Modal properties of dielectric bowtie cavities with deep sub-wavelength confinement

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***Abstract*—We present a quasinormal mode analysis of a dielectric bowtie cavity with deep sub-wavelength confinement. The cavity - which is based on an inverse design by topology optimization - exhibits a remarkable sensitivity to local shape deformations, which we show to be well described by perturbation theory.**

***Index Terms*—bowtie cavity, subwavelength, optical resonator, light-matter interaction, photonic crystal, Purcell factor**

I. Background

Dielectric bowtie cavities (DBCs) represent a relatively new class of optical resonators in which light can be concentrated to length scales much smaller than the wavelength in the material [1]–[3]. The key figure of merit for many applications in the weak coupling regime is the Purcell factor, which quantifies the relative enhancement in the radiative decay rate of a dipole emitter in an electromagnetic environment. For emitters in optical cavities, the Purcell factor can be written as [4]

$$F_p = \frac{3}{4\pi^2}\left(\frac{\lambda}{n}\right)^3 \frac{Q}{V}, \quad (1)$$

where λ/n is the effective wavelength in the material of refractive index n; the quality factor Q is a measure of the modal temporal confinement and the effective mode volume V is a measure of field localization. DBCs are notable for providing a means to achieve high Purcell factors by greatly enhancing the localization of the field while retaining relatively high Q. This regime of deep sub-wavelength confinement has interesting implications for light-matter interaction, and may enable single-emitter lasers and nonlinear devices operating with few photons.

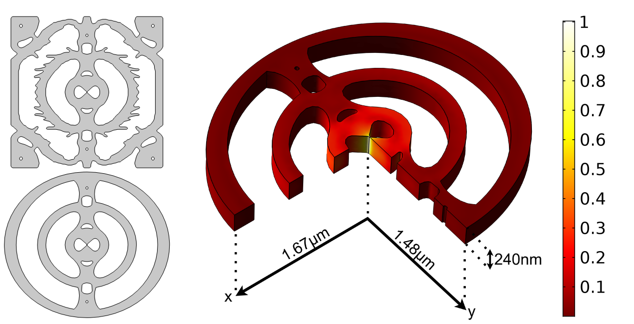

Fig. 1: Left: the topology optimization design from [3] (top) and the extracted simplified cavity analyzed in this work (bottom). Right: the simplified DBC (one quarter removed) with the relative field strength $|\mathbf{E}|$ of the QNM of interest on the surfaces.

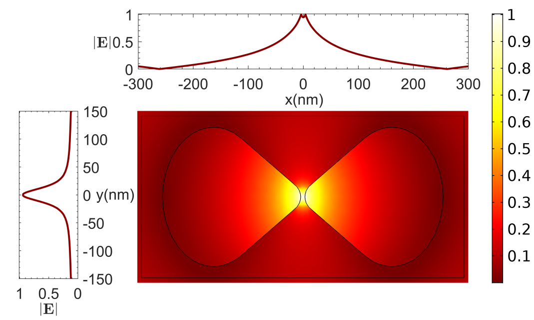

Fig. 2: Zoom-in of the central bowtie structure. Top and left line graphs show the field strength horizontally and vertically through the center.

The cavity used in this work is based on the layout of the DBC cavity in Ref. [3] which, in turn, was created by the inverse design framework of topology optimization (TO) [5] to maximize the local density of states in the cavity center [3]. Inspired by the design produced by the TO algorithm, we have developed a simplified design (see Fig. 1). Due to its smoother and more regular surfaces, the simplified design is more convenient for finite element modeling. The design has a footprint of $3\mu\text{m} \times 3\mu\text{m}$, and a bowtie bridge of 8nm, also shown in Fig. 2. To study the system, we use the finite element method as implemented in COMSOL Multiphysics.

We first show how one can study the local response of a DBC using a quasinormal mode (QNM) analysis [6], allowing us to describe the electromagnetic response of the system with just a single eigenmode. Next, as an application of the theory, we use perturbation theory to gauge the effect of shape deformations on the resonance and loss rate of the cavity. Such variability in the bowtie size is typical in fabrication from e-beam exposure [3], and understanding and quantifying its effects is therefore important. We find that a deformation by 1nm is enough to shift the resonance by several linewidths, which is quite significant, especially given current fabrication uncertainties. The simplification approach to TO designs coupled with the QNM analysis shown in this work provides a powerful modeling framework for studying this emerging class of cavities with interesting characteristics such as high sensitivity, strong confinement with low losses, and small footprints.

