Modeling of spontaneous parametric down-conversion in plasmonic structures as difference-frequency generation with realistic beams

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***Abstract*—Numerical study of plasmonic enhancement of the generation of the photon pairs by spontaneous parametric down-conversion is considered. The modeling is done by nonlinear transfer-matrix method which is extended to realistic waves. The results indicate that in the case of short range surface plasmon polaritons the main limiting factor is the short length of coherent buildup of the generated signal.**

I. Introduction

Spontaneous parametric down-conversion (SPDC) in a nonlinear crystal is one of the main processes of generation of entangled photon pairs: pump photons split to two due to the second-order nonlinearity of the medium. However, the process of SPDC is in general very weak, the yield of SPDC is only around 10^{-12} [1].

An idea to enhance the process by plasmonic enhancement is presented in Ref. [2]. It is shown, that it is possible to enhance the yield of SPDC up to 40 thousand times by usual Kretschmann configuration through field enhancement of surface plasmon polaritons (SPPs). However, only the effect of the field enhancement was estimated: no outcoupling efficiency, coherent buildup via phase-matching, effective nonlinearity or beam width effects were studied. The aim of this study is to fully model the process of SPDC in Kretschmann configuration by using nonlinear transfer-matrix method (NLTMM). It is done by approximating the process of SPDC as a difference frequency generation (DFG), where one of the input beam represents the vacuum fluctuations. In such way, it is possible to fully model the generation of photon pairs in any stratified medium.

II. Methodology

A. Modeling SPDC as DFG

The standard setup of DFG is shown in Fig.1a. Two input waves with frequencies ω_1 and ω_2 are incident from the left, due to the nonzero second order susceptibility $\chi^{(2)}$ the mixing between the frequencies occurs. On of the possible process is the generation of difference frequency $\omega = \omega_1 - \omega_2$. The characteristic property of this process is that only photons form the first input beam is absorbed. It is clearly visible form the energy diagram in Fig. 1a: the second input-beam is amplified instead. In addition to the conservation of the energy also

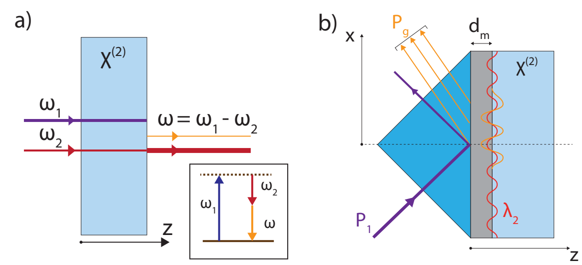

Fig. 1. a) Usual structure for DFG process. The frequency of two input beams are ω_1 and ω_2, respectively. b) SPDC generation in Kretschmann configuration. The pump beam power is P_1 and the power of generated beam is P_g.

phase must be matched for efficient generation of DFG (i.e momentum must be conserved).

In classical optics, generation of DFG is not possible without the second input beam. However, in quantum optics, vacuum fluctuations are always present. Clearly, the DFG is a classical analog to the SPDC process where the second input beam is replaced with a very weak fields, which represent the vacuum fluctuations.

B. Nonlinear transfer-matrix method for realistic waves

In linear optics a great method to study stratified medium is to use transfer-matrix method (TMM) [3], [4]. In order to study DFG in layered structure in similar manner an extension to the standard TMM was used, which allows to calculate any second-order nonlinear process [5].

Like standard TMM also NLTMM is limited to the calculation of infinite plane waves. In most cases, the plane waves approximate the properties of the realistic (e.g. Gaussian beam) waves very well. However, in our case we needed to expand the method for realistic waves for two main reasons. First, plasmonic structures, especially long-range SPPs, can have very narrow resonances, which cannot be efficiently excited by waves with finite dimensions. It means that in case of realistic beams only a part of the maximum enhancement is realistically available. Secondly, in order to estimate the phase-mismatch we need to have an access to the field profile of the coherent buildup of the DFG signal, which is not possible in the case of infinitely wide plane wave. For those reasons, the NLTMM was extended to realistic beams with arbitrary profile. The full code of NLTMM for realistic waves is available as a Python library (written in C++) at GitHub github.com/ardiloot/NonlinearTMM.

III. Results

The simplest structure supporting SPPs is Kretschmann configuration show in Fig. 1b. In consists of high refractive index prism $n_p = 2.2$, thin ($d_m = 50\,\mathrm{nm}$) metal silver film and nonlinear crystal. The refractive index of the crystal is taken to be equal to single crystal quartz and nonlinearity is defined as $\chi^{(2)}_{xxx} = \chi^{(2)}_{yyy} = \chi^{(2)}_{zzz} = 4.4\,\mathrm{pm/V}$ [6].

