Simulation of 2D Maxwell-Bloch Equations

NUSOD – Sep. 24, 2007

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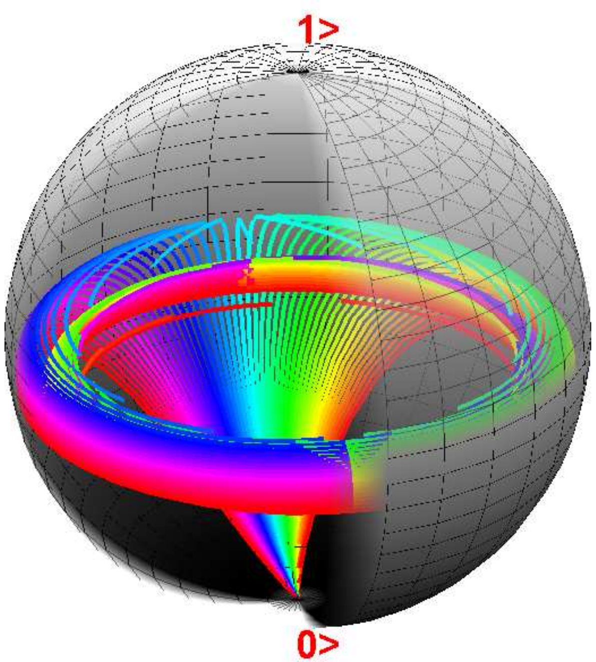
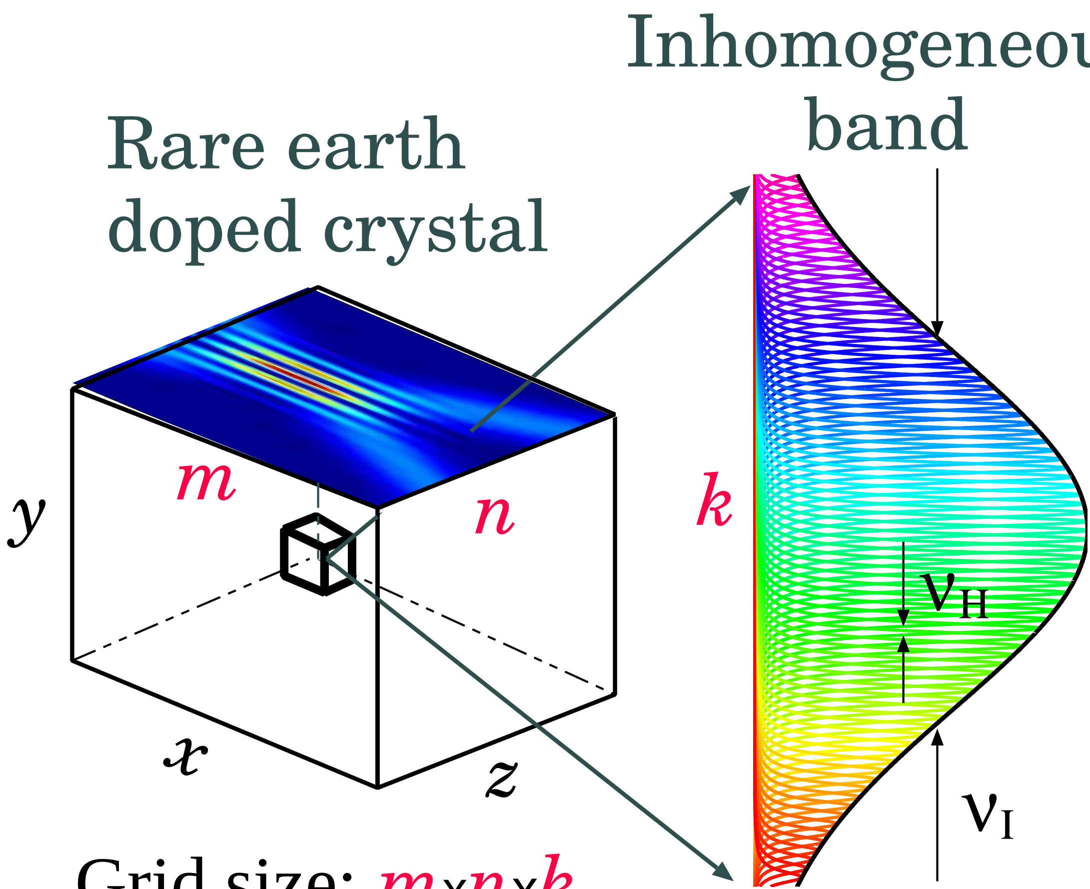


- *Material property*
- *Simulation schemes and results*
- *Experimental results*

Inhomogeneously broadened material



Grid size: mxnxk

Time domain parameters

- Life time $T_1 = 10$msec
- Decoherence time $T_2 = 5\,\mu$sec

Frequency domain parameters

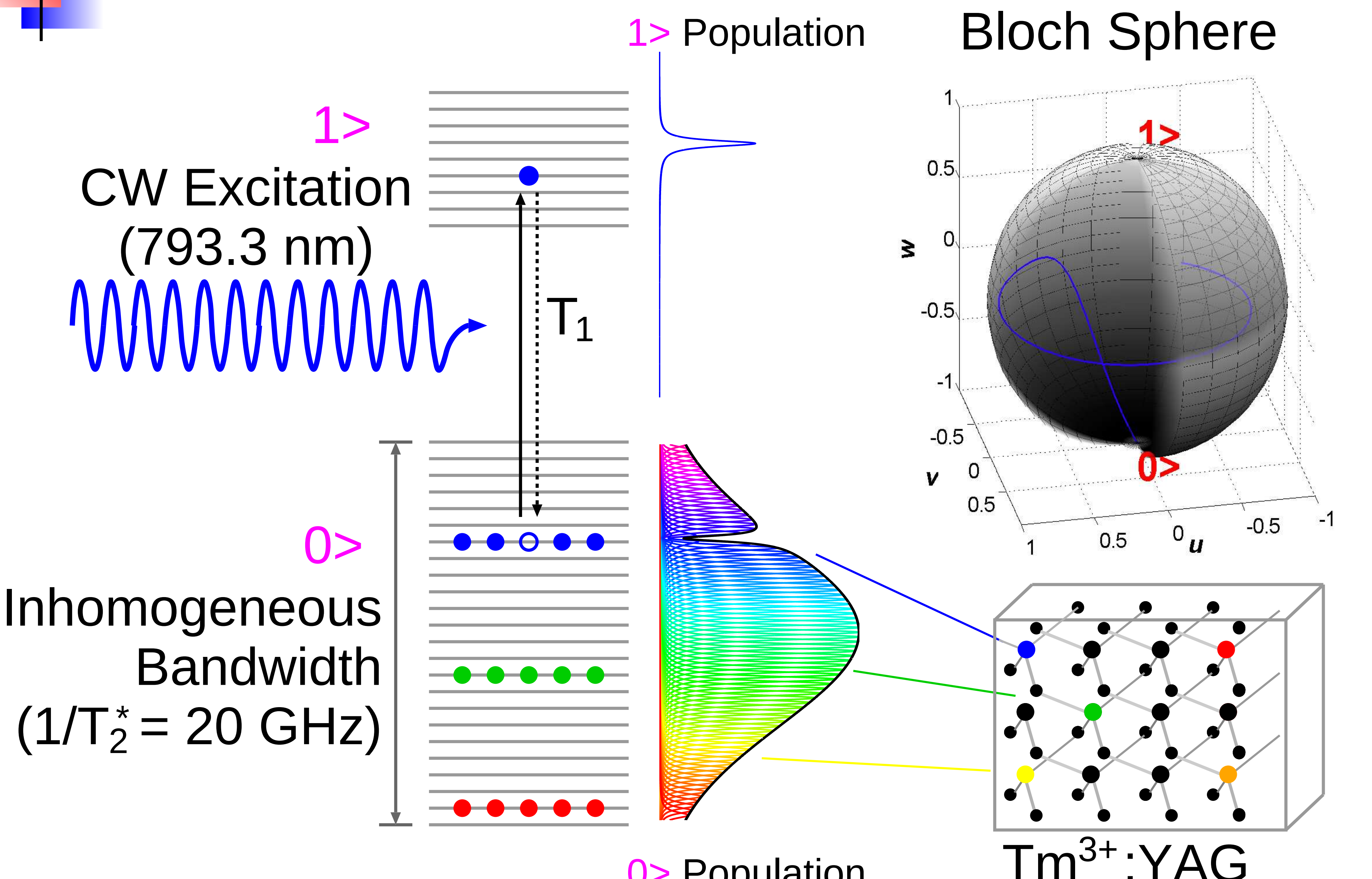
- Inhomogeneous BW $\nu_I = 20$GHz
- Homogeneous BW $\nu_H = 25$KHz
- Number of bins $N = \nu_I / \nu_H \approx 10^6$

