Steady states in dynamical semiconductor laser models and their analysis

(Invited Paper)

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***Abstract*—We present an algorithm for calculating steady states in the dynamic PDE model for SLs admitting gain compression, spatial hole burning, and multilevel carrier rate equations. Presented example simulations rely on 1(time)+1(space)-dimensional traveling-wave- and Lang-Kobayashi-type models.**

Edge-emitting (EE) semiconductor lasers (SLs) are used in many modern applications requiring specific dynamic or stationary emission characteristics. Modeling and analysis of nonlinear dynamics and calculation of stable and unstable steady states are crucial for understanding SLs [1], [2], predicting lasing characteristics [3], or designing SLs for specific purposes [4]. The complexity of dynamic SL models ranges from 1(time)+3(space)-dimensional (1+3-D) PDEs to simple rate equations. Complex models can give a deep insight into the spatio-temporal dynamics but rely on many not very well-known parameters and are computationally expensive. On the other hand, 1+1-D PDE and even simpler DDE/ODE models maybe lack quantitative precision but can be quickly resolved on standard computers, admit various analytic and semianalytic methods for their analysis, and can be successfully used for comprehensive parameter studies, qualitative analysis, and, thus, simulation and design of novel device concepts.

We seek to calculate and compare steady states in simple DDE and 1+1-D PDE models used to simulate SLs at well-above-threshold regimes. These models mimic dynamics of the complex optical field E and real carrier density N in m "active" sections S_a of the multisection device. Typically, the models for optically uninjected SL can be written as a rotationally-invariant w.r.t. E system

$$\frac{d}{dt}E = H(\beta(N,\epsilon P))E, \qquad \frac{d}{dt}N = \varepsilon\mathcal{N}(I,N,\epsilon P,E), \quad (1)$$

where I is the pump current and the field power P is a real function of E. In DDE and more advanced ODE models, $E(t)$ is defined by a single or several complex components, whereas the m'-dimensional real vector function $N(t)$ represents possibly multilevel carrier dynamics in different active sections. In the 1+1-D PDE model case, $E(z,t)$ is a (vector-) function determining field distribution along the entire SL device cavity. The vector function N can also depend on longitudinal coordinate z within corresponding S_a. In simpler approaches, one can neglect spatial hole burning and consider sectionally-uniform components of N and section-wise constant β and P. In this case, the carrier rate equations for ODE/DDE and PDE models are nearly identical. The operator H governs the optical fields and accounts for their delay or spatial derivatives and boundary/interface conditions in DDE and PDE cases, respectively. $\mathcal{N}$ is a real vector function with m'-components in both approaches. A (typically small) parameter ε represents the ratio of the photon and carrier lifetimes and indicates a slow-fast nature of the SL model, widely exploited for model analysis [1]. The gain compression factor $\epsilon \ll 1$, changes of β are slow, and the spectra of $H(\beta)$ calculated at some instant β provides useful information about dynamics of optical fields [5], [4]. Probably the most prominent representatives of DDE models for SLs are Lang-Kobayashi (LK)-type systems, originally used for lasers with delayed feedback. Different versions of the 1+1-D PDE traveling wave (TW) model [5] are used for simulation and analysis of spatio-temporal dynamics in an even larger variety of multisection SLs.

Each steady (or continuous wave, cw) state is defined by time-independent $\bar\beta$, $\bar P$, complex vector $\bar E$, real vector $\bar N$, and real optical frequency $\bar\omega$. To find cw states, one has to insert the ansatz $(\bar E e^{i\bar\omega t}, \bar N)$ into Eq. (1) and resolve the resulting system w.r.t. $\bar E$, $\bar N$, and $\bar\omega$. This procedure for ODE/DDE models, together with a rotational-invariance condition, leads to a system of algebraic equations whose roots are defining all stable and unstable cw states. In LK-type models, these states are known as external cavity modes, ECMs. The situation in the PDE model case is more tricky. Here, after the elimination of the time variable, we still deal with the system of algebro-differential equations

$$i\bar\omega\bar\Theta(z) = H(\beta(\bar N,\epsilon P))\bar\Theta(z), \quad (2)$$

