Numerical simulations of nonparaxial solitons and their interaction dynamics in coupled Helmholtz systems

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***Abstract*—The role of nonparaxiality provides a fertile ground for fabricating miniaturized nanoscale devices. In this work, we examine the existence of nonparaxial solitons in a dimensionless coupled nonlinear Helmholtz system, allowing the propagation of ultra-broad nonparaxial pulses in a birefringent optical waveguide. We analytically obtain a bright soltion solution by using standard Hirota's bilinearization method. Subsequently, we numerically investigate the scattering dynamics of two bright solitary waves by considering the obtained solution as the seed solution.**

***Index Terms*—Nonparaxial soliton, Hirota's bilinear method, split-step Fourier method.**

I. Introduction

Over the past four decades, the formation of vector solitons in birefringent optical media has been receiving widespread attention in nonlinear optics, due to their unique properties including stability and intriguing collision dynamics. Based on the nature of weakly (or strongly) birefringent optical media, it is possible to formulate coupled nonlinear Schorödinger equations (CNLSEs) to govern the propagation of optical pulses through a multimode fiber. The interaction dynamics between these vector solitons have been responsible for the development of many salient features, which include the long-distance high-bit-rate communications and ultra-fast all-optical switches [1]. The nature of the soliton interactions can be classified into coherent interactions (which rely on the relative phases of the interacting soliton at the input) and incoherent interactions (wherein the nonlinear response of the medium is decelerating than random fluctuations in the phase of the interacting soliton). In practice, the nature of inelastic collisions (on the ground of coupled versions of NLSE) has exhibited intriguing features compared to the elastic collisions, including shape-changing (alias energy sharing) collisions.

It is well known that this type of conventional models can be derived mathematically using the slowly varying envelope approximation (SVEA) or paraxial approximation from Maxwell's equations. While the SVEA has been extensively used to obtain many mathematical models supporting different nonlinear optical settings, including NLS-like equation, it fails to arrest the catastrophic collapse in higher dimensional systems as a consequence of losing the delicate balance between linear and nonlinear effects [1], [2]. It could be feasible to arrest the catastrophic collapse of higher dimensional NLS-type systems either by applying higher-order nonlinearities, such as quintic and saturable nonlinearities, or by invoking the inherent nonparaxiality term, where it is possible to ensure the stable propagation of optical pulses even in the higher dimensional NLS-like equations without altering the nonlinear profile. There exist a bunch of studies pertaining to the propagation of nonparaxial solitons in various nonlinear optical media [3] and rigorous investigation of elliptic waves in the coupled version of nonparaxial systems [4]. In a recent study, we have examined the existence and collision dynamics of nonparaxial solitons by using standard analytical and numerical techniques [5]. Despite there exit a number of studies in the single componet NLH systems, obtaining general solutions and analyzing their interaction dynamics in the two-component systems have not been explored yet. Hence, in this work, we first construct one bright solitary wave solution for the system by employing standard Hirota method though the system is a non-integrable one. We then carry out a detailed dynamics of coherent interactions between two bright solitary waves numerically.

II. The Model

Let us first consider the generation of non-slowly varying electric fields in a physical setting of birefringent optical fibers. It can be specified by the following dimensionless coupled equations

$$i\Psi_{j,z} + \Lambda\Psi_{j,zz} + \frac{s}{2}\Psi_{j,tt} + \gamma\Big(|\Psi_j|^2 + \sigma|\Psi_{3-j}|^2\Big)\Psi_j = 0. \tag{1}$$

Here, $\Psi_j, j = 1, 2$, denote the orthogonally polarized components of the optical modes and the parameters z, and t, respectively, represent longitudinal and transverse co-ordinates. The second parameter (Λ) in Eq. (1), is a nonparaxial parameter (NP) and it can vary from 10^{-2} to 10^{-4}. The term s indicates the value of group-velocity dispersion (GVD) and in this work it is assigned to be operating in the anomalous dispersion regime. The self- phase modulation and cross-phase

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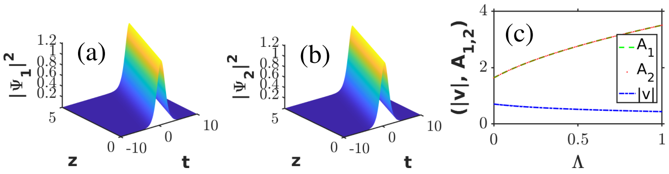

Fig. 1. (Color online) Propagation of solitary waves for (a) first and (b) second components, and (c) depicts the parametric plot between Λ and speed of the two components and first and second components of solitary wave versus nonparaxial parameter for the CNLH system. The parameters are $\alpha_1 = \alpha_1^* = \beta_1 = \beta_1^* = 1.5, \Lambda = 0.01$, and $\gamma = z = b_R = -b_I = 1$.

modulation parameters are, respectively, symbolized through the parameters γ and σ.