Applications

- Data storage: 10^{16} bits/cm^3
- Signal processing: $20+$GHz BW

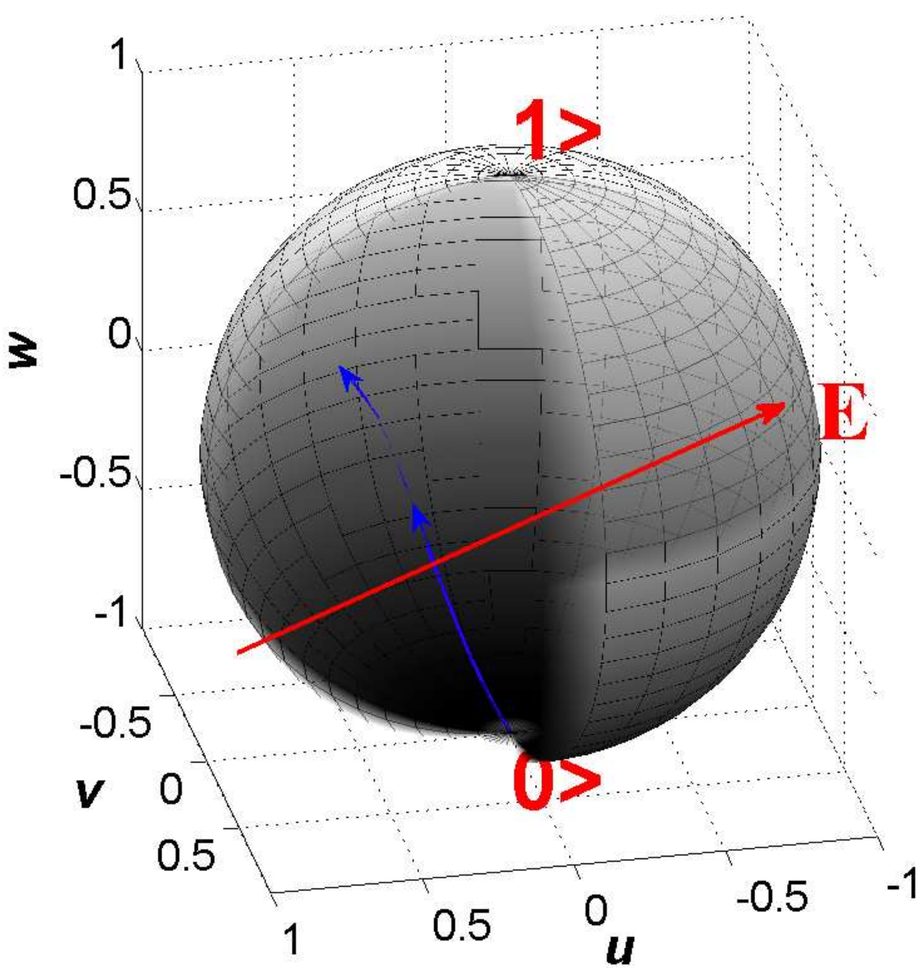
Sun, *et. al, J. of Luminescence*, Vol.98, p281, 2002

Two-level atoms



Bloch equations: Excitation

Excitation



$$\frac{du}{dt} = -\frac{u}{T_2} - \Delta \cdot v - \kappa \Im\{\mathcal{E}\} w$$
$$\frac{dv}{dt} = \Delta \cdot u - \frac{v}{T_2} + \kappa \Re\{\mathcal{E}\} w$$
$$\frac{dw}{dt} = \kappa \Im\{\mathcal{E}\} u - \kappa \Re\{\mathcal{E}\} v + \frac{w-1}{T_1}$$

