Self-pulsing and chaos in coupled ring resonators with non-instantaneous Kerr-nonlinear response

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***Abstract*—We present dynamical analysis of Kerr-nonlinear resonant structures consisting of two coupled ring resonators. We consider non-instantaneous Kerr response and the effect of loss. We demonstrate transitions from stable states to periodic (self-pulsing) and chaotic states. We identify parameters that can significantly affect onset of self-pulsing and chaos.**

***Keywords*—ring resonator; coupled cavities; self-pulsing; chaos; optical bistability; nonlinear optics**

I. Introduction

Bistability, self-pulsing (SP, i.e., generation of optical pulses from continuous wave input), and chaos are interesting effects that can occur in optical systems with nonlinear feedback. The behavior is related to the Ikeda instability that was observed in a single ring resonator filled with Kerr-nonlinear material [1]. In coupled-cavity systems [2], such as in Fig. 1, nonlinear interactions are enhanced and thus the systems, e.g. with instantaneous Kerr response, exhibit rich dynamics and offer more control over nonlinear switching, SP and chaos [3-5]. In particular, SP and chaos were observed in systems with two and three coupled microcavities [4]. For two-cavity systems, SP can be explained as a result of beating of modes [6]. However, SP can also be related to gap solitons [4].

In this work, we focus on analysis of systems with two coupled ring resonators. Compared with the previous studies [3-6], we take into account non-instantaneous Kerr response and the effect of loss. For the simulation, we use a numerical formalism that was developed recently [7] and demonstrate transitions from stable states to SP and chaotic states.

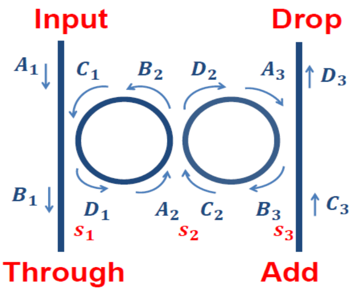

Fig. 1. Coupled-cavity system consisting of two ring resonators. Four ports are labeled. Arrows indicate propagation of the modes.

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II. Model

We consider a structure consisting of two coupled rings side coupled with two waveguides as shown in Fig. 1. The structure is excited at the input port only. The interaction in each coupler (in time *t*) is given by

$$B_j(t) = r_j A_j(t) + i s_j C_j(t), \tag{1}$$

$$D_j(t) = i s_j A_j(t) + r_j C_j(t). \tag{2}$$

Here, A_j, B_j, C_j and D_j represent the time-dependent (slowly-varying) mode amplitudes at different positions in the rings or waveguides. s_j and $r_j = \sqrt{1 - s_j^2}$ are respectively the coupling and transmission coefficients. The amplitudes are scaled to dimensionless form, defined by the relations such as $A_j = a_j \sqrt{2\pi n_2 L_{\mathrm{eff}} / \lambda}$, where a_j is the physical amplitude of the mode. In the stationary state, the nonlinear change of the effective mode index is given by the relation of type $n_2 |a_j|^2$, where n_2 is the effective nonlinear Kerr-index (it is assumed to be constant in all rings/waveguides). $L_{\mathrm{eff}} = [1 - \exp(\alpha L)]/\alpha$ is the effective length, α is the waveguide loss coefficient, L is the half of the ring circumference, and λ is the wavelength of light in vacuum. The propagation inside rings is described by the set of difference equations

$$A_{j+1}(t) = D_j(t-\tau)\exp(-\alpha L/2 + i\phi + i\delta_j(t-\tau)), \tag{3}$$

$$C_j(t) = B_{j+1}(t-\tau)\exp(-\alpha L/2 + i\phi + i\beta_{j+1}(t-\tau)), \tag{4}$$

where $\tau = L n_g / c$ is the group delay corresponding to propagation of the pulse over distance L, n_g is the mode group index, c is the velocity of light. Note that, the free spectral range, FSR, is related with the group delay by the relation $\tau \cdot \mathrm{FSR} = 1/2$. $\phi = 2\pi n_{\mathrm{eff}} L / \lambda$ is the linear phase shift acquired over distance L, n_{eff} is the linear effective mode index. The phase shift can be expressed as $\phi = \pi(m + \Delta f / \mathrm{FSR})$, where m is an arbitrary positive integer and Δf is the frequency detuning from resonance. The nonlinear phase shifts δ_j and β_j obey the Debye relaxation equations

$$T_{\mathrm{R}} \frac{d\delta_j(t)}{dt} + \delta_j(t) = \left|D_j(t)\right|^2, \tag{5}$$

$$T_{\mathrm{R}} \frac{d\beta_j(t)}{dt} + \beta_j(t) = \left|B_j(t)\right|^2, \tag{6}$$

where T_{R} is the medium relaxation time.

