Flexible, Process-Aware Compact Model of Effective Index in Silicon Waveguides for Commercial Foundries

Aneek James, Yuyang Wang, Anthony Rizzo, Keren Bergman
Department of Electrical Engineering,
Columbia University, New York, New York, 10027
Email: aej2149@columbia.edu

***Abstract*—We report the performance of a compact model for the effective index of SOI wire waveguides, showing exceptional agreement with simulated effective index and confinement factors. The development of such a model represents a potential pathway toward better modeling of silicon photonic devices in commercial foundry processes.**

I. Introduction

In the silicon photonic design cycle there is often a large gap between the predictive and actual performance of photonic devices, preventing successful designs on the first [1]. A key contributing factor is the lack of ability to effectively incorporate foundry-specific measurement data into the design phase. Consequently, there has been much work over the years trying to rectify that problem by either designing more variation-robust devices or extracting geometric data that can then be used in photonic simulation software for pre-calibrating device designs or predicting the yield of the next run [2]–[4]. However, there is a gap in the literature of work that moves towards making a compact model of photonic waveguides that enables yield prediction. In contrast to time-consuming FDTD or EME simulations to quantify the effect of variations on device performance, such compact models allow for direct behavioral modeling of a device through a set of analytical equations whose parameters are directly based on real device measurements [5], [6]. In this paper, we present a compact model of waveguide effective index that can be used for statstical evaluation of waveguides in commercial silicon photonic foundries. The objective of our model is to describe the performance of a waveguide as a function of design, process, and environmental variables: λ the wavelength, w the waveguide width, Δt the waveguide thickness variation. The ability of the model to capture the changes in effective index over a wide array of both design inputs and process variations is demonstrated through the fitting of the model to simulated index data in Lumerical MODE solver. A brief discussion of how this model can be connected to real foundry data is then presented.

II. Group Device Extraction-Based Compact Model

The change in effective index of a single mode waveguide over an arbitrary frequency range is well-approximated by a second-order Taylor-Expansion:

$$n_{\text{eff}}(\lambda) = \frac{1}{2}\frac{\partial^2 n_{\text{eff}}}{\partial \lambda^2}(\lambda-\lambda_0)^2 + \frac{\partial n_{\text{eff}}}{\partial \lambda}(\lambda-\lambda_0) + n_{\text{eff},0} \quad (1)$$

The fitting parameters $\frac{\partial^2 n_{\text{eff}}}{\partial \lambda^2}$, $\frac{\partial n_{\text{eff}}}{\partial \lambda}$, and $n_{\text{eff},0}$ are all dependent on the waveguide geometry and material. Thus, this dependence should be captured accurately over the geometry design space. With respect to width, all three fitting parameters are found to be well described by the behavioral model [7]:

$$p(w) = p_0 \cdot \frac{w^2 + p_1 w + p_2}{w^2 + p_2 w + p_4} \quad (2)$$

where $p \in \left[\frac{\partial^2 n_{\text{eff}}}{\partial \lambda^2}, \frac{\partial n_{\text{eff}}}{\partial \lambda}, n_{\text{eff},0}\right]$. Once n_{eff} vs frequency data is obtained for the range of waveguide widths of interest to designers, these fitting parameters can then be easily extracted using ordinary least squares (OLS). The power of a closed-form model that directly captures the behavior of effective index is that important design parameters such as the group index [8] and scattering loss [9] can either be directly calculated or extracted through measurement with this model.

III. Lumerical MODE Fitting Results

The effective indices for different SOI wire waveguides were simulated in Lumerical MODE (Fig. 1a.), varying width and thickness to evaluate how varied with geometry. Widths were swept from 0.4 – 1 um, and thicknesses were swept from 210 – 230 nm. All three fit parameters are matched well by the model over the entire range of the width sweep (Fig. 1b.). The model, therefore, accurately matches the behavior of the effective index both across a wide range of wavelengths (Fig. 1c.) and a wide range of waveguide widths (Fig. 1d.). By describing the width dependence of effective index with equations (1) and (2), the effect that a thickness variation Δt has on the model is captured in the sensitivity of each fitting coefficient p_i to Δt. The values and sensitivities for each parameter around a mean thickness of 220 nm are shown in Table I. It should be noted that though the MODE simulations took $\sim$ 6 hours to run, our model can reproduce the same values instantaneously.