where complex vector-eigenfunction $\bar\Theta$ provides a longitudinal distribution of the field amplitude $\bar E$, scaled by a complex factor $\bar f$. In the TW-modeling case [5] considered in the rest of this work, the initial and end values Θ_0 and Θ_L are (up to scaling factors) determined by the field reflection conditions at the cavity edges, $z=0$ and $z=L$. For any fixed $\beta(z)$ and ω, Eq. (2) is mainly determined by a couple of *linear* w.r.t. Θ first-order ODEs, which, at least for a piece-wise constant β, can be resolved using transfer matrices M. Thus, $\Theta(z)$ itself can be understood as a function of $\beta(z)$ and ω:

$$\Theta(z) = M(\beta,\omega)\Theta_0 = F_\Theta(z;\beta,\omega). \quad (3)$$

To match the reflecting conditions at the other end of the device, $z=L$, i.e., to satisfy the steady-state condition, one should find $\bar\beta$ and $\bar\omega$ solving a complex characteristic equation generated by the transfer matrix over the whole cavity:

$$F_\Theta(L;\bar\beta,\bar\omega) \propto \Theta_L \quad \Rightarrow \quad \chi(\bar\beta,\bar\omega) = 0, \quad (4)$$

where $\propto$ denotes proportionality of two vectors, and $\bar\beta$ is a function of, in general, z-dependent $\bar N$ and $\bar P$:

$$\bar\beta(z) = \beta(\bar N, \epsilon\bar P). \quad (5)$$

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$\bar{P}(z) = P(z', z'')$ represents the power of the field function $\bar{E} = \bar{f}\bar{\Theta}$ averaged over the interval $[z', z'']$ that contains z. In the simple TW model with sectionally-averaged N, β, and P, these intervals correspond to each of m sections S_a. In general case, $z' = z'' = z$, and $\bar{P}$ is the field power at each z of these S_a. For estimation of $P(z', z'')$ it is enough to know $\bar{\Theta}(z')$, $\bar{\Theta}(z'')$, $\bar{\omega}$, and $|\bar{f}|^2$. Thus, $\bar{P}(z)$ can be written as

$$\bar{P}(z)|_{z\in[z'z'']} = P(|\bar{f}|^2, \bar{\beta}, \bar{\omega}; z', z''). \qquad (6)$$

Finally, m' real carrier rate equations in (1) along with the rotational invariance implies further m' real relations,

$$\mathcal{N}(I, \bar{N}, \epsilon\bar{P}, |\bar{f}|F_{\Theta}(z; \bar{\beta}, \bar{\omega})) = 0. \qquad (7)$$

By inserting $\bar{\beta}$ from Eq. (5) into Eqs. (4), (6), and (7), we get a system of equations determining $\bar{\omega}$, $|\bar{f}|$, $\bar{P}$, and $\bar{N}$. For simple models relying on sectionally averaged N, P, and β, this system is equivalent to $2+m+m'$ real equations relating the same number of real factors $\bar{\omega}$, $|\bar{f}|$, P, and $\bar{N}$. In the general case, $\bar{\beta}$, $\bar{P}$, and $\bar{N}$ within each S_a are z-dependent, and Eqs. (5), (6), (7) are functional relations. Thus, we subdivide all S_a into m_D smaller subsections (e.g., 200 steps pro 1 mm-long all-active SL) and assume that N, β, and P are constant within each subsection. If $\bar{N}$ is defined by m'_D real numbers, the system to be considered is equivalent to $2 + m_D + m'_D$ real equations and relates the same number of real variables. For the numerical solution [6] of this system, we use Newton's iterations and the Homotopy method.

The algorithm for location of the cw states in general TW model was developed during the recent study of SL emission's linewidth in external-cavity diode lasers [3]. In many applications, however, simplified TW or even more simple DDE (such as LK-) models are sufficient. When spatial hole burning and gain compression are not important ($\epsilon = 0$), and only a single carrier rate equation per active section is used, the steady-state defining system is reduced to one complex and m real equations (4), (7), together defining real m-component vector $\bar{N}$ and real factors $\bar{\omega}$ and $|\bar{f}|^2$. Fig. 1 illustrating cw state

