Asymptotic and numerical analysis of semiconductor ring lasers with negative optoelectronic and incoherent optical feedback

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We study the dynamical behavior of semiconductor ring lasers (SRLs) with negative optoelectronic feedback (NOEF) or with incoherent optical feedback (IOF). As we vary the feedback strength, the devices under consideration in this work display both continuous wave operation and a period-doubling route to in-phase chaos. For delay times significantly longer than the period of the relaxation oscillations, the two counter-propagating modes of the SRL exhibit anti-phase chaotic oscillations in both systems. For these longer delay times, we use asymptotic methods to reduce the original set of five equations used to describe the dynamical behavior of SRL-NOEF or-IOF to two equations and one map. The equations of the reduced models turn out to be the same for both systems and provide us with insight into the appearance of the novel anti-phase chaotic oscillations.

SRLs are very attractive devices in photonic integrated circuits. Due to the bistability between two counter-propagating modes, the possibility exists to encode digital information in the emission direction. Therefore, they can be used for all-optical switching, gating, wavelength- conversion functions and all-optical memories [1]. Under external perturbations such as coherent optical feedback, SRLs as others semiconductor lasers often present instabilities in their optical output. Using coherent optical feedback for e.g. chaos synchronization schemes comes with strict requirements on the detuning between the free-running frequencies of the transmitter and receiver laser. However, for lasers with OEF or IOF [2], the schemes would not require fine-tuning of the optical frequencies. In this work, we focus on the dynamical analysis of SRL-NOEF or-IOF of which the schematic diagram of SRL-NOEF or-IOF is given in Fig. 1.

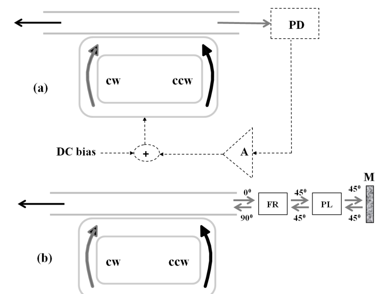

FIG. 1: Schematic diagrams of: (a) SRL-OEF and (b) SRL-IOF. In (a), the solid (dashed) lines indicate the optical (electronic) part. PD−photodetector, A−amplifier, FR−Faraday rotator, PL−polarizer, M−mirror.

SRL-OEF or-IOF operating in a single-longitudinal, single-transverse mode can be modelled by using the electric fields of the counter-propagating $E_{cw,ccw}$ and the carrier number N inside the cavity:

$$\dot{E}_{cw,ccw} = \kappa(1+i\alpha)[g_{cw,ccw}N-1]E_{cw,ccw} - ke^{i\phi_k}E_{ccw,cw}, \quad (1)$$

$$\dot{N} = \gamma[\mu - \zeta_{OE}|E_{cw}(t^{'}-\tau^{'}_{OE})|^2 - \zeta_{IO}N|E_{cw}(t^{'}-\tau^{'}_{IO})|^2 - N - Ng_{cw}|E_{cw}|^2 - Ng_{ccw}|E_{ccw}|^2]. \quad (2)$$

where $t^{'}$ is the time. $g_{cw,ccw} = 1 - s|E_{cw,ccw}|^2 - c|E_{ccw,cw}|^2$ are the differential gain functions. $\mu = 1.7$ is the injection DC current. $k = (k_c^2 + k_d^2)^{1/2}$ is the amplitude and $\phi_k = tan^{-1}(k_c/k_d)$ the phase. The parameters: $k_d = 0.0327ns^{-1}$, $k_c = 0.44ns^{-1}$, $\alpha = 3.5$, $\gamma = 0.2ns^{-1}$, $\kappa = 100ns^{-1}$, $c = 0.01$ and $s = 0.005$ are taken from Ref.[3]. For $\zeta_{IO} = 0$ and $\zeta_{OE} \neq 0$, Eqs. (1)–(2) describes the dynamics of SRL-NOEF while for $\zeta_{IO} \neq 0$ and $\zeta_{OE} = 0$, it describes the dynamics of SRL-IOF.
To investigate the dynamical behavior of SRL-OEF or-IOF, we shall vary the feedback strength $\zeta_{OE/IO}$ and delay time $\tau_{OE/IO}$. Since the solitary SRL has two characteristic timescales: the RO frequency and the alternate oscillations (AO) frequency, it is interesting to study the dynamical behavior of SRL-NOEF or-IOF at delay time $\tau_{OE/IO}$ comparable to the RO period (short delay time) and to the AO period (long delay time).
For delay time comparable to the period of ROs, the bifurcation diagrams depicting the local extrema of the counter-propagating mode intensities $I_{cw/ccw}$ of SRL-

NOEF or-IOF as function of $\zeta_{OE/IO}$ present continuous wave operation and a period-doubling route to in-phase chaos. The in-phase chaos is depicted in Fig.2.

