Passively mode-locked lasers subject to optical feedback: the role of amplitude-phase coupling

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***Abstract*—We investigate the impact of amplitude-phase coupling on the dynamics of a passively mode-locked laser subject to optical feedback. This is done using a delay differential equation model to calculate bifurcation diagrams in the plane of the feedback parameters. We find an increased complexity in the dynamics of the laser when the effective α-factors of the gain and absorber sections are different. With non-zero α-factors, quasiperiodic mode-locking can also be induced by resonant feedback and the extent of complex dynamical regions in the parameter plane increases.**

I. Introduction

Passively mode-locked (ML) lasers are of great interest due to their potential use as sources of high-frequency ultra-short light pulses in a wide range of applications. This is due to the relative ease of fabrication, as well as the high repetition rates and narrow pulse widths that can be achieved. However, due to the absence of an external reference clock, passively ML lasers have a relatively large timing jitter. It has been shown experimentally that optical feedback can reduce the timing jitter [1], [2]. Theoretical studies have been carried out to gain an understanding of the influence the optical feedback has on the laser dynamics [3], [4]. However a detailed study on the influence of the amplitude-phase coupling on the ML laser subject to optical feedback was still lacking. This is important, as the the amplitude-phase coupling greatly influences the dynamics of the free running laser and thus understanding this effect is crucial to predict stable dynamic regimes and to explain experimentally results. We therefore use the delay differential equation (DDE) model introduced in [5], and extended to include optical feedback [3], to investigate the effect of optical feedback on the dynamics of a passively ML laser, focusing on the impact of amplitude-phase coupling (α-factor).

II. Model

In Ref. [5] a DDE model describing a ring cavity, passively ML laser was introduced. This model was later extended to include optical feedback [3]. The set of three coupled delay differential equations describing the passively ML laser coupled to an external feedback cavity are

$$\gamma^{-1}\dot{\mathcal{E}}(t) + \mathcal{E}(t) = R(t-T)\,\mathcal{E}(t-T) + \sum_{l=1}^{\infty} K_l e^{-ilC} R(t-T-l\tau)\,\mathcal{E}(t-T-l\tau), \quad (1)$$

$$\dot{G}(t) = J_g - \gamma_g G(t) - e^{-Q(t)}\left(e^{G(t)}-1\right)|\mathcal{E}(t)|^2, \quad (2)$$

$$\dot{Q}(t) = J_q - \gamma_q Q(t) - r_s e^{-Q(t)}\left(e^{Q(t)}-1\right)|\mathcal{E}(t)|^2, \quad (3)$$

with

$$R(t) \equiv \sqrt{\kappa}\, e^{\frac{1}{2}((1-i\alpha_g)G(t)-(1-i\alpha_q)Q(t))}. \quad (4)$$

TABLE I. Parameter values used in numerical simulations.

symbol	value	symbol	value	symbol	value
γ	$2.5ps^{-1}$	r_s	25.0	T	$25ps$
γ_g	$1ns^{-1}$	κ	0.1	$T_{ISI,0}$	$1.015\,T$
γ_q	$75ns^{-1}$	α_g	varied	C	0
J_g	$4.8ps^{-1}$	α_q	varied	l	1
J_q	$2.5ps^{-1}$				

The dynamical variables are the slowly varying electric field amplitude $\mathcal{E}$, the saturable gain G and the saturable loss Q. The parameters in this equation are: the cold cavity roundtrip time $T \equiv v/L$ where L is the length of the ring cavity, the external cavity roundtrip time (delay time) τ, the full-width at half maximum γ of the Lorentzian-shaped filter function used to account for the finite width of the gain spectrum, the roundtrip number l dependent feedback strength K_l, the phase C of the fed back light, the unsaturated gain J_g in the gain section, the unsaturated absorption J_q in the saturable absorber section, the carrier lifetimes in the gain and absorber sections $1/\gamma_g$ and $1/\gamma_q$, the ratio of the saturation energies in the gain and absorber sections r_s, the non-resonant losses κ and the amplitude-phase coupling in the gain and absorber sections α_g and α_q.

In the equations above, zero detuning between the frequency of the maximum of the gain spectrum and the nearest cavity mode has been assumed. In the following we also restrict our study to small feedback strengths. This allows us to neglect all feedback contributions from multiple external cavity roundtrips. Table I lists the parameter values used in the simulations.

III. Results

Using the parameter values listed in Table I the solitary laser exhibits fundamental ML. This is true for a wide range of α-factors. Here we restrict α_g and α_q to values between zero and two, therefore focusing on nanostructured devices.