u: inphase component of $\mathcal{P}$

v: in-quadrature component of $\mathcal{P}$

w: population inversion

$\mathcal{E}$: E-field

Δ: detuning frequency

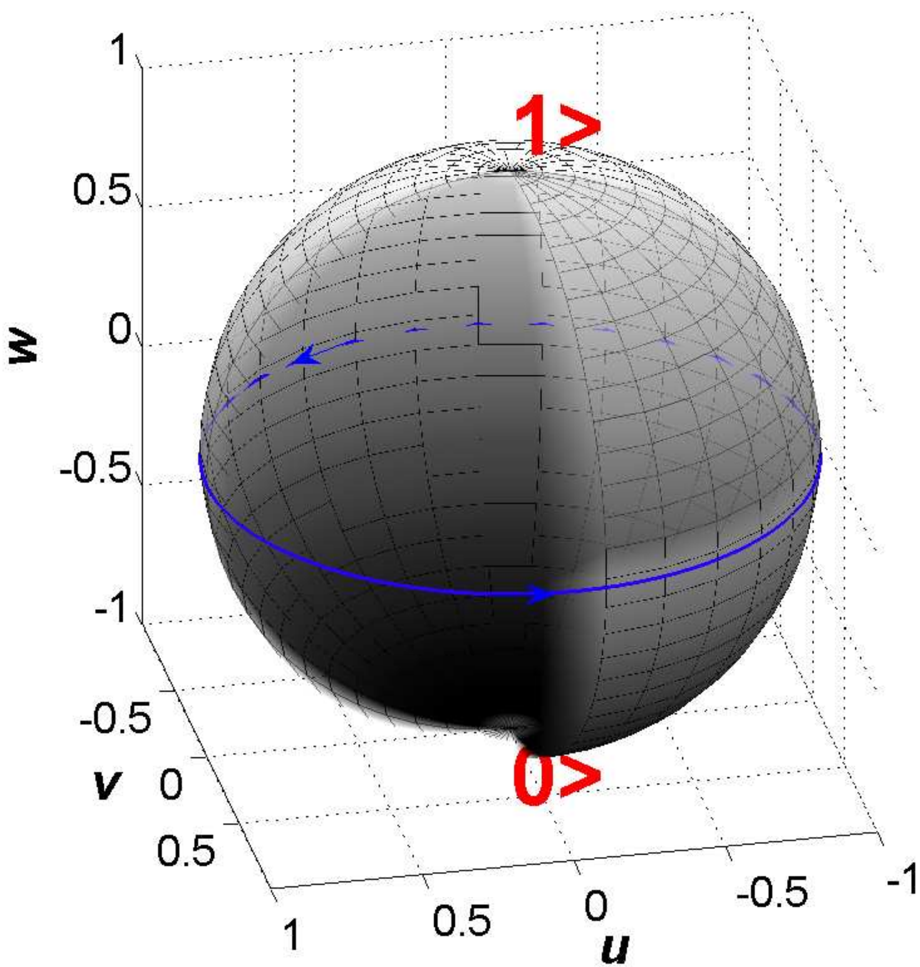
T_2: decoherence time

T_1: population lifetime

Allen and Eberly, *Optical resonance and two-level atoms*, 1975

Bloch equations: Detuning

Free precession



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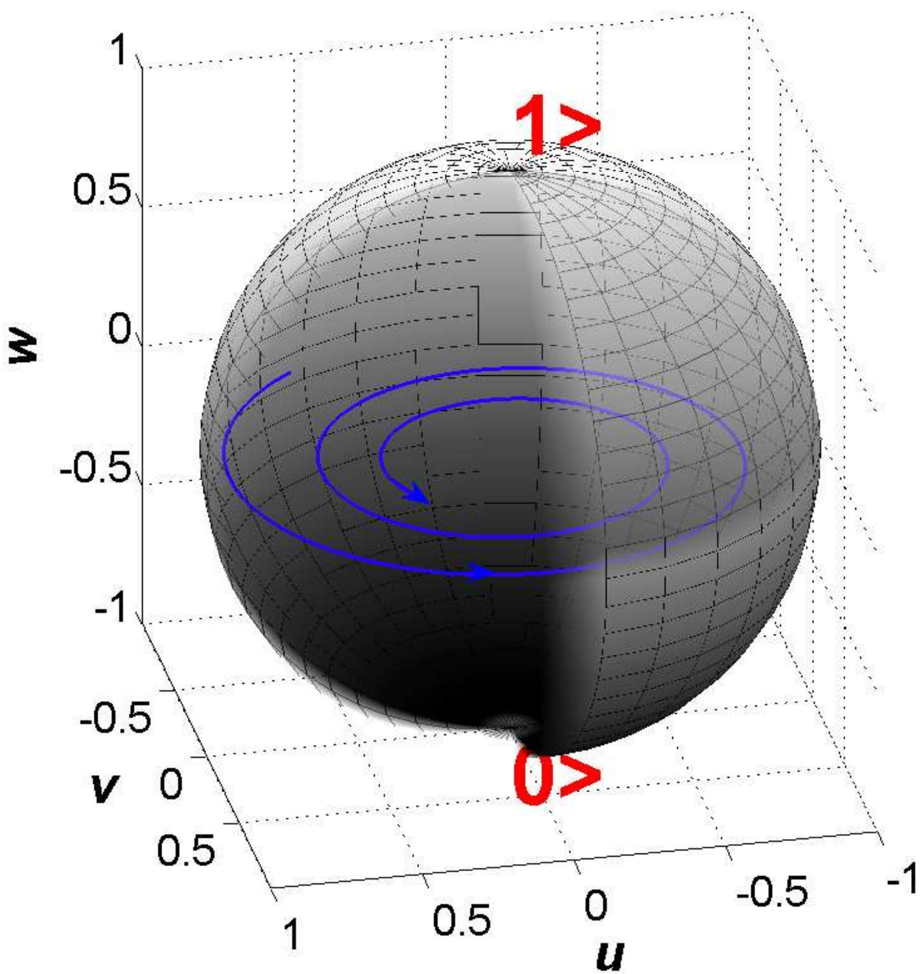
Bloch equations: Decoherence

Decoherence



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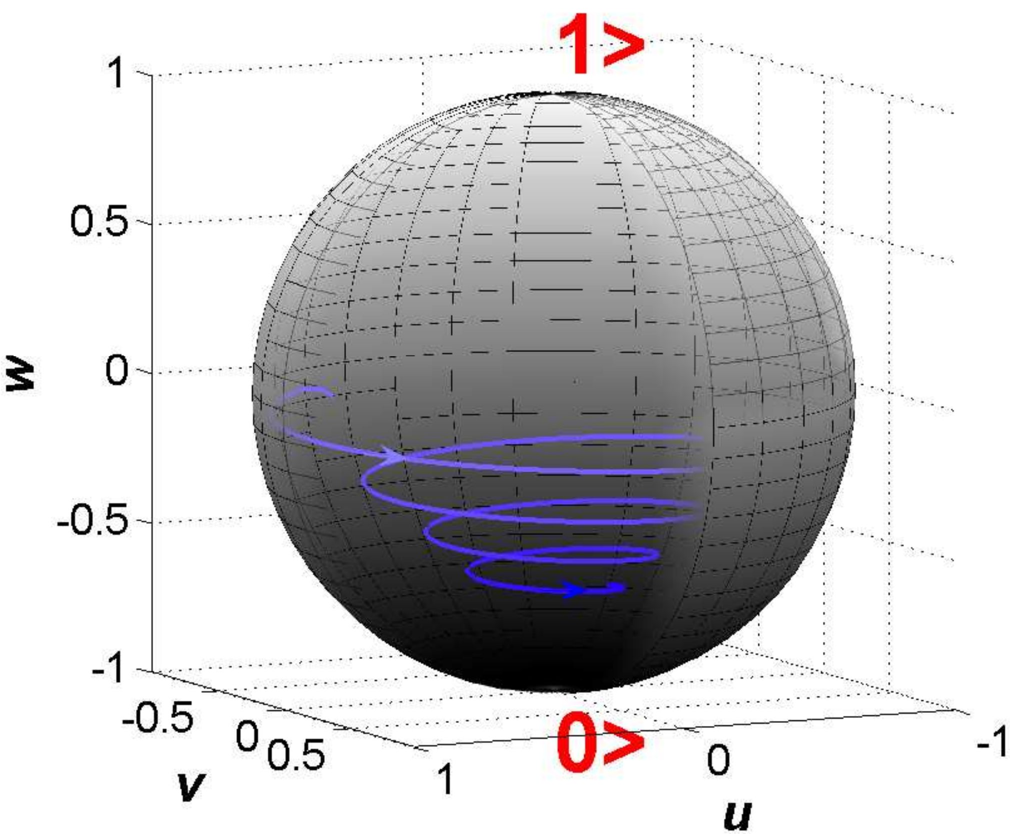
T_1: population lifetime