A value for the confinement factor based on our model can also be derived [10], and was compared to the confinement

TABLE I
EFFECTIVE INDEX FITTING RESULTS. SENSITIVITIES CALCULATED FOR A WAVEGUIDE THICKNESS OF 220 NM.

	Width Sub-Coefficients (p_i [units], $\partial p_i/\partial\Delta t$ [units / m])				
Fit Parameter	p_0	p_1	p_2	p_3	p_4
$\partial^2 n_{\text{eff}}/\partial f^2$ [THz^{-2}]	(-2.007e-06, -24.85)	(-1.470e-07, -1.161)	(-1.380e-14, 1.421e-06)	(-6.282e-07, 9.756e-01)	(1.183e-13, -1.878e-07)
$\partial n_{\text{eff}}/\partial f$ [THz^{-1}]	(3.767e-03, 1.878e+4)	(8.029e-08, -5.188)	(-1.082e-13, 3.014e-06)	(-6.481e-07, 7.209e-01)	(1.215e-13, -1.266e-07)
$n_{\text{eff},0}$	(2.908, 4.802e5)	(-7.332e-07, 1.020)	(1.484e-13, -3.732e-07)	(-6.465e-07, 7.474e-01)	(1.213e-13, -1.347e-07)

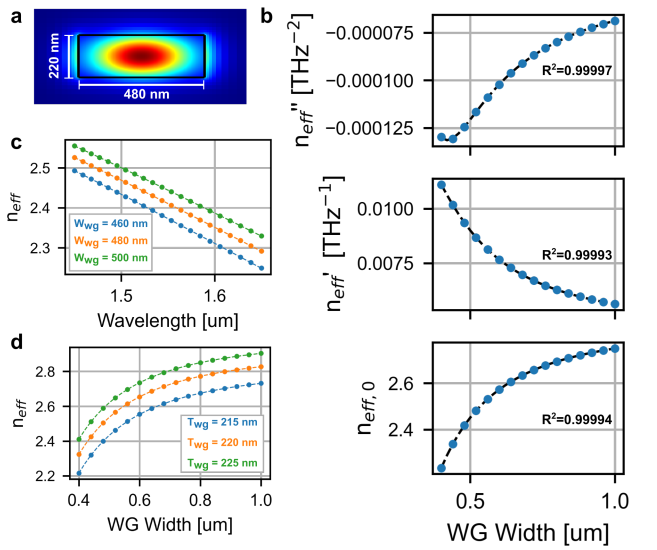

Fig. 1. **a,** TE Mode field distribution plot of simulated waveguide in Lumerical MODE. **b,** Plot of the n_{eff} parameters $\frac{\partial^2 n_{\text{eff}}}{\partial\lambda^2}$, $\frac{\partial n_{\text{eff}}}{\partial\lambda}$, and $n_{\text{eff},0}$ vs width. **c,** Comparison of model and simulated n_{eff} values over C- and L-bands. **d,** Comparison of model and simulated n_{eff} for different waveguide geometries at a wavelength of 1550 nm.