III. Nonparaxial solitons and their interactions

To obtain an analytical solution to the CNLH system, we use Hirota's standard bilinearization method. In order to proceed with a regular approach, we assume $\sigma = 1$ and we adopt the following rational solution form as $\Psi_j(z,t,x) = \frac{g^{(j)}(z,t,x)}{f(z,t,x)}$, $j = 1, 2.$, where g is a complex function and f is a real function. Substituting the above solution into Eq. (1), and after some mathematical manipulation, one can obtain

$$\Psi_j = A_j \; e^{i\eta_I} \; \text{sech}\left(\eta_{1R} + \frac{R}{2}\right), \; j = 1, 2., \qquad (2)$$

where the parameters $\eta_{1R}, \eta_{1I},$, R, and A_j are expreesed as $a_r t + b_r z$, $a_i t + b_i z$, and $2\log\sqrt{\frac{\gamma(\alpha_1\alpha_1^* + \beta_1\beta_1^*)}{(8\Lambda b_r^2 + 4a_r^2)}}$. Here the a_I and a_R can be determined by $\sqrt{-b_i + \Lambda(b_r^2 - b_i^2) \pm \sqrt{(b_r^2 + b_i^2)[1 + 2\Lambda b_i + \Lambda^2(b_r^2 + b_i^2)]}}$ and $-\frac{b_r(2\Lambda b_i + 1)}{2a_i}$, respectively. From Eq. (2), one can find the phase and amplitude as $a_i(t + \frac{b_i}{a_i}z)$ and $v = \frac{a_i}{(-2\Lambda b_i - 1)}$, respectively. Also, one can obtain the ideal soliton pulse propagation in both the components of CNLH system as shown in Figs. 1 (a) and (b), respectively. To elucidate the impact of NP on the obtained bright solitary wave for the proposed system (1), we reveal the intensity plots for the first and second components of the bright solitary wave of the system (1) as a function of the NP parameter Λ in Fig. 1 (c), wherein the stable propagation is witnessed. We then numerically study the collision dynamics of bright solitons of CNLH system numerically and use the analytical one bright solitary wave solution (2) as an initial condition by employing the split-step Fourier method based on Feit-Flock algorithm.

When we consider the in-phase solitary waves ($\phi = 0$), both the solitary wave components demonstrate perfectly coherent dynamics, where they display periodically repeated in-phase interaction dynamics that produce oscillating bound solitary waves as shown in Figs. 2 (a) and 2 (d). As the phase shift is tuned to $\pi/2$, its repetition nature of periodic bound solitons gets drastically decreased to almost two in both the components for the same propagation length when compared to the previous, see Figs. 2(b) and (e). The value of phase is

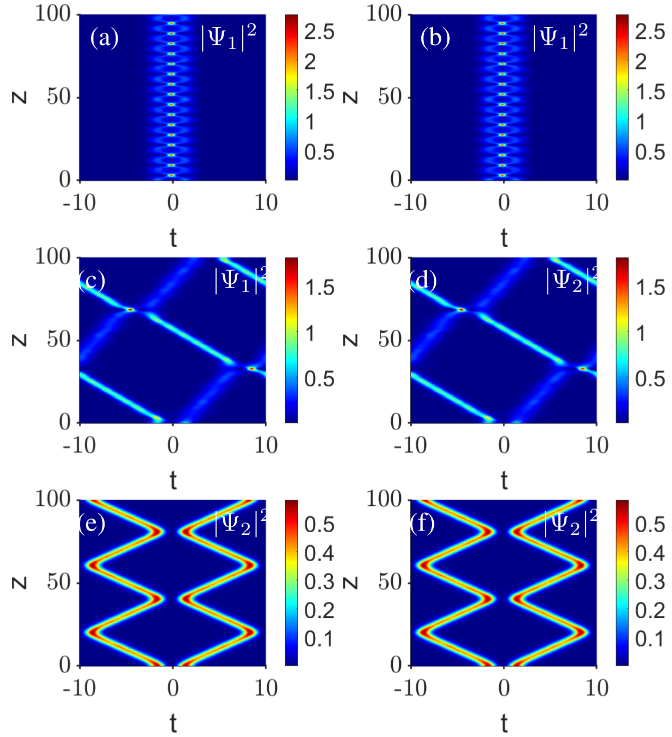

Fig. 2. Symmetric nature of interaction between two solitary waves of (1), including the first components (left panel) and second component (right panel). In this figure, the phase is rendered as $\phi = 0$ in the top panel, $\phi = \pi/2$ in the middle, and $\phi = \pi$ in the bottom panel, respectively. The remaining parameters are: $\Lambda = 0.001$, $\gamma = 1$, $\Delta t_0 = 1.5$, $b1_r = b1_i = 1$, and $\alpha_1 = \alpha_1^* = \beta_1 = \beta_1^* = 1.97$.

then increased to π, and it leads to an interesting interaction nature of two solitons from coherent to perfect incoherent, where the two individual solitons propagate in a zig-zag manner. All these plots are drawn for the similar values of α and β $(\alpha = \beta = 1.97)$.

IV. Conclusion

To conclude, we have constructed bright solitary wave solution for the CNLHE by using standard Hirota's bilinearization method. We have examined the impact of nonparaxiality on the physical parameters, speed and amplitudes of solitary waves and discussed in great detail. Following that we have numerically demonstrated the scattering dynamics of bright solitary waves, including two solitary waves, taking the obtained analytical solution as a seed solution, and emphasized their physical insights.

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