Bloch equations: Population relaxation

Population relaxation



$$
\begin{aligned}
\frac{du}{dt} &= -\frac{u}{T_2} - \Delta \cdot v - \kappa \Im\{\mathcal{E}\} w \\
\frac{dv}{dt} &= \Delta \cdot u - \frac{v}{T_2} + \kappa \Re\{\mathcal{E}\} w \\
\frac{dw}{dt} &= \kappa \Im\{\mathcal{E}\} u - \kappa \Re\{\mathcal{E}\} v + \frac{w-1}{T_1}
\end{aligned}
$$

u: inphase component of $\mathcal{P}$

v: in-quadrature component of $\mathcal{P}$

w: population inversion

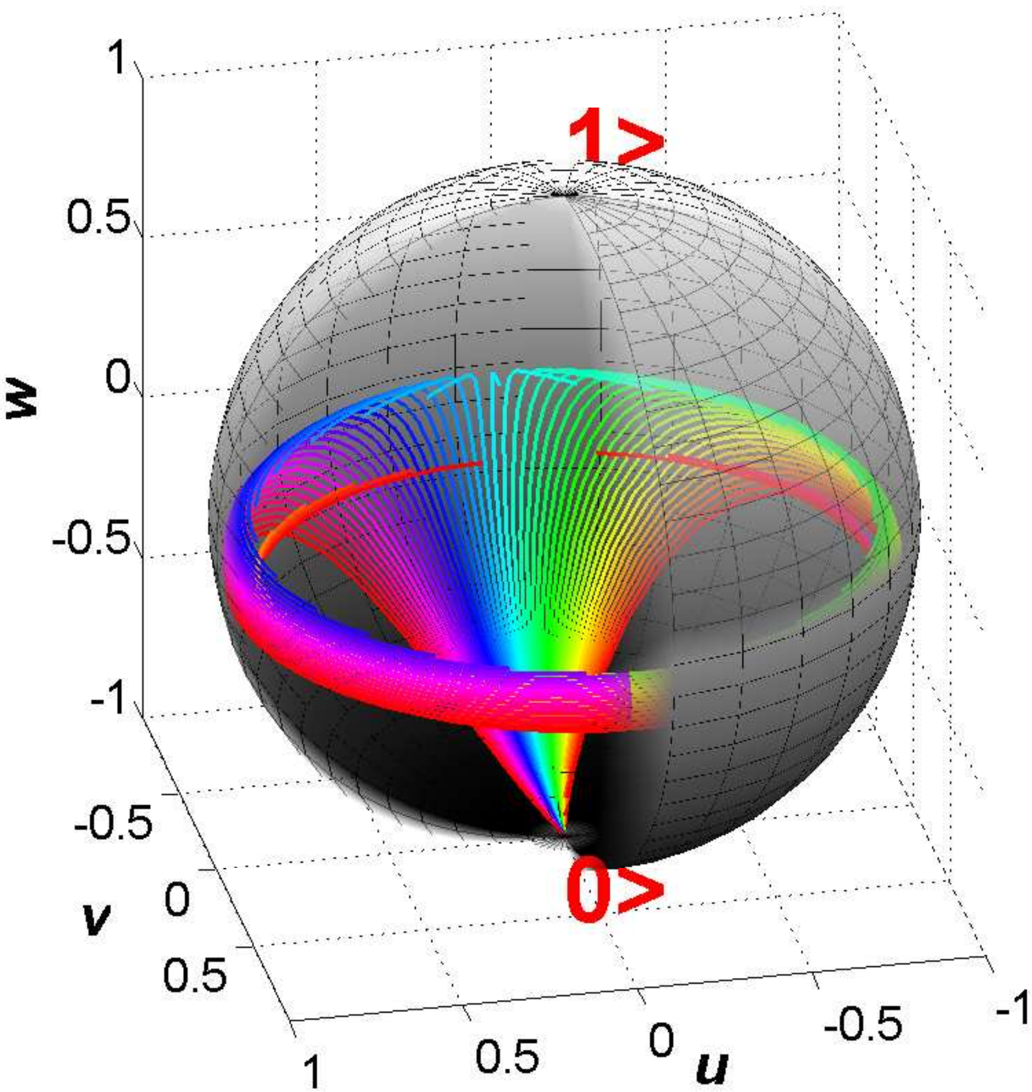
$\mathcal{E}$: E-field

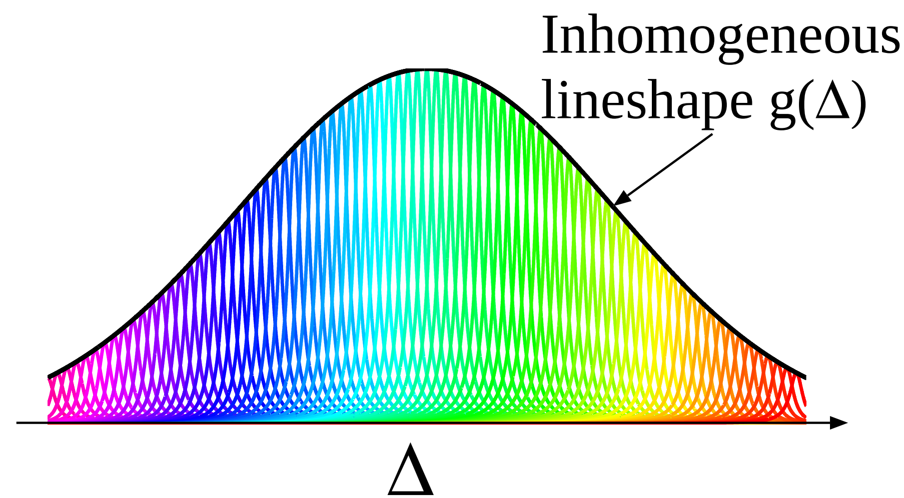
Δ: detuning frequency

T_2: decoherence time

T_1: population lifetime

Inhomogeneous band





Inhomogeneous averaging

$$\mathcal{P}(t) = \alpha \int_{-\infty}^{+\infty} g(\Delta)[u(t,\Delta) + iv(t,\Delta)]d\Delta$$

2D Maxwell-Bloch equations

The 2D SEVA wave equation is driven by polarization,

$$\frac{\partial \mathcal{E}(x,z,t)}{\partial z} + \frac{i}{2k}\frac{\partial^2 \mathcal{E}(x,z,t)}{\partial x^2} = -i\frac{\mu_0 \omega_0^2}{2k}\mathcal{P}(x,z,t)$$

Bloch equations under RWA driven by the E-field,

$$\frac{\partial}{\partial t}\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_2} & -\Delta & -\kappa\Im\{\mathcal{E}\} \\ \Delta & -\frac{1}{T_2} & \kappa\Re\{\mathcal{E}\} \\ \kappa\Im\{\mathcal{E}\} & -\kappa\Re\{\mathcal{E}\} & \frac{1}{T_1} \end{bmatrix}\begin{bmatrix} u \\ v \\ w \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -\frac{1}{T_1} \end{bmatrix}$$

Polarization given by summation over the inhomogeneous band

$$\mathcal{P}(x,z,t) = \alpha\int_{-\infty}^{+\infty} g(\Delta)[u(x,z,t,\Delta) + iv(x,z,t,\Delta)]d\Delta$$

Previous numerical solutions

- Vector form: FDTD[1,2]
  - Slow: $\Delta z = \frac{\lambda}{100} \sim \frac{\lambda}{50}$ corresponding to < 0.1fsec

2D spatial grid:	200μm$(x) \times 5$mm(z)		10^8 spatial steps
Time span:	2μsec	$\Rightarrow$	$2 \cdot 10^{10}$ time steps
Spectral lines:	250		250 lines

Using 2.25GHz computer, it would take about 3×10^4 years.