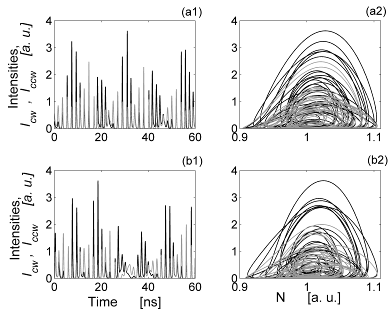

FIG. 2: Panels (a1) and (b1): time series of I_{ccw} (black) and I_{cw} (grey). Panels (a2) and (b2): phase portraits. Parameters are: $\zeta_{OE} = 1.1$ ($\zeta_{IO} = 0$) and $\tau_{OE} = 2.2$ ns (a), $\zeta_{IO} = 1.1$ and $\tau_{IO} = 2.2$ ns (b) with $\mu = 1.7$.

The chaotic time series of $I_{cw/ccw}$ shown in Figs. 2a1 and 2b1 are in-phase and their amplitudes are almost of the same order. The chaotic behavior is confirmed by their corresponding phase portrait shown in Figs. 2a2 and 2b2.
For long delay time $\tau_{OE/IO}$ (significantly longer than ROs period but comparable to AOs period), the bifurcation diagrams depicting the local extrema of SRL-NOEF or-IOF versus $\zeta_{OE/IO}$ show continuous wave bistable operation followed by a quasi-periodic route to anti-phase chaos. The counter propagating modes of SRL-NOEF or-IOF systems depict the same dynamical behaviors when their feedback strengths increased. In Fig. 3, we plot time series of $I_{cw/ccw}$ and the corresponding Poincaré section in chaotic regime.

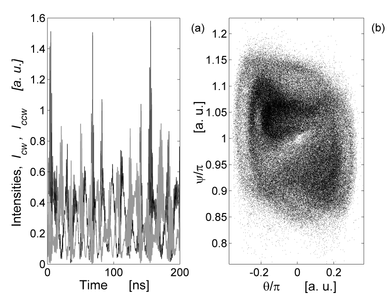

FIG. 3: Time series (a) of I_{ccw} (black) and I_{cw} (grey) and the corresponding Poincaré section (b) for $\zeta_{IO} = 0.8$ and $\tau_{IO} = 15$ ns.

From Fig. 3a, we find that the $I_{cw/ccw}$ exhibit anti-phase chaos. These anti-phase chaotic oscillations can be understood when we reduce the full model of Eqs. (1)-(2) using asymptotic methods valid at longer timescales. Let us define new variables and parameters:

$$\rho = \frac{\gamma}{\kappa}, N-1 = n\rho, s = S\rho, c = C\rho, \frac{k}{\kappa} = K\rho, t = \gamma t' \quad (3)$$

where n is assumed to be $O(1)$ and $\rho = \gamma/\kappa$ is a small parameter. The parameters S, C and K are of $O(1)$. After substituting Eq.(3) in Eqs.(1)–(2) with the limit $\rho \to 0$, we obtain the reduced set of equations:

$$\dot{\theta} = -2\sin\phi_k \sin\psi + 2\cos\phi_k \cos\psi \sin\theta + J[1 - \frac{\zeta|E_{cw}(t-\nu)|^2}{\mu - 1}]\sin\theta\cos\theta \quad (4)$$

$$\dot{\psi}\cos\theta = \alpha J[1 - \frac{\zeta|E_{cw}(t-\nu)|^2}{\mu - 1}]\sin\theta\cos\theta + 2\cos\phi_k \sin\psi + 2\sin\phi_k \cos\psi \sin\theta \quad (5)$$

$$|E_{cw}(t)|^2 = [\mu - 1 - \zeta|E_{cw}(t-\nu)|^2]\sin(\theta/2 + \pi/4) \quad (6)$$

with $\nu = K\tau$ and $\psi \in [0, 2\pi]$. The chaotic behavior can now be analysed in the Poincaré section in the phase space of the reduced model, as shown in Fig. 3b.

This work dealt with the dynamical behavior of SRLs with NOEF or-IOF. For short delay times, we have shown that by increasing the feedback strength, the SRL-NOEF or-IOF exhibit both continuous wave operation and a period-doubling route to in-phase chaos similar to single mode semiconductor lasers. For long delay time, however, we have found that by increasing the feedback strength, the SRL-NOEF or-IOF display both continuous wave bistable operation followed by a quasi-periodic route to anti-phase chaos. We have demonstrated that by using asymptotic methods the original model of five differential equations for SRL-NOEF or-IOF can be reduced to two differential equations and one map valid on time-scales longer than the ROs. The reduced model accurately describes the anti-phase chaos and allows for more insight into their origins.

ACKNOWLEDGMENTS

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[1] Wang, Z., Yuan, G., Verschaffelt, G., Danckaert J., and Yu, S., "Storing 2 bits of information in a novel single semiconductor microring laser memory cell," IEEE Photon. Technol. Lett. 20, 1228 (2008).
[2] Erneux, T., "Applied delay differential equations," Springer New York, (2009).
[3] Kingni, S. T., Van der Sande, G., Ermarkov, I. V. and Danckaert, J., "Semiconductor ring lasers with negative optoelectronic and incoherent optical feedback: asymptotic, bifurcation and numerical analysis," Submitted to Chaos (2014).