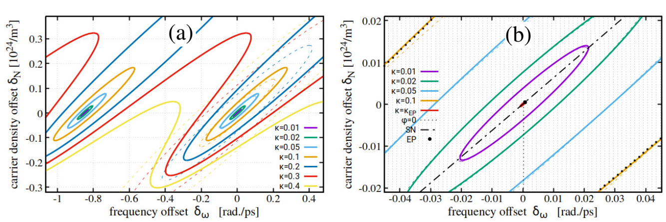

Fig. 1. Steady states in a FP laser with optical feedback in relative frequency-carrier density domain. (a): States for several feedback levels κ and arbitrary phase φ close to two resonances of the solitary SL. Solid and thin dashed: TW and LK models, respectively. (b): Same states for several $\kappa \le 0.1$ (solid) and fixed φ but arbitrary κ (dotted) in vicinity of the solitary SL resonance. Thick dots at $\kappa = 0.01$ curves: cw states for $\varphi = 0$. Black dash-dotted: saddle-node bifurcation. Thick black bullet at $\kappa = \kappa_{EP} \approx 3.8 \cdot 10^{-4}$: exceptional point.

calculations in the simplest nontrivial device, a Fabry-Perot (FP) laser ($m = 1$) with optical feedback from the external mirror, uses a simple TW and corresponding LK models. Steady states are defined by real-valued triples $(\bar{\omega}, |\bar{f}|^2, \bar{N})$ ($|\bar{f}| \equiv \bar{E}$ in the LK case). Let $\kappa e^{i\varphi}$ be the ratio of emitted and back-reflected delayed field amplitude just outside the front facet (κ and φ: feedback level and phase shift). By setting $\kappa = 0$ (vanishing feedback) and resolving Eq. (4) (which is a standard roundtrip condition for FP lasers), we locate an infinite number of pairs $(\bar{\omega}_s, \bar{N}_s)$ determining steady states of the solitary laser with the same $\bar{N}_s$ and $(2\pi/\tau_{FP})$-separation of adjacent state frequencies (τ_{FP}: field roundtrip time in the FP diode). Fig. 1 shows the surrounding of a pair of such solitary FP laser resonances. $\delta_N = \bar{N} - \bar{N}_s$ and $\delta_\omega = \bar{\omega} - \bar{\omega}_s$, denoted on x- and y-axes, are carrier densities and frequencies relative to one of the FP resonances. For FP laser, Eq. (4) splits into two real equations [2], [5],

$$\chi_1(\bar{\beta}, \bar{\omega}; \kappa) = 0, \qquad \chi_2(\bar{\beta}, \bar{\omega}; \varphi) = 0. \qquad (8)$$

For fixed κ, the first equation defines by φ-parametrized curves in the $(\bar{\omega}, \bar{N})$ domain; see colored solid curves in Fig. 1. Parameter φ corresponding to each point of such curve is determined by the second equation in (8). With an increase of κ, these *fixed κ level* curves, initially surrounding all solitary FP resonances, expand until they touch each other at a critical $\kappa_c = \sqrt{R_f} \approx 0.224$ ($R_F = 0.05$: front facet reflection) and split afterward, forming a continuous curve at lower values of $\bar{N}$, as shown in Fig. 1(a) for $\kappa \ge 0.3$. Similarly, for each fixed φ, function χ_2 provides κ-parametrized curves (dotted in Fig. 1(b)). κ for each point on these curves is determined χ_1. Minimal but still positive κ on each but one fixed-φ curve occurs on the dash-dotted line, indicating the creation/annihilation of a saddle- and node-type pair of the cw states. A single fixed-φ curve terminates at the origin, where κ vanishes. Intersections of the *fixed* κ and *fixed* ϕ curves define cw states (ECMs in LK approach). A subset of such states calculated [6] for a TW model with included material gain dispersion [5] for $\kappa = 0.1$ and $\varphi = 0$ is shown by black dots on the orange curve in Fig. 1(b). In more complex lasers (DBR or DFB SLs, for example) representations of Fig. 1, estimation of the critical κ_c at which initially closed fixed-κ loops are splitting, or mode degeneracy or exceptional points (EP) where $\chi(\omega, \bar{N}) = \partial_\omega \chi(\omega, \bar{N}) = 0$ require numerical calculations [6].

In conclusion, we discussed challenges arising in the calculation of (stable and unstable) cw states in the TW model of SLs. In the numerical example, we compared cw states in TW and LK models for the FP laser with optical feedback. Constructed using a first-order approximation of $\chi(\beta, \omega)$ at the solitary SL resonance [7], the LK model is in good agreement with the TW model when feedback is small, and a single-mode operation of the solitary SL is pronounced. For multimode SLs or SLs with κ^2 exceeding the front facet reflectivity, this good agreement is lost, and the usage of the LK model becomes questionable. Even though adding higher-order feedback terms with multiple delays improves the approximation of the fixed-κ loops, this works only up to $\kappa \le \kappa_c$.

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