- Scalar form (wave equation)
  - Cornish[3] simulated 1-D
  - Chang[4] simulated two symmetric angled plane waves but didn't give specifics of numerical technique

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1. Ziolkowski, *et. al*, *Phys. Rev.*, Vol 52, p3082, 1995
2. Schlottau, *et. al*, *Optics Express*, Vol. 13, p182, 2005
3. Cornish, Ph.D. thesis, University of Washington, 2000
4. Chang, *et. al*, *J. of Luminescence*, Vol. 107, p138, 2004

FFT-BPM I

The 2D SEVA wave equation is driven by polarization,

$$\frac{\partial \mathcal{E}(x,z,t)}{\partial z} + \frac{i}{2k}\frac{\partial^2 \mathcal{E}(x,z,t)}{\partial x^2} = -i\frac{\mu_0 \omega_0^2}{2k}\mathcal{P}(x,z,t)$$

Expand $\mathcal{E}(x,z,t)$ and $\mathcal{P}(x,z,t)$ in Fourier domain,

$$\mathcal{E}(x,z) = \int_{-\infty}^{+\infty} \hat{\mathcal{E}}(k_x,z)\exp(-ik_x x)dk_x,$$

$$\mathcal{P}(x,z) = \int_{-\infty}^{+\infty} \hat{\mathcal{P}}(k_x,z)\exp(-ik_x x)dk_x$$

$$-k_x^2\hat{\mathcal{E}}(k_x,z) - 2ik\frac{d\hat{\mathcal{E}}(k_x,z)}{dz} = -\mu_0\omega_0^2\hat{\mathcal{P}}(k_x,z)$$

Feit, M. D.*et. al*, *Appl. Opt.* Vol. 17 p3990, 1978

FFT-BPM II

SEVA wave equation we want to solve is

$$\frac{\partial \hat{\mathcal{E}}}{\partial z} = \frac{i}{2k}(k_x^2 \hat{\mathcal{E}} - \mu_0 \omega_0^2 \hat{\mathcal{P}})$$

Use trapezoidal rule to evolve wave quation

- Since delayed source terms inconsistant with SS-BPM frame work

For an ODE $y' = f(x, y)$, the trapezoidal rule is

$$y_{n+1} = y_n + \frac{1}{2} h \left[f(x_n, y_n) + f(x_n + h, y_{n+1}) \right]$$

Apply trapezoidal rule and solve for $\hat{\mathcal{E}}_{n+1}$,

$$\hat{\mathcal{E}}_{n+1} = \frac{1}{1 - \frac{i}{4k} k_x^2 dz} \left[\left(1 + \frac{i}{4k} k_x^2 dz \right) \hat{\mathcal{E}}_n - \frac{i}{4k} dz \cdot 2\mu_0 \omega_0^2 \hat{\mathcal{P}}_n \right]$$

Numerical scheme for Bloch equations

Bloch equations under RWA driven by the E-field,

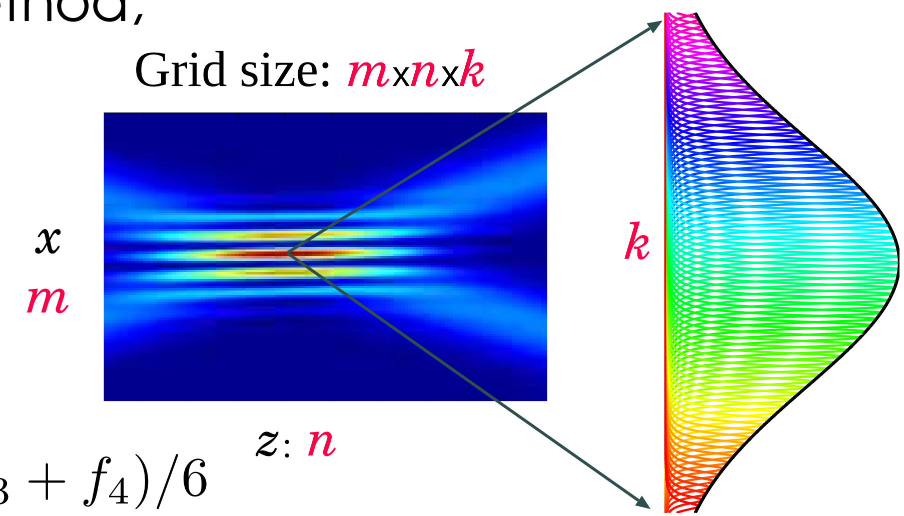
$$\frac{d}{dt}\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_2} & -\Delta & -\kappa\Im\{\mathcal{E}\} \\ \Delta & -\frac{1}{T_2} & \kappa\Re\{\mathcal{E}\} \\ \kappa\Im\{\mathcal{E}\} & -\kappa\Re\{\mathcal{E}\} & \frac{1}{T_1} \end{bmatrix}\begin{bmatrix} u \\ v \\ w \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -\frac{1}{T_1} \end{bmatrix}$$

Write in matrix-vector form, it becomes $\underline{y}' = \underline{\underline{A}}\underline{y} + \underline{b}$

Applying the 4th order Runge-Kutta method,

$$\begin{aligned} f_1 &= \underline{\underline{A}}\underline{y}_{l,m,n,k} + \underline{b}, \\ f_2 &= \underline{\underline{A}}(\underline{y}_{l,m,n,k} + \tfrac{f_1}{2}) + \underline{b} \\ f_3 &= \underline{\underline{A}}(\underline{y}_{l,m,n,k} + \tfrac{f_2}{2}) + \underline{b}, \\ f_4 &= \underline{\underline{A}}(\underline{y}_{l,m,n,k} + f_3) + \underline{b} \end{aligned}$$

