \frac{\partial H_y}{\partial x} - \sigma_e E_{sxz} - J_z \quad (1)$$

$$\mu \frac{\partial H_{sxz}}{\partial t} + \frac{\mu}{\epsilon}\sigma_x H_{sxz} = -\frac{\partial E_y}{\partial x} - \sigma_m H_{sxz} - M_z \quad (2)$$

$$s_i = 1 - j\frac{\sigma_i}{\varpi\varepsilon} \quad (3)$$

Here s_i (*i=x, y, z*) are called stretching factors. Attenuation coefficient σ_i is set to zero outside the PML. Also σ_m, σ_e, $\vec{M}$ and $\vec{J}$ to be zero in PML regions. Taking the advantage of the duality relations, one could think of switching the equations from TM_z mode Eq. (1) to TE_z mode Eq. (2) or the other way around. However, the coefficients of the second terms on the left hand side in Eq. (1) and Eq. (2) are not symmetric. Hence, if one wants to switch the code for TM_z mode to TE_z mode, in addition to writing codes to define duality relation, asymmetric nature of the coefficients should be taken care of, which results in writing additional lines in coding.

Here, a modified definition of stretching factors denoted by s_i^* (*i=x, y, z*) is introduced.

$$s_i^* = 1 - j\frac{\sigma_i^*}{\varpi\varepsilon\mu} \quad (4)$$

Note that σ_i^* is set to zero outside PML in this case as well. By following the same steps, Eq. (1) and Eq. (2) can be written as Eq. (5) and Eq. (6) respectively.

978-1-4244-7017-4/10/$26.00 ©2010 IEEE

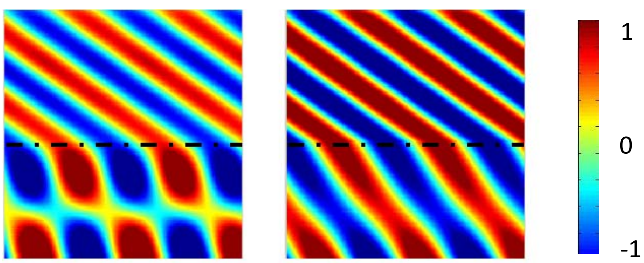
$$\epsilon \frac{\partial E_{sxz}}{\partial t} + \frac{1}{\mu}\sigma_x^* E_{sxz} = \frac{\partial H_y}{\partial x} - \sigma_e E_{sxz} - J_z \quad (5)$$

$$\mu \frac{\partial H_{sxz}}{\partial t} + \frac{1}{\epsilon}\sigma_x^* H_{sxz} = -\frac{\partial E_y}{\partial x} - \sigma_m H_{sxz} - M_z \quad (6)$$

Now, use of the duality relations immediately changes TM_z mode equations to TE_z mode equations and vice versa. When writing a code for TM_z mode and TE_z mode, the modified definitions only requires writing a set of equations for one polarization. The other polarization can be expressed using duality relations. Hence, there is no need to write codes for two polarizations separately.

III. RESULTS

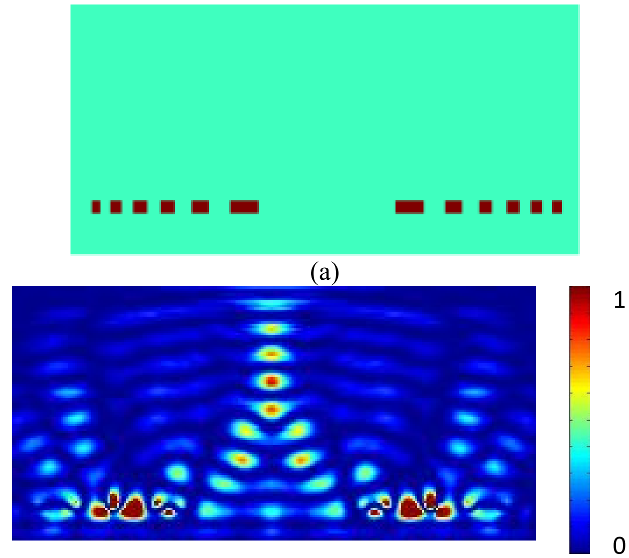
Commercial software, MATLAB (MathWorks), is used for programming. Concerning our code, only a set of equations for TM_z mode is written and the duality relation gives TE_z mode. For simulation we use input wavelength 700 nm. Matching layers are PML (for Brewster angle in horizontal direction, PBC is used) which are 1.5 times the wavelength. The functionality of the polarization switching can be easily understood by observing the wave incident at Brewster angle. By definition, no reflection occurs when TE_z polarized plane-wave is incident at Brewster angle fig 1(b), however, it is not the case for TM_z wave fig 1 (a). The simulated plots for TM_z mode and TE_z mode are shown in Fig. 1. Here, wave incidents from air to a media with 1.45 refractive index.



(a) (b)

Fig. 1. Wave incident at Brewster angle (a) E_z in the TM_z mode and (b) H_z in the TE_z mode. In the fig, dash dot line indicates the material interface- upper media with 1.45 refractive index and lower one with 1.

Next wave propagation through the Fresnel zone plate is shown [3]. Fresnel zone plate is designed to focus light based on diffractive optics. Fresnel zone plate consists of alternating rings which have alternating refractive indexes n_1 and n_2. Light hitting the zone plate will diffract around the ring. The rings can be spaced so that the diffracted light constructively interferes at the desired focus, creating an image there. Here n_1 is 1 and n_2 is 2.33. From fig 2(b) we can easily see that light is focused.



(b)

Fig. 2. (a) Schematic diagram of the Fresnel zone plate. Here boxes are the zone plate with refractive index 2.33 and surrounding media is air. (b) E_z^2 profile in the TM_z mode. Here we can see the focus point.

IV. CONCLUSION

A modified definition of stretching factors for 2D FDTD method with PML is presented. By following our definition and using the duality relations, FDTD codes for both TM_z mode and TE_z mode can be realized by only writing the codes for one polarization. Based on the proposed definition, we have shown simulations of the wave incident at Brewster angle and wave propagation through the Fresnel zone plate.

ACKNOWLEDGMENT

We thank Prof. Dan Jiao, ECE, Purdue University for the fruitful discussions.

REFERENCES

[1] J.P. Berenger, "A perfectly matched layer for the absorption of electromagnetic waves," *J. Comput. Phys*., vol. 114, pp. 185-200, October 1994.

[2] W. C. Chew and W. H.Weedon, "A 3D perfectly matched medium from modified Maxwell's equation with stretched coordinates," *Microwave Opt. Tech. Lett.*, vol. 7, pp. 599–604, 1994.

[3] K. Lee, et al., "Scalar FDTD Analysis of Fresnel Zone-Plate Lenses," *J. Opt. Comm.*, vol 27, pp 11-13, 2006.