Analysis of a Metal Disc-Type Terahertz Surface Wave Splitter Using the Cylindrical LOD-FDTD Method

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***Abstract*—To efficiently analyze the electromagnetic problems in cylindrical structures, we develop a locally one-dimensional finite-difference time-domain (FDTD) method in cylindrical coordinates. The Sherman-Morrison formula is used to solve a cyclic matrix, and the image theory is introduced to treat a perfect electric conductor. As an application, we analyze a metal disc-type terahertz surface wave splitter. The computation time is reduced to approximately half that of the explicit FDTD method, while maintaining the comparable accuracy.**

I. Introduction

The finite-difference time-domain (FDTD) method in cylindrical coordinates has often been used to treat the electromagnetic problems in cylindrical structures. Note, for the cylindrical FDTD method, that the circumferential mesh size close to the center axis becomes inevitably small. This gives rise to a small time step restricted by the Courant-Friedrichs-Lewy (CFL) condition, resulting in a long computation time. To remove this restriction, the implicit locally one-dimensional (LOD) scheme [1] was tried to be introduced into the cylindrical FDTD method [2]. However, no discussion has been made on the connection of the fields at a specific plane in the circumferential direction. In addition, the treatment of a perfect electric conductor (PEC) has not been described. These issues are quite important for the application of the implicit scheme to the cylindrical FDTD method.

In this article, we develop an LOD-FDTD method in cylindrical coordinates. A cyclic matrix, resulting from the application of the implicit scheme to cylindrical coordinates, is solved using the Sherman-Morrison formula. The PEC condition, which cannot directly be incorporated into the implicit scheme, is imposed using the image theory. As an application, we analyze a metal disc-type terahertz surface wave splitter. Numerical results are compared with those obtained from the traditional explicit FDTD method. It is found that the computation time is reduced to approximately half that of the FDTD method.

II. Discussion

We formulate the cylindrical LOD-FDTD method. The basic formulas for the 1st step are as follows:

$$E_\rho^* = E_\rho^n + \frac{c\Delta t}{2\varepsilon_r \rho}\left(\frac{\partial H_z^*}{\partial \varphi} + \frac{\partial H_z^n}{\partial \varphi}\right)$$

$$E_\varphi^* = E_\varphi^n + \frac{c\Delta t}{2\varepsilon_r}\left(\frac{\partial H_\rho^*}{\partial z} + \frac{\partial H_\rho^n}{\partial z}\right)$$

$$E_z^* = E_z^n + \frac{c\Delta t}{2\varepsilon_r \rho}\left(\frac{\partial (\rho H_\varphi^*)}{\partial \rho} + \frac{\partial (\rho H_\varphi^n)}{\partial \rho}\right)$$

$$H_\rho^* = H_\rho^n + \frac{c\Delta t}{2\mu_r}\left(\frac{\partial E_\varphi^*}{\partial z} + \frac{\partial E_\varphi^n}{\partial z}\right)$$

$$H_\varphi^* = H_\varphi^n + \frac{c\Delta t}{2\mu_r}\left(\frac{\partial E_z^*}{\partial \rho} + \frac{\partial E_z^n}{\partial \rho}\right)$$

$$H_z^* = H_z^n + \frac{c\Delta t}{2\mu_r \rho}\left(\frac{\partial E_\rho^*}{\partial \varphi} + \frac{\partial E_\rho^n}{\partial \varphi}\right)$$

where c is the speed of light in a vacuum, ε_r is the relative permittivity, and μ_r is the relative permeability. In cylindrical coordinates, the fields are regarded as periodic in the circumferential direction. Therefore, it is necessary to connect the fields at a specific plane in the circumferential direction using the periodic boundary condition. Unfortunately, this gives rise to a cyclic matrix, which cannot directly be solved using the conventional Thomas algorithm. Then, we take advantage of the Sherman-Morrison formula [3], [4] that allows us to use the Thomas algorithm. Furthermore, the convolutional perfectly matched layer is incorporated as an absorbing boundary [5].

Fig. 1 shows the metal disc-type terahertz surface wave splitter [6]. The depths of gratings are chosen to be h_{R}=50 μm and h_{L}=30 μm on the right and left sides at separate frequencies of 1.0 and 1.5 THz, respectively. The number of grating periods is chosen to be 9. Other structure parameters are w_{B}=480 μm, w_{T}=200 μm, l=400 μm, r=80 μm, p=50 μm, d=20 μm, φ_{R}=φ_{L}=135° and φ_m=45°. We analyze this structure using the explicit FDTD and LOD-FDTD methods. The metal is treated as a PEC. Here, we resort to the image theory for the LOD-FDTD method [7], i.e., the two magnetic fields, between which the electric field on the metal surface is sandwiched, are set to be equal. This forces the electric field to be zero on the metal surface.