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II. Quasinormal Mode Expansion and Shape Deformations

The QNMs are solutions to the source-free electromagnetic wave equation subject to a suitable radiation condition valid far from the resonator [6]. The resulting eigenfrequencies are complex, $\tilde{\omega}_m = \omega_m - i\gamma_m$, from which we can calculate the quality factor Q of each mode as $Q_m = \omega_m/2\gamma_m$. Figure 3 shows the single-QNM approximation to the Purcell factor at a position $5\mathrm{nm}$ above the surface as well as the relative error when comparing to an independent reference calculation. Close to resonance, the relative error is on the order of 0.3%. Also shown in Fig. 3 is the spectrum of complex QNM frequencies in the fourth quadrant of the complex plane. Although more QNMs are seen to be present, they evidently contribute insignificantly to the Purcell factor at the spatial position of interest.

Having established the validity of the single-QNM approximation, we use perturbation theory to analyze the effect on the QNM of interest from shape deformations. Specifically, we consider the case of shrinking or expanding the holes defining the bowtie, as illustrated in Fig. 4, and we consider different perturbations $\Delta h = \{-2, -1, 0, 1, 2, 3\}\mathrm{nm}$ (negative values denote an enlargement of the air holes). The results for the eigenfrequency shift and the perturbation theory predictions can be compared with those of a conventional L1 photonic crystal cavity with similar resonance frequency and Q value, but for which the field confinement is much weaker. The complex eigenfrequency shifts are markedly more pronounced in the case of the bowtie cavity, which is to be expected from QNM perturbation theory [6] as shown by the straight lines in the plot. The vertical gray shading indicates the linewidth of the unperturbed structures. Owing to the sub-wavelength confinement, a shift of the sidewall of a single nanometer is enough to shift the resonance by several linewidths.

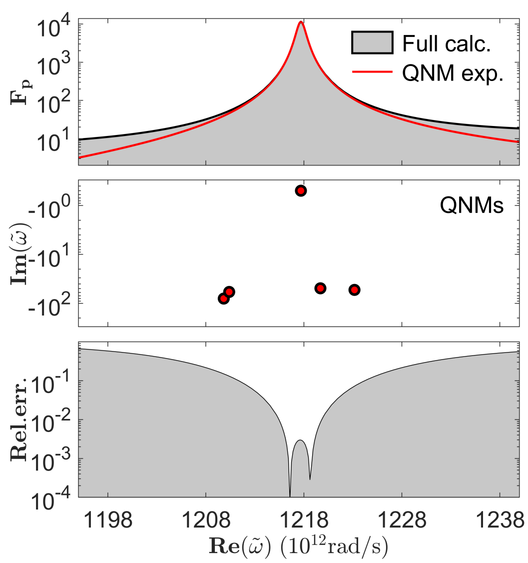

Fig. 3: Single-QNM approximation of the Purcell factor, QNM spectrum and relative error.

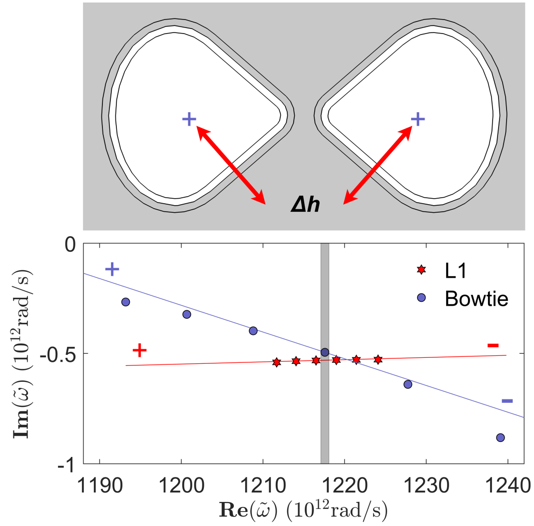

Fig. 4: Shape deformations of the bowtie (top) and effect on the complex eigenfrequency (bottom), along with perturbation theory predictions (lines). For comparison, also shown are the results for an L1 cavity under variation of its hole sizes. Gray shading indicates the linewidth of the cavity.

III. Conclusion

We have presented a QNM analysis of a DBC with deep sub-wavelength confinement. We have shown that the Purcell factor of the structure can be very well approximated by use of a single QNM. With this QNM, we have used perturbation theory to analyze the effect of shape deformation to the central bowtie structure, showing greatly increased sensitivity of the complex QNM frequency compared to a conventional photonic crystal cavity.

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