Beam Propagation Analysis Using Higher-Order Full-Vectorial Finite-Difference Method

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***Abstract*—We develop a full-vectorial higher-order finite-difference formulation to model optical waveguides. Our proposed scheme yields improved convergence and field calculation results. Applications to beam propagation analysis of a photonic crystal coupler is demonstrated.**

I. Introduction

Accuracy and efficiency are major concerns in numerical optical simulation of photonic devices. Finite-difference method (FDM) has been very popular among various computer-aided design (CAD) tools due to its ease of implementation. Numerical error convergence of FDM can be improved by various method, where higher-order scheme is a straight-forward solution. Moreover, proper modeling of field behavior near structure interface requires treatment with full-vectorial feature for good accuracy, while higher-order FDM requires interface continuity conditions of field and its derivatives up to corresponding differential order. This technique has been implemented in mode analysis of optical waveguides [1], where report shows that 6-, 15-, and 28-point FDM yields second-, fourth-, and sixth-order error convergence. Application in photonic crystal fiber and pipe waveguide has also been demonstrated. In this paper, a dual-core photonic crystal fiber coupler is modeled as an further demonstration of improved convergence performance and capability for complex structure simulation. We also further apply the full-vectorial higher-order FDM to beam propagation method (BPM) and visualize the coupling phenomenon of coupler. Results show that superior accuracy can be achieved with higher-order scheme even when using coarser grids. Its application in BPM also shows potential usefulness in future photonic design.

II. Formulation and Numerical Results

For z-invariant waveguide mode eigenvalue problem based on electric field, we solve the vector Helmholtz equation

$$\nabla_{\perp}^2 \bar{\psi} + k_0^2 n^2 \bar{\psi} = \beta^2 \bar{\psi} \tag{1}$$

where $\bar{E} = \bar{\psi}\exp(-j\beta z)$ and β denotes the propagation constant of waveguide mode. Numerical calculation and assessment of this eigenvalue problem problem using full-vectorial higher-order finite-difference method has been demonstrated in [1]. To apply this technique to beam propagation method, we first specify a reference refractive index $\bar{n}$. By using slowly-varying wave packet approximation, $\bar{E} = \bar{\varphi}\exp(-jk_0\bar{n}z)$, we have

$$(\nabla_{\perp}^2 + k_0^2 n^2)\bar{\varphi} = -\frac{\partial^2 \bar{\varphi}}{\partial z^2} + 2jk_0\bar{n}\frac{\partial \bar{\varphi}}{\partial z} + k_0^2 \bar{n}^2 \bar{\varphi}. \tag{2}$$

We also use wide-angle BPM technique with order $(1,1)$ Padé approximant based on forward propagation operator $\Theta = (\nabla_{\perp}^2 - k_0^2\bar{n}^2)/k_0^2\bar{n}^2$ [2], which yields

$$(I + a_1^*\Theta)\varphi(z_0 + \Delta z) = (I + a_1\Theta)\varphi(z_0) \tag{3}$$

where I is identity matrix and complex conjugate pair a_1 and a_1^* are functions of $k_0\bar{n}$.

We simulate a dual-core air-hole photonic crystal fiber coupler as assessment. Wavelength λ_0 and hole pitch Λ are 1.55 and 1.8 μm, respectively. Ratio of air hole diameter d and hole pitch is $d/\Lambda = 0.52$. Perfectly-matched layers (PML) of 20 layers are used at computation boundary. Arrangement of air holes and core and mode power distribution of even and odd modes are shown in Fig. 1. The calculated effective indices and corresponding coupling lengths using different approximation scheme and grid size $\Delta x = \Delta y = \Delta$ are listed in Table II. Calculation based on 28-point scheme yields extremely high convergence performance with variation of $\Re[n_{\text{eff}}]$ less than 10^{-8}.

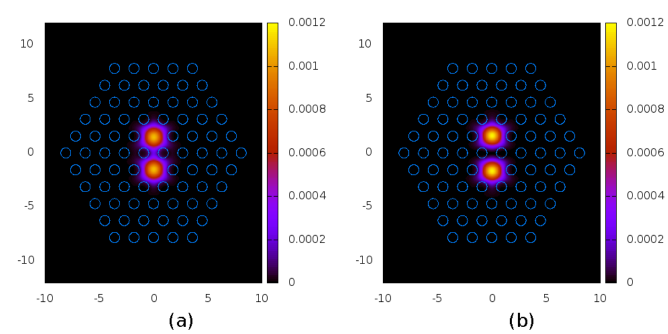

Fig. 1. Simulated E_x power distribution of (a) even and (b) odd modes.

