Green's function integral equation methods for modeling of optical devices

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***Abstract*—Green's function integral equation methods are presented that can be applied for modeling of optical devices in cases where the problem can be formulated as a scattering problem. The methods are applied to study in three dimensions the effect of a cylindrical micro-lens on radiation emitted from a THz photoconductive antenna, and for studying the effect of scatterers on the front-side of thin-film silicon solar cells with the aim of increasing the solar cell efficiency.**

I. Introduction

Green's function integral equation methods (GFIEMs) in optics or electromagnetics are concerned with finding solutions to Maxwell's equations by solving integral equations [1], and are suitable for modeling of optical device problems that can be formulated as a scattering problem. The starting point is a reference structure, where the properties are governed by a Green's function or Green's tensor, $\mathrm{G}(\mathrm{r},\mathrm{r}')$, which gives (in this paper) the electric field at a position $\mathbf{r}$ due to a point source at a position $\mathbf{r}'$. In addition, a known reference field, $\mathrm{E}_0(\mathrm{r})$, is considered, which is a solution for the electric field in the reference structure. This field may, for example, correspond to the field generated by an antenna or any given current sources, or it can be the resulting field solution when the reference structure is illuminated e.g. by a plane wave or a Gaussian beam. The GFIEMs consider the case where the reference structure is modified by introducing a scattering object. For positions outside the scatterer the methods give the resulting total field as the sum of the reference field and a scattered field. The Green's function can be chosen to satisfy for example the radiating boundary condition or other boundary conditions exactly. The computational domain is often very small since it is sufficient to discretize either the inside or the surface of the scattering object depending on whether volume integral equation methods or surface integral equation methods are considered. Contrary to other popular methods, such as the Finite-Element-Method or Finite-Difference-Time-Domain method, it is not necessary to discretize a region outside the scatterer, and not necessary to use additional resources on boundary conditions. For example, you do not need a perfectly matched layer since boundary conditions are already taken care of via the choice of Green's tensor.

In this short paper we consider GFIEMs for two examples of optical device problems, namely for studying the effect of a cylindrical micro-lens on the emission from a THz photoconductive antenna [2], and for studying the optics of thin-film silicon solar cells with scatterers on the front-side [1, 3]. In the first case the lens (scatterer) can be designed to reduce coupling of radiation into guided modes of a thin film, and thus improve the extraction efficiency, and further it can be designed to also collimate the radiation into a pencil-beam. In the latter case the purpose of the scatterer is the opposite, since here coupling of light into guided modes of the thin-film geometry should be increased instead in order to improve the absorption of light in the thin-film solar cell.

II. Cylindrical micro-lens for THz photonics

As a first example we consider a geometry from Ref. [2] (see schematic in Fig. 1) with a THz photoconductive antenna placed on the back-side of a semiconductor slab of thickness $t \approx 400\ \mu$m and refractive index 3.418 (silicon). The width of the slab is treated as infinite. For such a reference geometry the Green's tensor is known analytically in terms of certain integrals [1]. The antenna is modeled simply as an oscillating electric dipole with dipole moment pointing in the x-direction and with a frequency of 1 THz (wavelength 300 μm). In that case the reference field is itself directly given from the Green's tensor with $\mathbf{r}'$ at the position of the dipole. On top of the slab a cylindrical micro-lens lens is placed consisting of a bottom semiconductor cylinder with radius r and thickness h, and with another glass cylinder on top of the same radius and thickness a. The glass layer gives an antireflection effect.