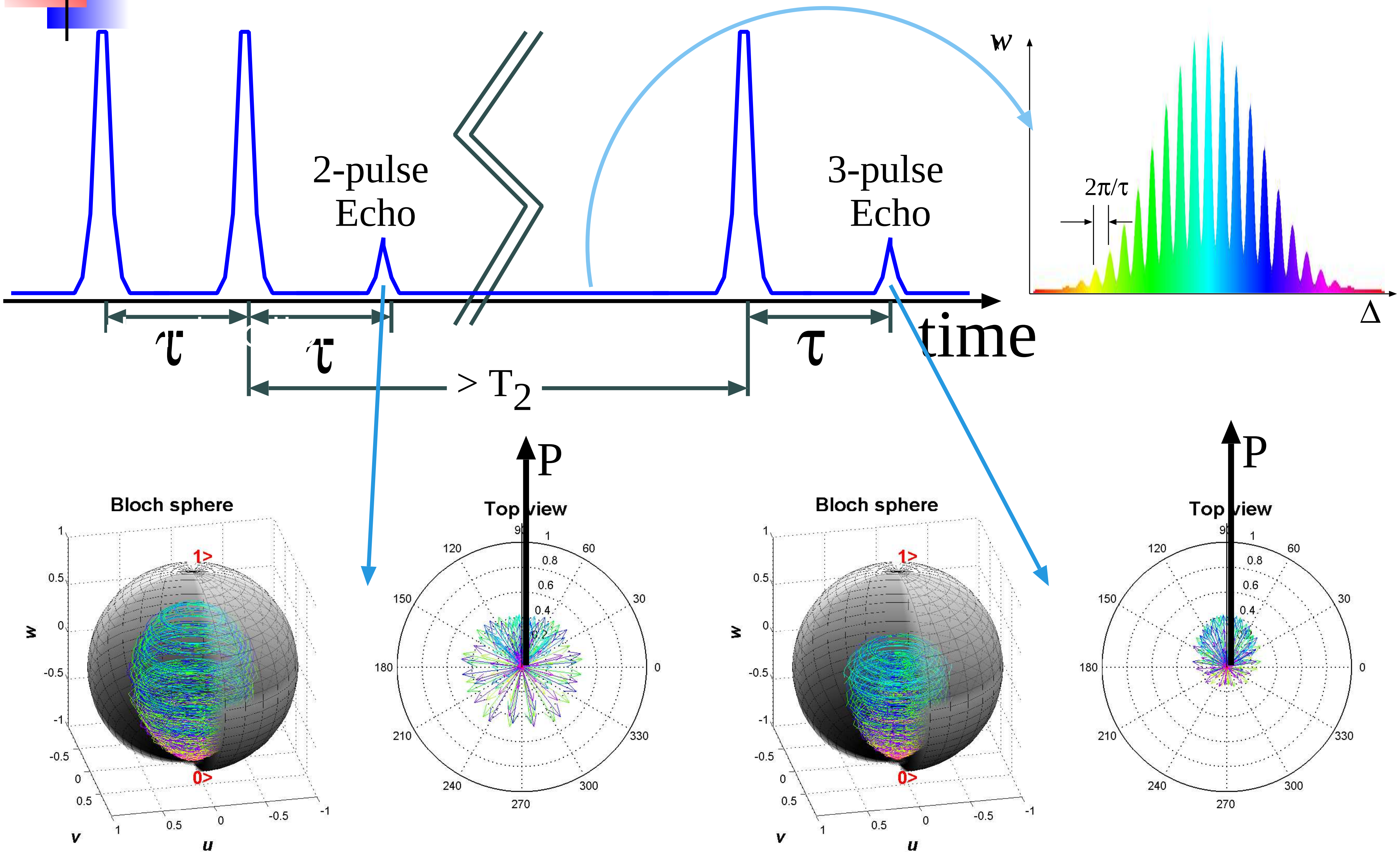
$$\underline{y}_{l+1,m,n,k} = \underline{y}_{l,m,n,k} + \Delta t(f_1 + 2f_2 + 2f_3 + f_4)/6$$



l: time step, m: x sampling step, n: z step, k: spectral lines

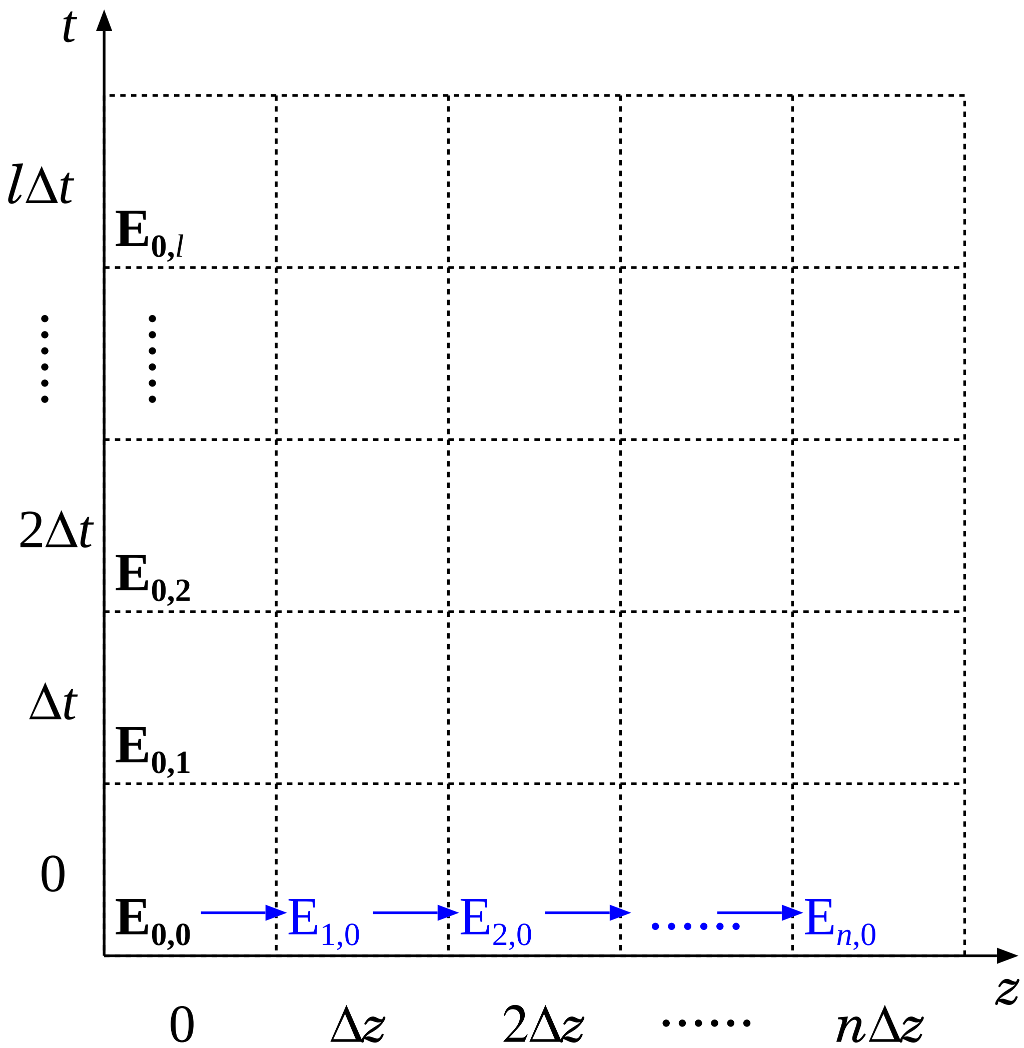
Gross and Manassah, *Opt. Lett.*, Vol.17, p340, 1992

