# Graphene Multilayers for Quantum Microwave Signal Up-Conversion to the Optical Domain

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Abstract—We propose a new scheme for tunable quantum microwave signal upconversion to the optical frequency domain using graphene multilayers. The graphene layers are electrically connected, biased by a quantum level microwave signal, and subjected to intensive optical pump. The principle of operation is based on modifying the graphene conductivity by the biasing microwave signal. It then follows that an upper optical sideband is generated, while the lower sideband is resonantly attenuated by the destruction of the layered medium . Our calculations show large number of upconverted photons for microvolt driving microwave signal.

## I. INTRODUCTION

Combining quantum optical and microwave devices is indispensable for future quantum systems, enabling efficient quantum computations, large bandwidths, as well as transmitting quantum information over long distances. Recently, several proposals are reported to achieve such integration using electrooptic (i.e., EO)- based techniques [1]. However, given the very small EO coefficients, the required driving voltages are large (in millivolts) even using best existing EO materials. This is impractical in quantum systems. Furthermore, limited tunability is usually achieved in such techniques, as resonators are typically incorporated to enhance the weak EO effects [1].

In this work, a graphene multilayers is proposed for quantum microwave upconversion to the optical frequency domain. Thanks to the dispersion of the graphene multilayers and to the graphene conductivity, it is demonstrated that large number of upconverted photons can be obtained for few microvolt biasing microwave signal. Moreover, a frequency tunable upconversion process is attainable by controlling the optical pump frequency.

### **II. PROPOSED STRUCTURE**

The proposed structure is compressed of interdigital electrically connected graphene layers, as shown in Fig. 1. A microwave signal  $\nu_m$ , of frequency  $f_m$ , is biasing the graphene layers while an optical input pump of frequency  $f_1$  is normally subjected. For N layers, the structure can be seen from electrical perspective as shunted 2N - 2 identical capacitors with an equivalent capacitance of  $C_T = \frac{(2N-2)\epsilon_0\epsilon}{d}$ . While from optical standing point, the structure can be seen as periodic layers with dispersion relation given by  $\cos(d\beta) = \cos\left(d\sqrt{\epsilon}\frac{2\pi f}{c}\right) - i\frac{Z_0}{2\sqrt{\epsilon}}\sin\left(d\sqrt{\epsilon}\frac{2\pi f}{c}\right)\sigma_s$ , where  $\sigma_s$  is the graphene conductivity.



Fig. 1. The proposed graphene multilayers structure.

## A. Perturbation Analysis

Consider a microwave signal,  $v_m = \nu e^{-i2\pi f_m t} + c.c.$ , biasing the graphene layers. For weak amplitude, such that  $\nu \ll \frac{q\pi n_0}{2C_T}$ , the graphene chemical potential can be approximated (up to the first order) by  $\mu_c = \mu'_c + \nu \mu''_c e^{-i2\pi f_m t} + c.c.$ , where  $\mu'_c = \hbar V_f \sqrt{\pi n_0}$ , and  $\mu''_c = \hbar V_f \frac{C_T}{q\sqrt{\pi n_0}}$ . Here qis the charge of electron and  $n_0$  is the doing concentration. Consequently, for  $\nu \mu''_c \ll \mu'_c$ , the graphene conductivity is approximated by [2]:

$$\sigma_s = \sigma'_s + \nu \sigma''_s e^{-i2\pi f_m t} + c.c., \tag{1}$$

where 
$$\sigma'_s = \frac{iq^2}{4\pi\hbar} ln \left( \frac{2\mu'_c - (f+i\tau^{-1})\hbar}{2\mu'_c + (f+i\tau^{-1})\hbar} \right) + \frac{iq^2K_BT}{\pi\hbar^2(f+i\tau^{-1})} \left( \frac{\mu'_c}{K_BT} + 2ln \left( e^{-\frac{\mu'_c}{K_BT}} + 1 \right) \right)$$
, and  $\sigma''_s = \frac{iq^2}{\pi\hbar} \frac{(f+i\tau^{-1})\hbar}{4(\mu'_c)^2 - (f+i\tau^{-1})^2\hbar^2} \mu''_c + \frac{iq^2K_BT}{\pi\hbar^2(f+i\tau^{-1})} tanh \left( \frac{\mu'_c}{2K_BT} \right) \frac{\mu''_c}{K_BT}$ . It then follows that, using the graphene conductivity in Eq.(1), the propagation constant also can be by approximated up to the first order. Thus, the effective permittivity of the graphene multilayers can be given by [3]:

$$\varepsilon_{eff_j} = \varepsilon'_{eff_j} + \nu \varepsilon''_{eff_j} e^{-i2\pi f_m t} + c.c., \tag{2}$$

where  $\varepsilon'_{eff_j} = \left(\frac{\beta'_j}{k_{0_j}}\right)^2$ ,  $\varepsilon''_{eff_j} = 2\frac{\beta'_j\beta''_j}{k_{0_j}^2}$ ,  $\beta'_j$  is the solution of the dispersion relation, and  $\beta'' = i\frac{Z_0}{2d\sqrt{\varepsilon}}\frac{\sin\left(d\frac{2\pi f\sqrt{\varepsilon}}{c}\right)}{\sin(d\beta')}\sigma''_s$ .

