Plane wave analysis of bull's eye gratings

Debdatta Ray and Anil Prabhakar

***Abstract*—We investigated the transmittivity of a bull's eye grating using the plane wave expansion method. The parameters of the grating were optimized at 1550 nm. The grating has annular grooves with alternating layers of aluminum and air. The dispersion relation was obtained by expressing Maxwell's equations in cylindrical co-ordinates and solving the eigenvalue problem. We observe flat bands for frequencies below plasma frequency (ω_p). COMSOL simulations were used to validate the eigenfrequencies at $k \rightarrow 0$ in the band diagram.**

***Index Terms*—bull's eye grating, surface plasmon, plasma frequency**

I. Introduction

THERE is a constant effort to scale down device dimensions without compromising on device performance. But photonic components are constrained by the diffraction limit of $\frac{\lambda}{2}$. Unlike photonic components plasmonic components can provide functionality in the nm regime, even with μm excitation. Bull's eye gratings, using Al incorporated on photodetectors, is one such device that can help to scale down the active area of the diode from $(\mu\text{m})^2$ to hundreds of $(\text{nm})^2$, without compromising the responsivity and quantum efficiency of the photodetectors [1]. The grating has a central subwavelength hole flanked by periodic corrugations of air and Al. Surface plasmons can be excited in the radial direction on the air metal interface by grating coupling [2]. These fields propagate towards the central hole and interfere constructively to generate high $\mathbf{E}$ fields on the central hole, yielding high values of transmittivity [1]. The transmittivity values also depend on a number of other grating factors like period, groove depth and duty cycle [3].

In this paper we use the plane wave expansion method (PWEM) to find the dispersion relation for a bull's eye structure [4]. The characteristic frequencies were obtained after reducing Maxwell's wave equations to an eigenvalue problem [5][6]. The group velocity and transmission resonances of the grating can be explained by the dispersion curves. We have also carried out finite element COMSOL simulations at different frequencies. The simulated results were used to validate the eigenfrequencies obtained from PWEM.

II. Device structure

A bull's eye grating is shown schematically in Fig.1 and Fig.2. The grating parameters were optimized for excitation at 1550 nm [3]. We assume a = 1550 nm and the diameter of the central hole to be $\frac{a}{2}$. We also assume the metal to be aluminum, of height 400 nm. The aspect ratio of the grating is $\frac{h}{w} = \frac{200}{775} = 0.26$, shown in Fig. 2.

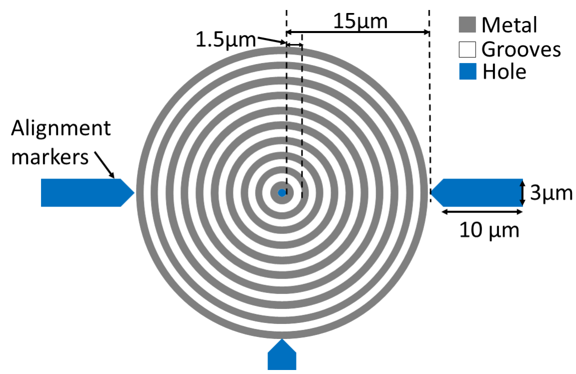

Figure 1. Top view of the bull's eye grating. a is the period of the grating along the radial direction. The solid circle at the centre represents a hole, and the grey annular rings represents metallic regions whereas the white regions represent grooves. The structure is periodic in ϕ with a periodicity of 2π.

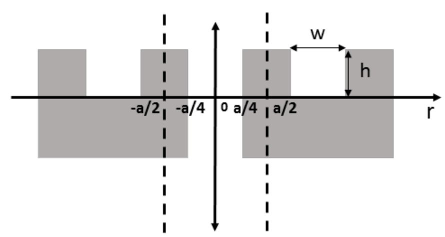

Figure 2. Cross section of the bull's eye eye grating. It shows a cavity filled with air for $\frac{-a}{4}$ to $\frac{a}{4}$. The periodicity of the grooves starts from $\pm\frac{a}{4}$. $\dfrac{a}{4} < r < \dfrac{3a}{4}$ is metal, and $\dfrac{3a}{4} < r < \dfrac{5a}{4}$ is air.

III. Results and Discussions

We solve Maxwell's equations in cylindrical co-ordinates to benefit from the circular symmetry of the grating. The direction of propagation of the surface plasmons has been assumed to be along r and z. As we are interested in analyzing the effects of surface plasmon resonance (SPR), we investigate the spatial profile of E_r. (1) shows the wave equations in cylindrical co-ordinates. For the analysis we have assumed $\mathbf{E}_r \sim \mathbf{E}_r e^{j\omega t}$ and $\mathbf{E}.\nabla\epsilon(r,\omega) \simeq 0$.

$$\left[\frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2}\right] E_r(r,z,\phi) = \frac{-\omega^2\epsilon(r,\omega)}{c^2} E_r(r,z,\phi) \tag{1}$$

In the PWEM, we express each term in (1) as a Fourier series.

Debdatta Ray is with the Department of Electrical Engineering, Chennai, India, e-mail: debdattaray.ece@gmail.com.