The Gaussian pump beam ($\lambda_1 = 532\,\mathrm{nm}$) is incident through prism. The angle of incidence is defined by the normalized tangential wave vector $\beta_1 = \sin(\theta_1)\, n_p(\lambda_1)$, the power of the beam is $P_1 = 100\,\mathrm{mW}$ and the waist size of the beam is $w_0 = 1.0\,\mathrm{mm}$ (see Fig. 2c).

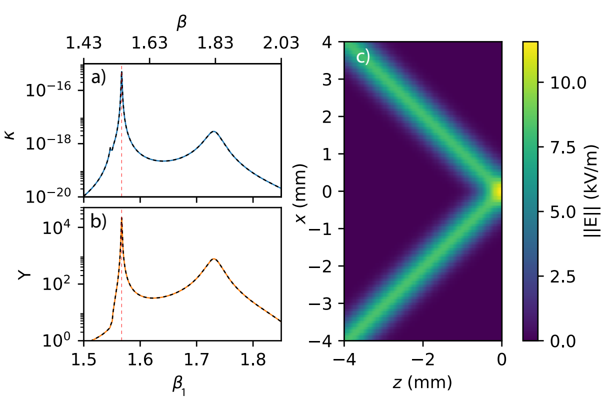

Fig. 2. The dependence of the SPP-enhanced yield κ (a) and enhancement factor Υ (b) on $\beta_1 = \sin(\theta_1)\, n_p(\lambda_1)$. The electrical field norm of the pump beam is given in (c).

This study focus to the process of “fpp”, which means that pump photon splits to two due to the plasmons (vacuum fluctuations in plasmonic mode) and another plasmon is generated. Such process, also studied in Ref. [2], is potentially highly enhanced, as two participating waves are plasmonically enhanced. To model such process, the second input beam is tuned to the plasmonic resonance. In current study, only equal splitting is studied, thus the wavelength of the fluctuations is fixed to $\lambda_2 = 532\,\mathrm{nm}$. The amplitude of the vacuum fluctuations are set from the knowledge, that the yield of SPDC in case of phase matched $3\,\mathrm{mm}$ long BBO crystal is in order of 10^{-12} [1], [7].

The results of SPP-enhanced SPDC calculations as DFG is presented Fig. 2. In Fig. 2b the total field enhancement of SPDC is shown and it is up to $\Upsilon = (\eta_1\eta_2\eta)^2 \approx 40 \cdot 10^3$ and accounts for field enhancement (η) of all participating fields. In Fig. 2a the yield of SPDC κ is shown and it clearly follows the same trends as the field enhancement factor Υ. It means, that the enchantment of SPDC through plasmonic fields enhancement works really well. However, the yield itself is low in comparison to the 10^{-12} from the ordinary phase-matched SPDC. Reasons for low yield of SPDC might be low out-coupling of the generated plasmons (i.e generated plasmons are absorbed), non-perfect phase-matching (i.e. no coherent buildup of the signal), low effective nonlinearity or wrong dimensions of the input beam.

The calculations reveal, that the out-coupling is as high as $\eta_{oc} = 71\,\%$ and is hardly an limiting factor. Also, the effective nonlinearity $\chi^{(2)}_{eff} = 2.5\,\mathrm{pm/V}$ is approximately equal to the half of the maximal value, thus is an expected result, because only part of the fields of SPP-mode is in the nonlinear medium. The main limiting factor turns out to be the coherent buildup of the signal. Although, the real part of the phase-mismatch is zero (i.e phase is perfectly matched), it has nonzero imaginary part $Im\,[\Delta k] = 7700i$ accounting to the losses. It turns out to be the main limiting factor of the generation of the signal, as the coherent buildup is limited by the distance $64\,\mu\mathrm{m}$. It is possible to increase the distance by use of long-range SPPs or dielectric modes (results not shown here).

IV. Conclusions

The study focuses to the modeling of plasmon-enhanced SPDC in classical limit as DFG, where one input beam represents the vacuum fluctuations. To do that, the NLTMM is extended to realistic waves and the code is made publicly available at github.com/ardiloot/NonlinearTMM. In addition to the field enhancement effect, also studied before, the out-coupling efficiency, coherent buildup via phase-matching, effective nonlinearity and beam width effects are studied. It is revealed, that the main limiting factor is the coherent buildup of the signal, despite the real part of the phase-mismatch is zero, it has imaginary part limiting the buildup.

Acknowledgment

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