The above system of the difference-differential equations (1)-(6) is a generalization of the Ikeda equations for single ring [1] and fully describes the time evolution of the system. For the solution, we use the numerical technique developed in [7].

III. Numerical Example

Fig. 2 presents a bifurcation diagram for the selected structure. In our calculation (green curve/area), we gradually increased the input power P_{in} and searched for the corresponding stable, SP or chaotic state. As an initial condition, we used the state found for the previous value of P_{in}. It should be noted that other solutions can be found for different ways of changing P_{in}. In addition, we calculated steady-states (red curve) and applied the linear stability analysis (gray shaded regions).

We observe a stable region followed by a Hopf bifurcation at $P_{\mathrm{in}} \approx 0.021$. Above the Hopf bifurcation, we observe SP states, and, with further increase of input power, period doubling (e.g. at $P_{\mathrm{in}} \approx 0.054$). An example of such solution is illustrated in Fig. 3 (see the results for $P_{\mathrm{in}} = 0.030$). Finally, at $P_{\mathrm{in}} \approx 0.062$, the limit cycle breaks down, SP states bifurcate into chaotic states and an attractor is observed (Fig. 3, the results for $P_{\mathrm{in}} = 0.065$).

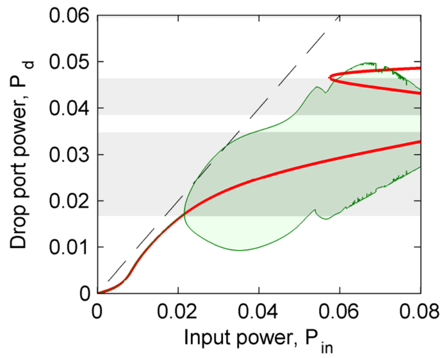

Fig. 2. Power at the drop port P_{d} vs. input power P_{in}. Structural parameters: $s_1 = s_3 = 0.42$, $s_2 = 0.2$, $\Delta f/\mathrm{FSR} = -0.05$, $\exp(-\alpha L/2) = 0.99$, $\tau/T_{\mathrm{R}} = 2$. Green curve (area) labels solutions calculated for adiabatic increase of P_{in}. Superimposed red curve shows steady-state solutions. Horizontal gray shaded regions mark intervals of P_{d} in which the steady-state solutions are unstable. Dashed line denotes 100% power transfer, i.e., $P_{\mathrm{d}} = P_{\mathrm{in}}$.

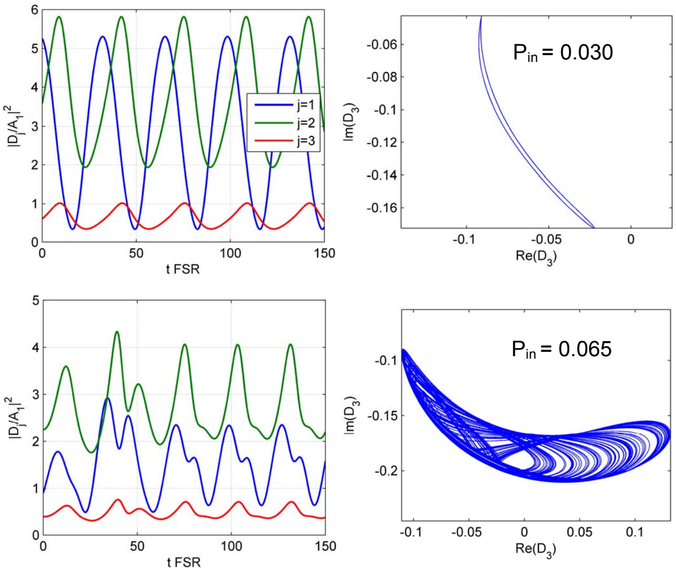

Fig. 3. Left panel: Relative powers $\left|D_j/A_1\right|^2$ in rings ($j = 1,2$) and at the drop port ($j = 3$) vs. normalized time $t \cdot \mathrm{FSR}$. Right panel: Corresponding phase portraits of the amplitude D_3. The dependencies were calculated for various input powers P_{in}.

IV. Conclusion

We presented a simple model for investigation of the dynamical behavior of Kerr-nonlinear resonant structures consisting of coupled ring resonators. The model includes both the effect of loss and non-instantaneous Kerr response. As an example, we considered a structure with two resonators; we presented steady-state solutions and demonstrated evolution from stable states to self-pulsing and chaotic states. The conference paper will also include more details about influence of loss and finite relaxation time on the dynamical behavior.

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