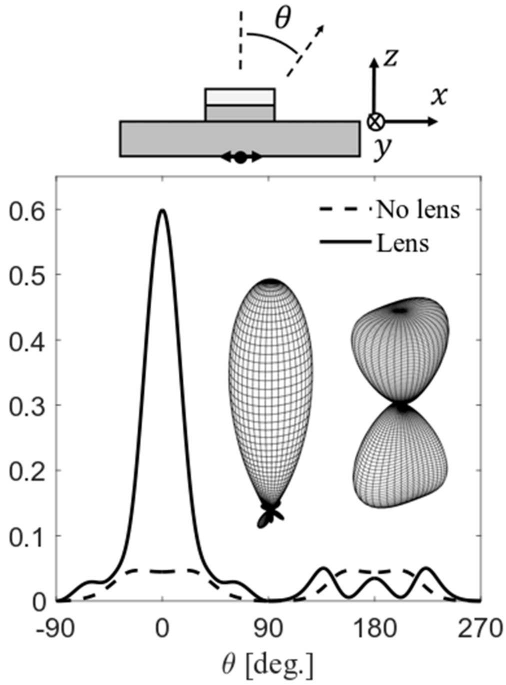

Fig. 1. Top: Schematic of a dipole antenna (1 THz) and a cylindrical micro lens on front- and backside, respectively, of a semiconductor slab of thickness app. 400 μm. The lens consists of a semiconductor cylinder of radius 200 μm and thickness 55 μm, and with a glass cylinder on top of thickness 40 μm. Bottom: radiation patterns for the cases with and without the lens.The total emitted power including also into guided modes is normalized to unity.

In the absence of the lens the geometry consists of only the semiconductor slab (dielectric constant $\boldsymbol{\varepsilon}_{\mathrm{ref}}(\mathbf{r})$). As the lens (scatterer) is added the resulting total electric field, $\mathrm{E}(\mathrm{r})$, can, for example, be found by solving the volume integral equation

$$\mathrm{E}(\mathrm{r}) = \mathrm{E}_0(\mathrm{r}) + \int \mathrm{G}(\mathrm{r},\mathrm{r}') k_0^2 \big(\boldsymbol{\varepsilon}(\mathrm{r}') - \boldsymbol{\varepsilon}_{\mathrm{ref}}(\mathrm{r}')\big) \cdot \mathrm{E}(\mathrm{r}') d^3 r', \tag{1}$$

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where $\boldsymbol{\varepsilon}(\mathbf{r})$ is the dielectric constant of the total structure including lens and slab, and k_0 is the free-space wave number corresponding to 1 THz. The integral equation (1) expresses the total field as the sum of the reference field and a scattered field, where the latter propagates away from the scatterer (the lens in this case) due to the properties of the Green's tensor. Note that Eq. (1) can be solved by considering at first only positions $\mathbf{r}$ that are inside the lens, since $\left(\boldsymbol{\varepsilon}(\mathbf{r}') - \boldsymbol{\varepsilon}_{\text{ref}}(\mathbf{r}')\right)$ vanishes for positions $\mathbf{r}'$ that are outside the lens. The computational domain is thus very small with this method. In addition, it is possible to exploit cylindrical symmetry when solving Eq. (1) and effectively reduce the computational problem from a 3D problem to a 2D problem [1]. Alternatively, if a surface-integral equation method is applied instead, and this is combined with taking advantage of cylindrical symmetry, it is even sufficient to discretize in only one dimension [1]. The different variants of GFIEMs will be discussed in more detail at the conference.

Once the electric field has been calculated inside the lens Eq. (1) can be applied to calculate the field at any other position by direct integration. In particular, the far-field, and thereby also the Poynting vector in the far-field region, are easily obtained by using a simple analytic far-field approximation to the Green's tensor [1]. This leads to a calculation of the differential emission being the emission per unit solid angle for different directions in the far-field. The differential emission from the antenna is shown in Fig. 1 for a case with and without a lens both as 3D radiation patterns and versus the angle θ in the *yz*-plane. In both cases the total radiated power from the antenna, which also includes radiation trapped in the semiconductor slab, is normalized to unity. The lens as designed in Ref. [2] is extremely compact with a radius of 200 μm and a total thickness of 95 μm.