First, we show the filed distributions on the upper surface of the splitter. Figs. 2 and 3 illustrate the H_φ field distributions at f=1.0 and 1.5 THz, respectively. In these figures, the results of the FDTD and the LOD-FDTD methods are compared, in which $\Delta t/\Delta t_{\mathrm{CFL}}$ is defined as CFLN (Δt_{CFL} is the upper limit of the time step for the FDTD method). The waves at f=1.0 and 1.5 THz are shown to mainly propagate in the

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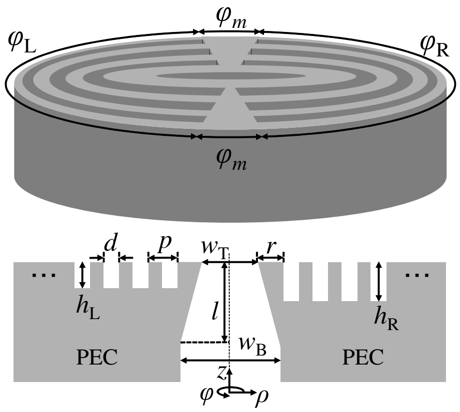

Fig. 1. Configuration.

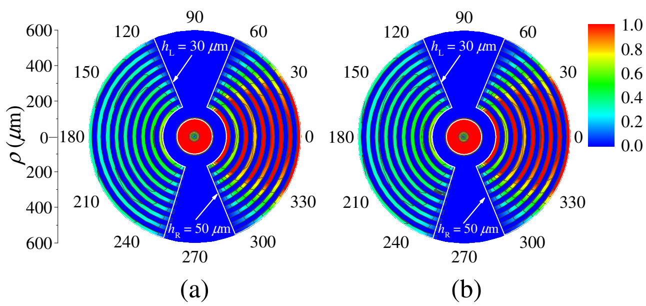

Fig. 2. H_φ field distributions at f=1.0 THz. (a) Explicit FDTD method for CFLN=1. (b) LOD-FDTD method for CFLN=10.

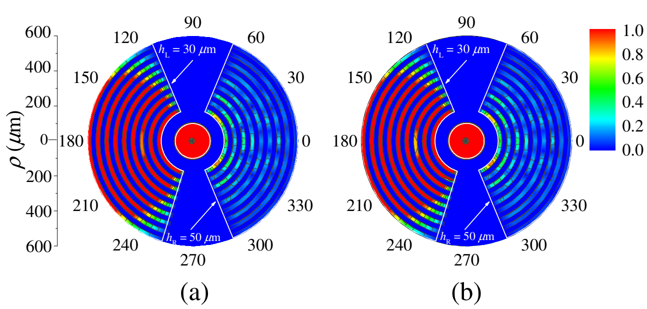

Fig. 3. H_φ field distributions at f=1.5 THz. (a) Explicit FDTD method for CFLN=1. (b) LOD-FDTD method for CFLN=10.

right and left directions, respectively. It is found, even for CFLN=10, that the LOD results are in good agreement with the FDTD counterparts.

Next, we evaluate the power propagating in each direction using the Poynting vector calculations. The following values are calculated: the radiation power propagating in the $+z$ direction above the disc, the reflection power propagating in the $-z$ direction under the disc, and the power propagating in the right and left directions along the gratings. Table. 1 shows the normalized power at f=1.0 THz. For CFLN=1, the LOD-FDTD and FDTD results agree well with each other. Although as CFLN is increased the LOD-FDTD results gradually deviate from the FDTD ones, the differences are acceptably small even for CFLN=10.

TABLE I
NORMALIZED POWER

Method	Explicit	LOD-FDTD		
CFLN	1	1	5	10
Radiation	0.076	0.076	0.077	0.077
Reflection	0.848	0.847	0.847	0.848
Right	0.048	0.048	0.049	0.049
Left	0.035	0.035	0.036	0.036
Total	1.007	1.006	1.009	1.010

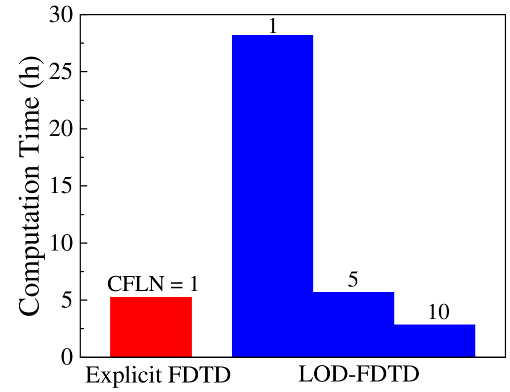

Fig. 4. Computation times

The computation times of the FDTD and LOD-FDTD methods are shown in Fig. 4. For CFLN=10, the computation time of the LOD-FDTD method is reduced to approximately 50% of that of the FDTD method.

III. Conclusion

We have developed a cylindrical LOD-FDTD method and analyzed a terahertz surface wave splitter. The Sherman-Morrison formula is introduced to treat a cyclic matrix, and the image theory is applied to imposing the PEC condition. It is shown that the computation time is reduced to approximately half that of the FDTD method, while maintaining the comparable calculation accuracy.

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