We also apply the full-vectorial higher-order finite-difference scheme to BPM and model the same problem. An x-polarized Gaussian beam with waist of 1.0 μm is launched

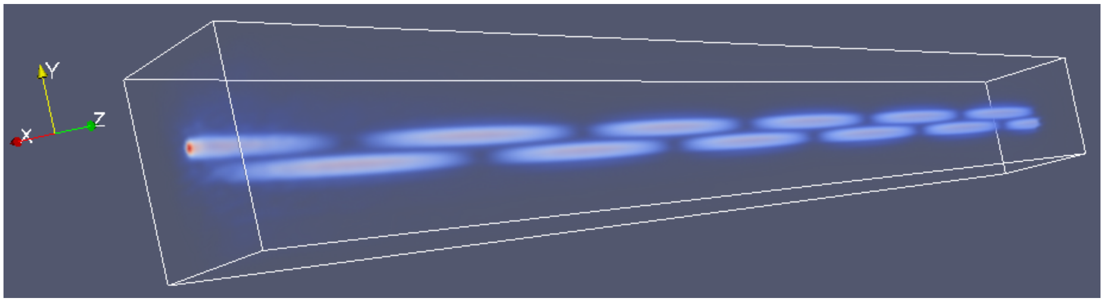

Fig. 2. 3D visualization of BPM simulation of photonic crystal fiber coupler. Computation domain is 20 μm $\times$24 μm $\times$2500 μm. Scaling along z-direction is 1 : 10.

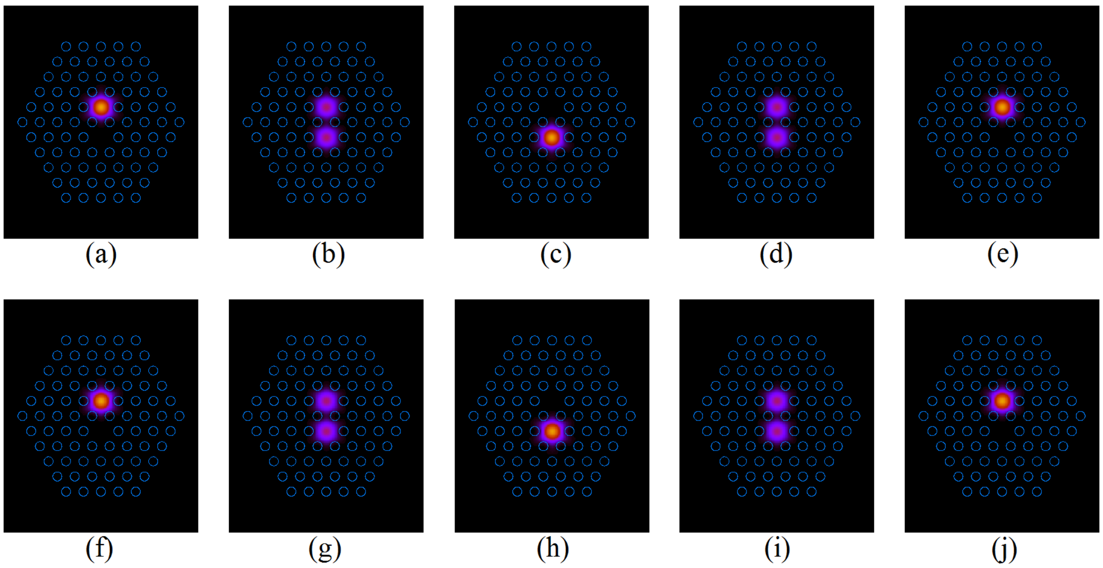

Fig. 3. E_x power distribution at $z = 457, 571, 685, 800$, and 914 μm using (a-e) 15-point scheme with $\Delta = 0.04$ μm and (f-j) 28-point scheme with $\Delta = 0.08$ μm.

TABLE I
CALCULATED $\Re[n_{\text{EFF}}]$ OF EVEN AND ODD MODES FOR SCHEMES OF DIFFERENT ORDERS

Δ (μm)	6-point, even	15-point, even	28-point, even
0.02	1.411776100	1.411768212	1.411768194
0.04	1.411800721	1.411768276	1.411768202
Δ (μm)	6-point, odd	15-point, odd	28-point, odd
0.02	1.408383421	1.408376332	1.408376316
0.04	1.408406993	1.408376384	1.408376322

at upper core and propagates along z-direction for 2500 μm. The propagation step size $\Delta z = 1.0$ μm. Reference refractive index $\bar{n}$ is set to 1.41. From the previous mode analysis results, we have coupling length $L = \pi/(\beta_{\text{even}} - \beta_{\text{odd}}) = 228.49$ μm. The 3D visualization of $|E_x|$ distribution is shown in Fig. 3, where field coupling effect is correctly modeled. To demonstrate the calculation capability, we use 15-point scheme with $\Delta x = \Delta y = 0.04$ μm and 28-point scheme with $\Delta x = \Delta y = 0.08$ μm and plot the power distribution. Calculation results are consistent with each other and agree with [3].

III. CONCLUSION

We have successfully implemented full-vectorial higher-order finite-difference method for optical waveguide analysis. Results show good convergence performance compared with conventional method. The formulation can be applied to beam propagation method for advanced design. We will present more detailed results in the conference including the numerical property of full-vectorial beam propagation analysis.

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