# Investigation for the Quantum Goos-Hänchen Spatial Shift and Angular Shift of Reflected Guided-Mode

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*Abstract*—In a waveguide corner mirror structure the quantum properties of Goos-Hänchen (GH) spatial and angular shifts of reflected guided-mode are theoretically demonstrated at first. Then, the quantum jumps of both spatial shift and angular shift, and their relationship are modelled. Finally, with a silicon-oninsulator (SOI) waveguide corner mirror (WCM) structure both the quantum spatial shift and quantum angular shift are simulated and analyzed, and their intrinsic relationship is found, and with a tapered multimode interference (MMI) structure and free-carrier dispersion (FCD) based optic refractive-index (RI) modulation the total displacement of over 3-wavelengths for is implemented via a ~10<sup>19</sup> cm<sup>-3</sup> concentration variation.

Keywords: quantum spatial shift, quatum angular shift, SOI waveguide-based WCM; FCD RI modulation, total displacement

## I. INTRODUCTION

Demand for next-generation optical networks capable of high-capacity, high-speed and high-agility has been driving the research and development of new ultrahigh-speed optical switches and matrix switches as the key building blocks [1]. Silicon-on-insulator (SOI) waveguide based photonic integrated circuits (PIC) with the free-carrier dispersion (FCD) optical modulation has been accepted to be an essential technology for the next-generation optical networks since it has a high compatibility with the micro-electronic infrastructure and its ultrahigh-speed FCD-based refractive index modulation (RIM). Thereby, the SOI-PIC technology has been attracting the intensive research in the field of PIC components and applications in the past decade [3].

In 2013 we theoretically demonstrated this quantum effect of the GH spatial shift with the consistent solutions of Maxwell equation and Schrödinger equation, and then proposed a digital electro-optic (EO) switching regime on the SOI-PIC platform where a new MOS-capacitor type EO modulation method was also proposed to realize an interface FCD-based RIM [4]. In this work, with waveguide corner mirror (WCM) structure we first investigate the coherence of the spatial shift and angular shift of the reflected guidedmodes, then analyze the quantum processes of GH shifts responding to an incident angle with the eigenstates of guidedmodes of a multimode interference (MMI) structure, furthermore simulate the distribution of the total displacement versus the concentration change of free-carrier holes.

## II. GOOS-HÄNCHEN SPATIAL AND ANGULAR SHIFTS

# A. Analysis for all the guided-modes in the GH reflection

A few impressive establishments in research on GH shifts of guided-mode have shown an angular shift of reflected guided-mode, namely, the reflection angle of guided-mode is no longer equal to the incident angle, which was first found by C. C. Chan and T. Tamir in 1980's, then after passing about 30 years, until 2009 this angular shift of reflected beam was again investigated by A. Aiello [5,6]. Figure-1 shows the relationship of all three beams – the incident mode, reflected mode and transmitted mode at the interface between two media having the refractive indices  $n_1$  and  $n_2$ . The GH spatial shift  $\Delta$  and angular shift  $\Theta$  that are determined by the optical phase of reflected guided-mode should be defined by Eq. (1):

$$\Delta = \frac{1}{k_{in}} \frac{\partial \phi}{\partial \theta} = \operatorname{Im}(\ln r), \Theta = -\frac{2}{(k_{in}^2 \cdot w_o^2)} \frac{\partial R}{\partial \theta} = \operatorname{Re}(\ln r) \quad (1)$$



Figure 1 Schematic relations of the incident, reflected and transmitted modes at the interface and the concepts of both GH spatial and angular shifts

## B. Demonstrations for the two quantum GH shifts

In Fig. 1, the optical reflected guide-mode must meet the independent property, implying  $N_{eff}$  is only the eigenvalue of single-mode. Thus, if  $\varepsilon_r(x)$  is the relative dielectric constant of waveguide material, the Maxwell wave equation for the TE/TM-mode are defined by (2a) and (2b), respectively: [7]

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$$2j\beta \frac{\partial \phi(x,z)}{\partial z} = \frac{\partial^2 \phi(x,z)}{\partial x^2} + k_0^2 (\varepsilon_r^2 - N_{eff}^2) \phi(x,z)$$
(2a)  
$$2j\beta \frac{\partial \phi(x,z)}{\partial z} = \varepsilon_r \frac{\partial}{\partial x} (\frac{1}{\varepsilon_r} \frac{\partial \phi(x,z)}{\partial x}) + k_0^2 (\varepsilon_r^2 - N_{eff}^2) \phi(x,z)$$
(2b)

Similarly, if U(x) and m(x) stand for the arbitrary potential and the effective mass, respectively, with the effective mass approximation and the Plank constant  $\hbar$ , the wave function can be expressed by a Schrödinger equation as

$$j\hbar \frac{\partial \psi(x,t)}{\partial t} = -\frac{\hbar^2}{2} \frac{\partial}{\partial x} \left(\frac{1}{m(x)} \frac{\partial \psi(x,t)}{\partial x}\right) + U(x)\psi(x,t)$$
(3)

(2) and (3) can yield the correspondences as

$$\frac{\partial^2}{\partial x^2} \leftrightarrow -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \text{ for TE-mode}$$
(4a)

$$\mathcal{E}_r \frac{\partial}{\partial x} \left( \frac{1}{\mathcal{E}_r} \frac{\partial}{\partial x} \right) \leftrightarrow -\frac{\hbar^2}{2} \frac{\partial}{\partial x} \left( \frac{1}{m(x)} \frac{\partial}{\partial x} \right)$$
 for TM-mode (4b)

Where the parameters in Maxwell and Schrödinger equations have a correspondence as  $k_0^2(\varepsilon_r^2 - N_{eff}^2) \leftrightarrow U(x)\psi(x,t)$ . It turns out that the spatial and angular shifts for either TE or TM mode are quantum selections of eigenstates. If an MMI structure having a length  $L_{mm}$ , the total displacement is:

$$S_{T} = (\Delta + L_{mm}\Theta) / \cos\theta \tag{5}$$

# III. SIMULATION FOR GH SPATIAL AND ANGULAR SHIFTS

 $W_r = 350nm$  with Eq. (1) we simulate the corresponding dependences of both the GH spatial shift and angular shift on the incident angle as shown in Fig. 2. Note that there is a common linear area of incident angle from  $38.91^\circ$  to  $38.95^\circ$ .



Figure 2. The incident angle dependences of the GH spatial and angle shifts for 350nm rib width in which the characteristics of GH shifts are categorized

With (5) and a tapered MMI structure having an input width 2.5µm and a length  $L_{1m} \approx n_1 W_i^2 / \lambda_0$ , we simulate the relationship between the total displacement at the MMI end and the hole carrier concentration (HCC) variance as shown in Fig. 3. The total displacement is sensitive to the FCD-RIM.



Figure 3. Dependence of the total displacement on the HCC for waveguide width of 350nm in the moderate response area with respect to three different incident angles where the input end width of MMI waveguide is 2.5µm.

### IV. CONCLUSION

We can conclude that the inherent relationship and the mutual influence between the GH spatial shift and angular shift are dependent of the incident angle based on a quantum process. The two shifts contribute to the final total displacement of the reflected beam. Thus, with the high-speed FCD RIM, this phenomenon can realize a digital optical switching function.

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