Bloch simulation results

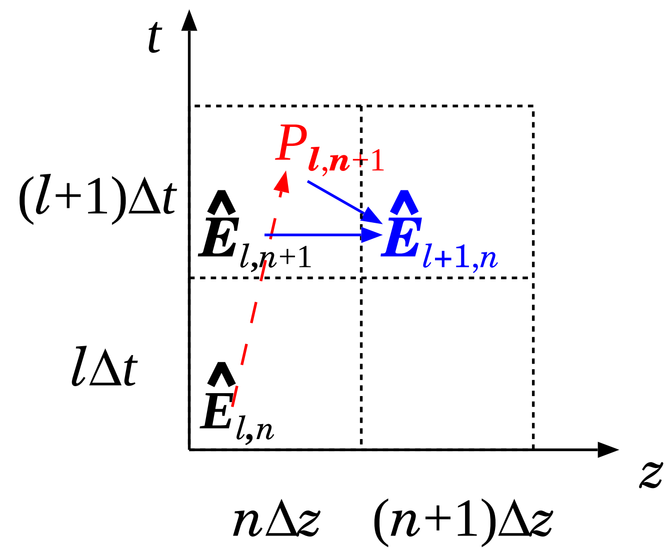


Advancing the Maxwell-Bloch equations

Initial Value Condition

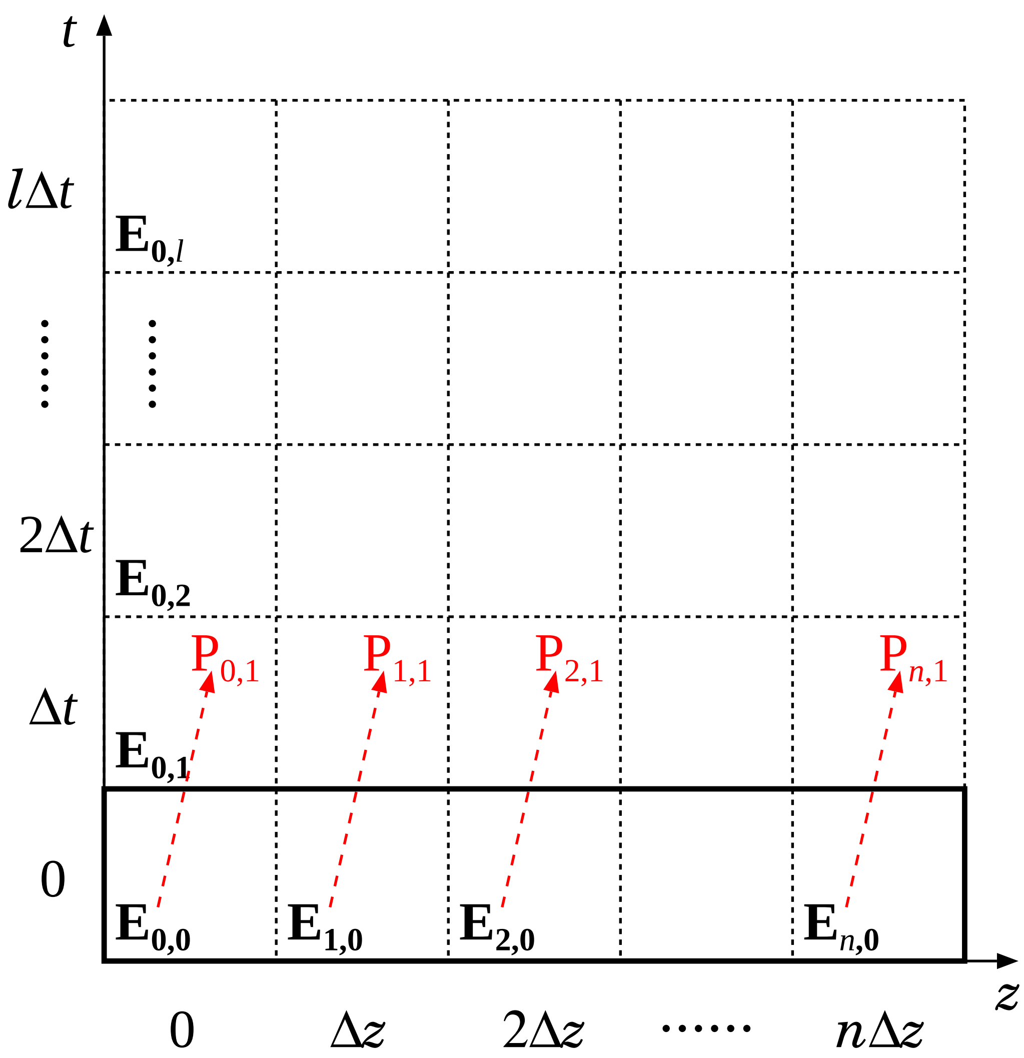


- Calculate **Maxwell's** equation
- Propagate E-field from boundary into crystal