Important for the study of the dynamics of the ML laser subject to optical feedback is the feedback resonance condition,

$$p\tau = qT_{ISI}, \text{ for } p, q \in \mathbb{N}. \quad (5)$$

Here T_{ISI} is the interspike interval time between pulses for the solitary laser. When this condition is fulfilled and $p = 1$ feedback is resonant and there is only one pulse within the cavity. Higher order resonances occur for larger integer values of p. In the higher order resonance regime p pulses travel within the laser cavity for $p < 6$. In fig. 1 bifurcation diagrams

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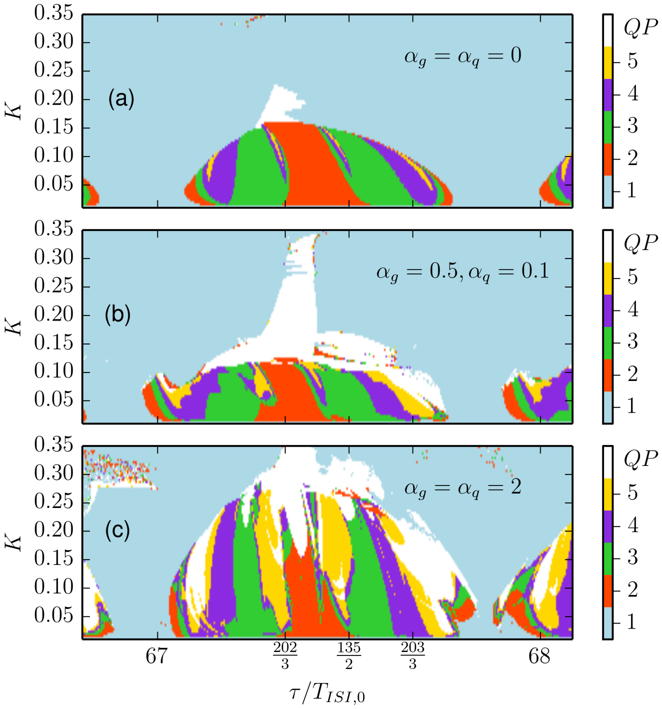

Fig. 1. Bifurcation diagrams as a function of the feedback strength K and external delay τ (in units of the interspike interval of the solitary laser with $\alpha_g = \alpha_q = 0$, $T_{ISI,0}$). Color code: 1-5 indicate the number of pulses in the cavity, regions in white indicate quasiperiodic (QP) ML. Parameters: as in Table I.

of the laser output are plotted as a function of the feedback strength K and the feedback cavity delay time τ which is in units of the interspike interval of the solitary laser with $\alpha_g = \alpha_q = 0$ ($T_{ISI,0}$). Regions in blue indicate fundamental ML, white regions indicate quasiperiodic ML (QP), the remaining colored regions indicate ML with additional feedback induced pulses. In subplot (a) with $\alpha_g = \alpha_q = 0$, fundamental ML is observed for resonant feedback. For low feedback strengths higher order resonances occur. For larger feedback strengths the width of the main resonance regions increase. This is due to locking between the pulse traveling in the laser cavity and the pulse traveling in the feedback cavity. In subplots (b) and (c) of fig. 1, fundamental ML occurs at the main resonances, however for small feedback strengths the extent of the main resonances is reduced. The structure of the higher order resonance regions for low K is more complex and the regions of quasiperiodic ML are larger. For $\alpha_g = \alpha_q = 2$ quasiperiodic ML is even observed at the main resonance. Meaning that the pulse stream is destabilized by resonant feedback.

To explore the multi-stability of solutions we vary the initial conditions. In fig. 2 bifurcation diagrams are shown as a function of τ for $K = 0.1$ and $K = 0.3$. The points were obtained by up-sweeping τ (green), down-sweeping τ (red) and using the same initial conditions as used in fig. 1 (blue). For low K only small regions of bistability are observed, however for larger K the system becomes bi- or multi-stable. In the central region of subplot (d) the laser exhibits quasiperiodic ML, here the system is very sensitive to the initial condition, therefore we observe three different solutions.

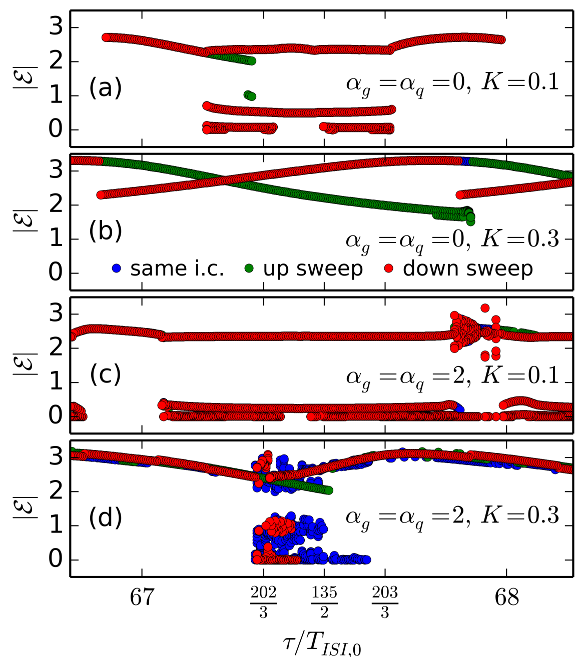

Fig. 2. Bifurcation diagrams as a function of the external delay τ (in units of the interspike interval of the solitary laser with $\alpha_g = \alpha_q = 0$, $T_{ISI,0}$). Color code: up sweep in τ (green), down sweep in τ (red), same initial conditions (blue). Parameters: as in Table I.

IV. Conclusion

For non-zero amplitude-phase coupling the extent of fundamental ML in the plane of the feedback parameters is reduced and even at the main resonances feedback can destabilize the pulse stream. Complex dynamics occur even at low feedback strengths. Where the system does not exhibit fundamental ML it is more susceptible to noise fluctuations, which is important for real semiconductor devices. For the purpose of timing jitter reduction the feedback would have a detrimental effect in these regions.

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References

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