Anil Prabhakar is with the Department of Electrical Engineering, Chennai, India, e-mail anilpr@gmail.com.

$$\frac{1}{r} = \sum_{m=-\infty}^{\infty} G_{1m}\mathrm{e}^{-\mathrm{j}\frac{2\pi mr}{a}} \text{ and } \frac{1}{r^2} = \sum_{m=-\infty}^{\infty} G_{2m}\mathrm{e}^{-\mathrm{j}\frac{2\pi n}{a}}$$

$$\epsilon(r,\omega) = \sum_{m=-\infty}^{\infty} B_m\mathrm{e}^{-\mathrm{j}\frac{2\pi mr}{a}} = 1 + [\epsilon(\omega)-1]\,S(r)$$

$$E_{qr}(r,z) = \sum_{n=-\infty}^{\infty} C_n\mathrm{e}^{-\mathrm{j}\frac{2\pi nr}{a}}\mathrm{e}^{-\mathrm{j}k_{qz}z}\mathrm{e}^{-\mathrm{j}k_{qr}r}$$

where $S(r)$ is shape factor of value 1 in metal and 0 in air. We equate the coefficients to arrive at an eigenvalue equation

$$\sum_{n=-\infty}^{\infty} Q_{p,n}(k_{qr})C_n = \frac{\omega^2}{\mathrm{c}^2}\,[C_p] \tag{2}$$

where

$$Q_{p,n} = \left[\left[(k_{qr}+\frac{2\pi n}{a})^2 + k_{qz}^2\right]\delta_{n,p}\right] + \left[q^2 G_{2(p-n)}\right] + \mathrm{j}\left[G_{1(p-n)}(k_{qr}+\frac{2\pi n}{a})\right] + \frac{\omega_p^2}{ac^2}\int_{\frac{a}{4}}^{\frac{5a}{4}} S(r)e^{\mathrm{j}\frac{2\pi(p-n)r}{a}}\mathrm{d}r$$

where k_{qr} is the momentum vector along r for the q^{th} mode. The eigenvalues of (2) are shown in Fig. 3. We also carried out a 2D finite element simulation to plot the E fields for different ω. Fig. 4 is a surface plot of $|E|$ for $\frac{\omega}{\omega_p} = 0.09$.

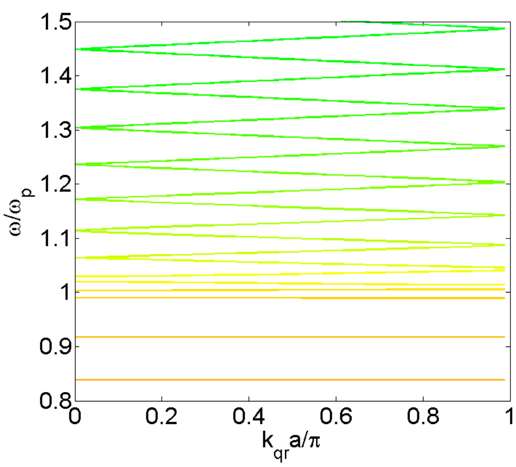

Figure 3. Band diagram of the grating for the first Brillouin zone

Using a parametric sweep of the frequency for $\omega < \omega_p$ to $\omega > \omega_p$, we estimate $\int_0^{\frac{21a}{4}} |E|\mathrm{d}r$. In Fig. 5, we plot the result of the parametric sweep in COMSOL, and label the eigenfrequencies obtained from the PWEM. We observe peaks that correspond to the eigenfrequencies that were calculated from the PWEM.

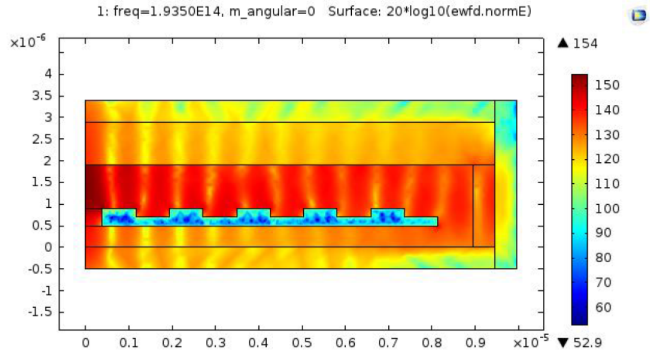

Figure 4. Surface plot of the grating at 1550 nm computed in COMSOL, corresponding to $\frac{\omega}{\omega_p} = 0.1$

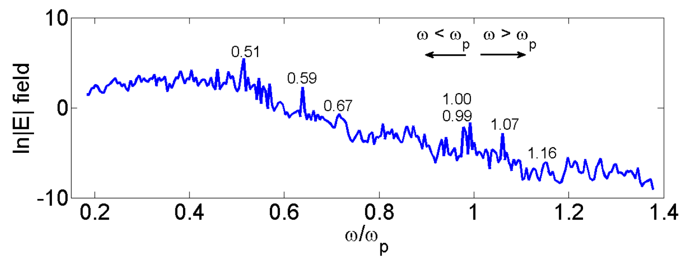

Figure 5. Transmittivity of the bull's eye grating. The values above the peaks were obtained as eigenfrequencies from the PWEM.

Summary

We used a plane wave expansion to analyze the transmittivity of a bull's eye grating. The eigenfrequencies obtained after solving Maxwell's equations in cylindrical co-ordinates were validated against finite element simulations.

Acknowledgment

We acknowledge DAAD Sandwich Master's programme and University of Stuttgart for helping us to fabricate the gratings.

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