In the lens geometry the emission is predominantly going into directions near the z-axis in the upper half-plane (pencil beam), while in the case without a lens almost the same amount of radiation goes into the lower half-plane as into the upper half-plane. In the case without a lens the shape of the radiation pattern is, however, rather sensitive to the slab thickness. Without the lens a much higher fraction of emitted radiation will be trapped in guided modes in the semiconductor slab, which explains the small differential emission in that case [2]. In addition, the lens is seen to collimate the light.

III. SCATTERING INTO A THIN-FILM SILICON SOLAR CELL

The second example that we will consider is a 50 nm layer of (amorphous) silicon on silver. This represents from an optical point of view a thin-film silicon solar cell. A cylindrical silicon scatterer of radius 30 nm and height 25 nm is placed on top of the solar cell (schematic in Fig. 2). The structure is illuminated by a normally incident plane wave, which leads to scattering out-of-the plane, and scattering into guided modes of the silicon-on-silver geometry. This problem can also be modeled using Eq. (1). The excitation of guided modes is calculated by dividing the far-field Green's tensor into components that govern the excitation of s- and p-polarized guided modes and out-of-plane propagating radiation [1]. In order to make a clean calculation of the contribution from each mode the absorption in silicon and silver is neglected. Light can be scattered into guided modes with both *s*- and *p*-polarization. For the present case the *p*-polarized guided mode is a type of mode bound to the silicon-silver interface, which is known as a surface-plasmon-polariton (SPP). From Fig. 2 it is clear that the main contribution to the scattering peak near the wavelength of 1000 nm is due to the scattering of light into the SPP. The *s*-polarized guided mode is an ordinary waveguide mode, which only exists for wavelengths below app. 1000 nm. Scattering into this mode is much smaller compared with the SPP (in this example). The scattering cross section (Fig. 2) is scattered power normalized with incident power per unit area.

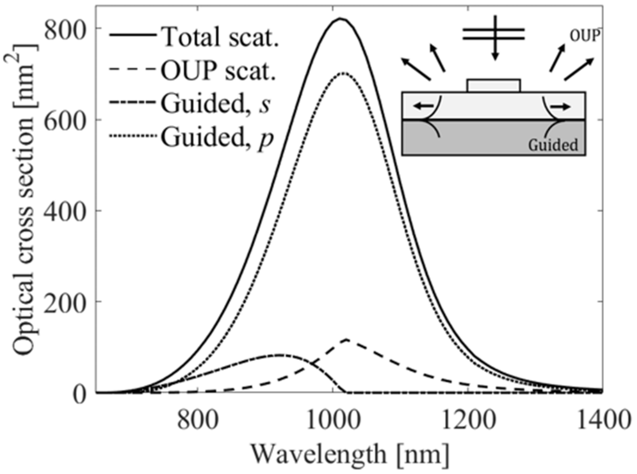

Fig. 2. Scattering cross section spectrum for light normally incident on a geometry with a cylindrical scatterer (radius 30 nm and height 25 nm) on top of a 50-nm-silicon-film-on-silver waveguide. The scattering cross section is divided into contributions *s*- and *p*-polarized guided modes, and out-of-plane scattering.

The scattering peak nearly coincides with the cut-off-wavelength of the *s*-polarized guided modes, and the wavelength of the peak is thus more a property of the waveguide than a property of the scatterer. The wavelength can be tuned by changing the thickness of the silicon film. The idea is that without a scatterer most of the incident light is reflected back out of the solar cell at wavelengths with weak absorption. However, light scatterered into guided modes will propagate long distances in the solar cell which increases the probability of absorption, which is a possible approach to improve the efficiency of thin-film solar cells.

IV. SUMMARY

To summarize, a range of GFIEMs have been described that can be applied for modeling of optical devices in cases where the problem can be expressed as a scattering problem. The methods were applied for studying radiation patterns from a photoconductive THz antenna and for scattering of light into a thin-film silicon solar cell. Further discussions of GFIEMs and device modeling will be presented at the conference.

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