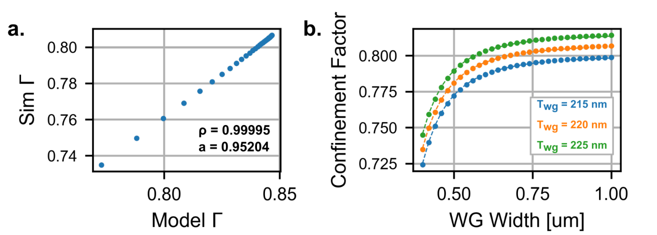

Fig. 2. **a,** Correlation between simulated and modeled confinement factors. **b,** Comparison plot of confinement factor vs width for simulated and modeled values.

factor reported by our simulations. There was a near perfect correlation between the model and simulated confinement factor, showing that the overall behavior is captured by the model (Fig. 2a). The modeled confinement values were off by a correction factor a of 0.95204. Though the origin of the correction factor is unclear, the factor was consistent across all studied waveguide thicknesses and widths. This implies that at the very least the correction factor is independent of the waveguide geometry itself (Fig. 2b).

IV. Discussion & Conclusion

We have introduced a compact model that has potential to incorporate process waveguide performance of commercial foundries more tightly into the device design phase. By carefully designing metrology devices, the mean compact model parameters for a given foundry process can be extracted through by choosing appropriate devices [4], [8]. Afterwards, an optimization algorithm would estimate exact parameter deviations for each device [6], [7]. Finally, a circuit-based simulation software can be used to model exact statistics and estimate device yield for a given silicon photonic circuit [11]. Future work should look at incorporating the coupling and loss into the model. The behavior of waveguide index, coupling and loss fully modeled, variation-aware compact models for most silicon photonic building blocks such as micro-resonators and Mach-Zehnder Interferometers can easily then be constructed.

References

[1] W. Bogaerts and L. Chrostowski, "Silicon photonics circuit design: methods, tools and challenges," *Laser & Photonics Reviews*, vol. 12, no. 4, p. 1700237, 2018.

[2] A. Rizzo, U. Dave, A. Novick, A. Freitas, S. P. Roberts, A. James, M. Lipson, and K. Bergman, "Fabrication-robust silicon photonic devices in standard sub-micron silicon-on-insulator processes," 2022. [Online]. Available: https://arxiv.org/abs/2205.11481

[3] Y. Xing, J. Dong, S. Dwivedi, U. Khan, and W. Bogaerts, "Accurate extraction of fabricated geometry using optical measurement," *Photonics Research*, vol. 6, no. 11, pp. 1008–1020, 2018.

[4] W. A. Zortman, D. C. Trotter, and M. R. Watts, "Silicon photonics manufacturing," *Optics express*, vol. 18, no. 23, pp. 23 598–23 607, 2010.

[5] R. Woltjer, L. Tiemeijer, and D. Klaassen, "An industrial view on compact modeling," in *2006 European Solid-State Device Research Conference*. IEEE, 2006, pp. 41–48.

[6] N. Moezi, "Statistical compact model strategies for nano cmos transistors subject of atomic scale variability," Ph.D. dissertation, University of Glasgow, 2012.

[7] A. E. James, A. Wang, S. Wang, and K. Bergman, "Evaluating regression-based techniques for modelling fabrication variations in silicon photonic waveguides," in *Applications of Machine Learning 2021*, vol. 11843. International Society for Optics and Photonics, 2021, p. 1184305.

[8] W. Bogaerts, P. De Heyn, T. Van Vaerenbergh, K. De Vos, S. Kumar Selvaraja, T. Claes, P. Dumon, P. Bienstman, D. Van Thourhout, and R. Baets, "Silicon microring resonators," *Laser & Photonics Reviews*, vol. 6, no. 1, pp. 47–73, 2012.

[9] D. Melati, A. Melloni, and F. Morichetti, "Real photonic waveguides: guiding light through imperfections," *Advances in Optics and Photonics*, vol. 6, no. 2, pp. 156–224, 2014.

[10] A. Yariv and P. Yeh, *Photonics: optical electronics in modern communications*. Oxford university press, 2007.

[11] A. E. James, X. Meng, A. Gazman, N. Janosik, and K. Bergman, "Time-efficient photonic variability simulator for uncertainty quantification of photonic integrated circuit," in *2019 IEEE Optical Interconnects Conference (OI)*. IEEE, 2019, pp. 1–2.