# B. Qunatum Mechanics Analysis

The optical fields in the medium include the input pump at frequency  $f_1$  and the upper and lower sidebands at frequencies  $f_2 = f_1 + f_m$  and  $f_3 = f_1 - f_m$ , respectively. The associated electric fields are given by  $\vec{E}_k = u_k (e^{-i2\pi f_k t + i\beta'_k x} + c.c.)\hat{e}_y$ , where  $k \in \{1,2,3\}$ . The total energy is given by  $\mathcal{H} = \frac{1}{2} \int_{\mathcal{V}} (\varepsilon_0 \varepsilon_{eff} \vec{E}^{t-2} + \mu_0 \vec{H}^{t-2}) \partial \mathcal{V}$ . Here  $\vec{E}^t = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$ ,  $\vec{H}^t$  is the corresponding magnetic field, and  $\mathcal{V} = \mathcal{A}L$  is the medium's volume. Both the optical and the microwave fields can be quantized through the relations:  $u_k = \hat{a}_k \sqrt{\frac{\hbar f_k}{\varepsilon'_{eff_k} \varepsilon_0 \mathcal{V}}}$ and  $\nu = \hat{b} \sqrt{\frac{\hbar f_m}{C_T \mathcal{A}}}$ , where  $\hat{a}_k$  and  $\hat{b}$  represent the annihilation of the  $k^{th}$  optical and microwave fields, respectively. It then follows that, on substituting the quantum operators into the total energy expression, the quantum Hamiltonian is given by [3]:

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_0 + \hat{\mathcal{H}}_1, \tag{3}$$

where  $\hat{\mathcal{H}}_0 = \hbar f_m \hat{b}^{\dagger} \hat{b} + \hbar f_1 \hat{a}_1^{\dagger} \hat{a}_1 + \hbar f_2 \hat{a}_2^{\dagger} \hat{a}_2$ ,  $\hat{\mathcal{H}}_1 = \hbar g (\hat{a}_2^{\dagger} \hat{b} \hat{a}_1 + h.c.)$  and  $g = \varepsilon_{eff_2}^{\prime\prime} \sqrt{\frac{f_{1f_2}}{\varepsilon_{eff_1}^{\prime} \varepsilon_{eff_2}^{\prime}}} \sqrt{\frac{f_m}{C_T \mathcal{A}}}$  is the upconversion rate. We note here that the optical pump is considered intensive and treated classically. Also, the lower sideband is considered resonantly attenuated (suppressed) by the layered medium destruction; achieving a low noise upconversion [3].



Fig. 2. The upconversion rate against the microwave signal frequency.

## **III. RESULTS**

In the following numerical evaluations,  $f_3 = 193.5484$  THz,  $d = \frac{c}{f_3} = 1.55 \mu m$ , T = 3mK, air is the filling material,  $\mathcal{A} = 10^{-4}m^2$  and  $n_0 = 10^{12}m^{-3}$ . While  $f_3$  (the lower side band frequency) is fixed, the pump optical frequency  $f_1$  is varied according to the microwave frequency  $f_m$ .

In Fig. 2, the upconversion rate is presented against the microwave frequency. Different microwave voltages are considered. In Fig. 3, the upconversion rate is calculated against the biasing microwave voltage.

To evaluate the evolution of the operators, one may substitute the quantum Hamiltonian of Eq. (3) in the Heisenberg equation  $\frac{\partial \hat{x}}{\partial t} = \frac{i}{\hbar} [\hat{\mathcal{H}}, \hat{x}]$ . Consequently, it can be shown that



Fig. 3. The upconversion rate against the driving microwave voltage.

the motion equations are given by:  $\frac{\partial \hat{A}_2}{\partial t} = -\frac{\Gamma}{2}\hat{A}_2 - igA_1\hat{B} + \sqrt{\Gamma}N_2$  and  $\frac{\partial \hat{B}}{\partial t} = -\frac{\Gamma_m}{2}\hat{B} - igA_1^*\hat{A}_2 + \sqrt{\Gamma_m}N_m$ . Here,  $\Gamma$  and  $\Gamma_m$  are the optical and microwave decay coefficients, respectively, and  $N_2$  and  $N_m$  are the noise operators. The number of upconverted photons can be calculated by solving these motion equations. In Fig. 4, the mean number of upconverted photons are shown agianst the microwave frequency. Different number of layers are considered.



Fig. 4. The mean number of upconverted photons against the microwave frequency.

#### **IV. CONCLUSION**

A novel modality using graphene multilayers is proposed for quantum microwave upconversion to the optical frequency domain. Our calculations show that large number of photons can be upconverted over vast microwave frequency range.

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