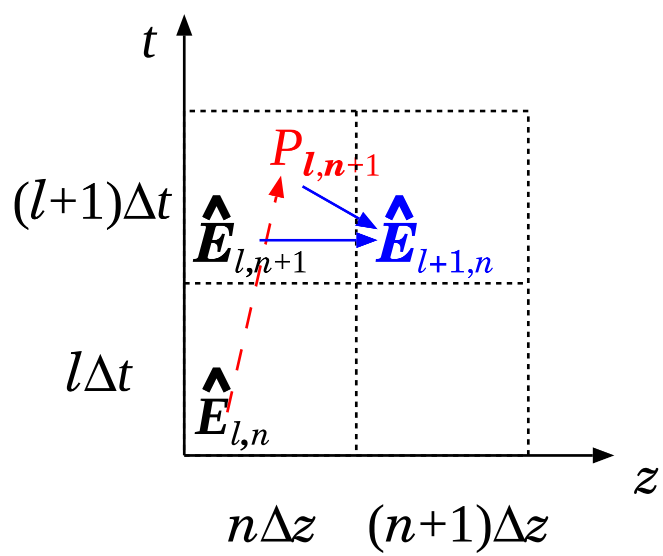


Advancing the Maxwell-Bloch equations

Δt later

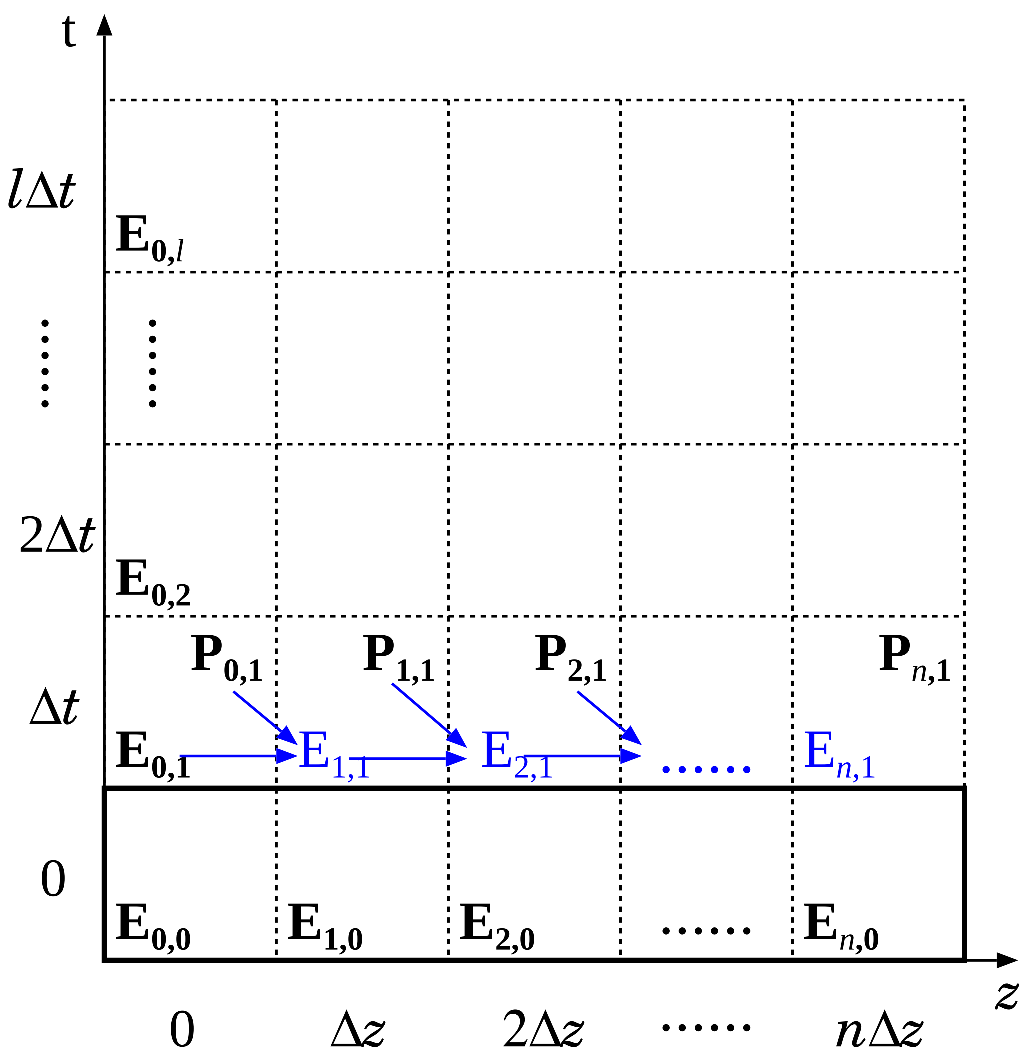


- Calculate **Bloch** equation
- E-field drives Bloch vectors to get polarization

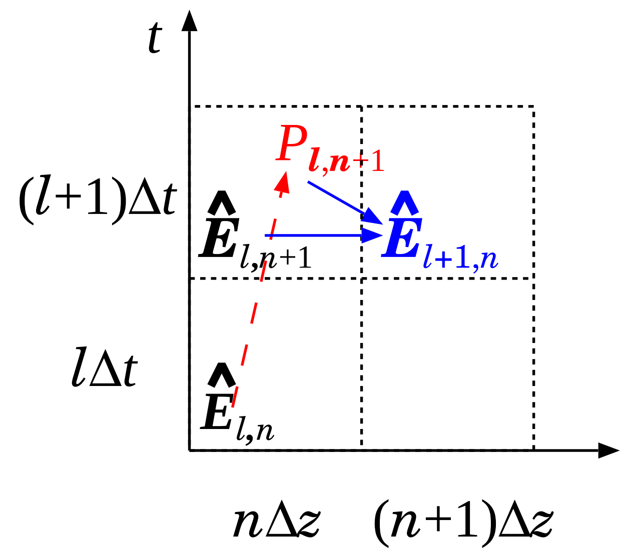


Advancing the Maxwell-Bloch equations

Δt later

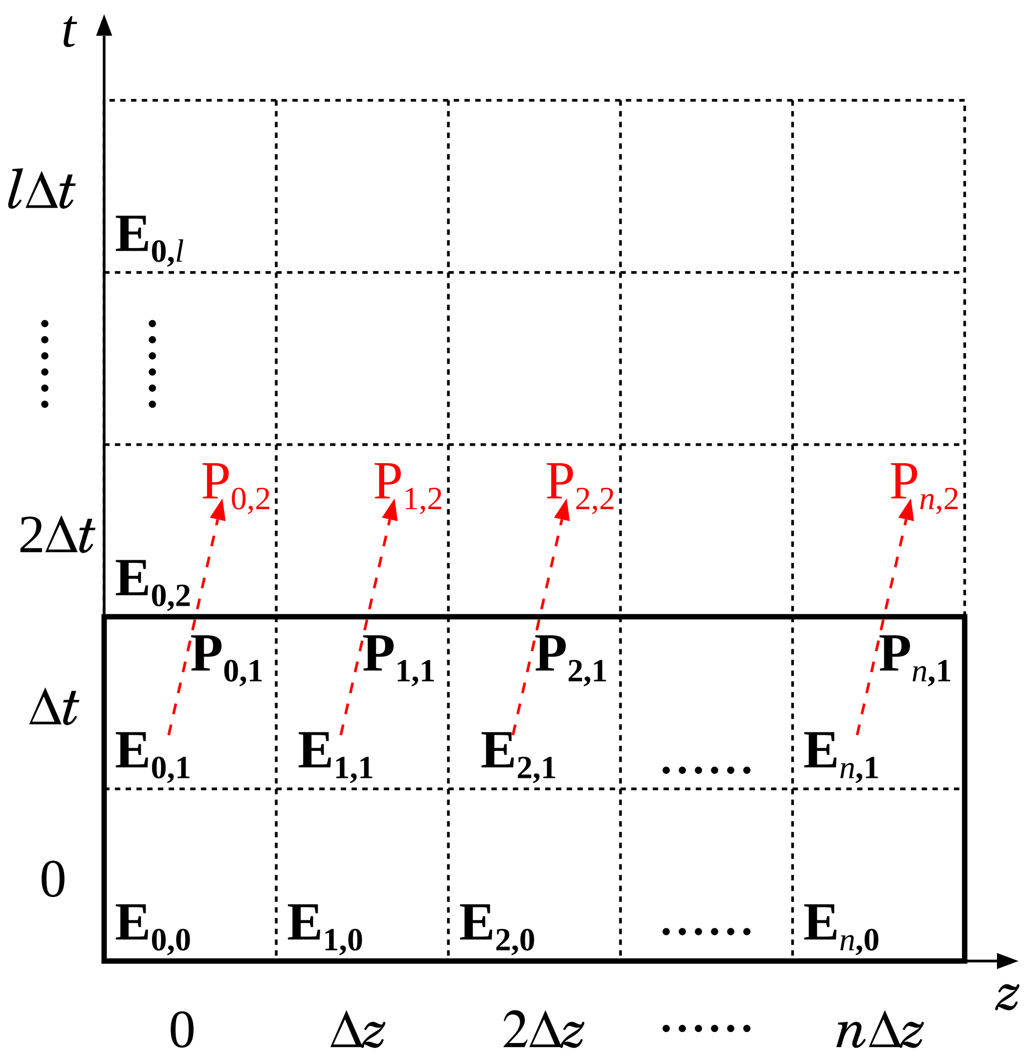


- Calculate **Maxwell's** equation
- Contribute polarization to propagate E-field

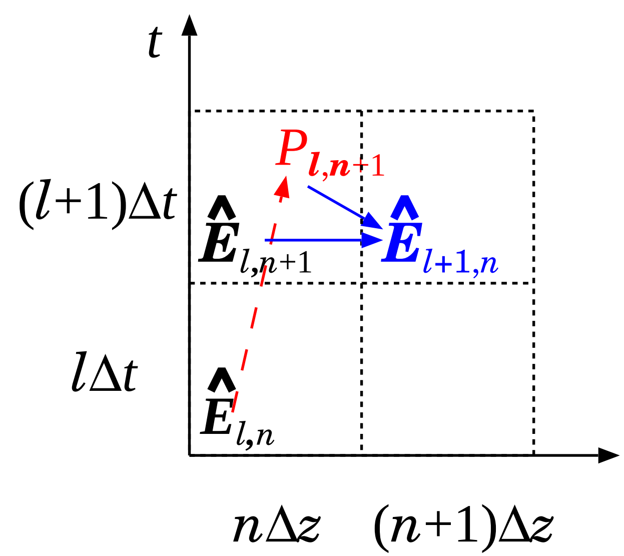


Advancing the Maxwell-Bloch equations

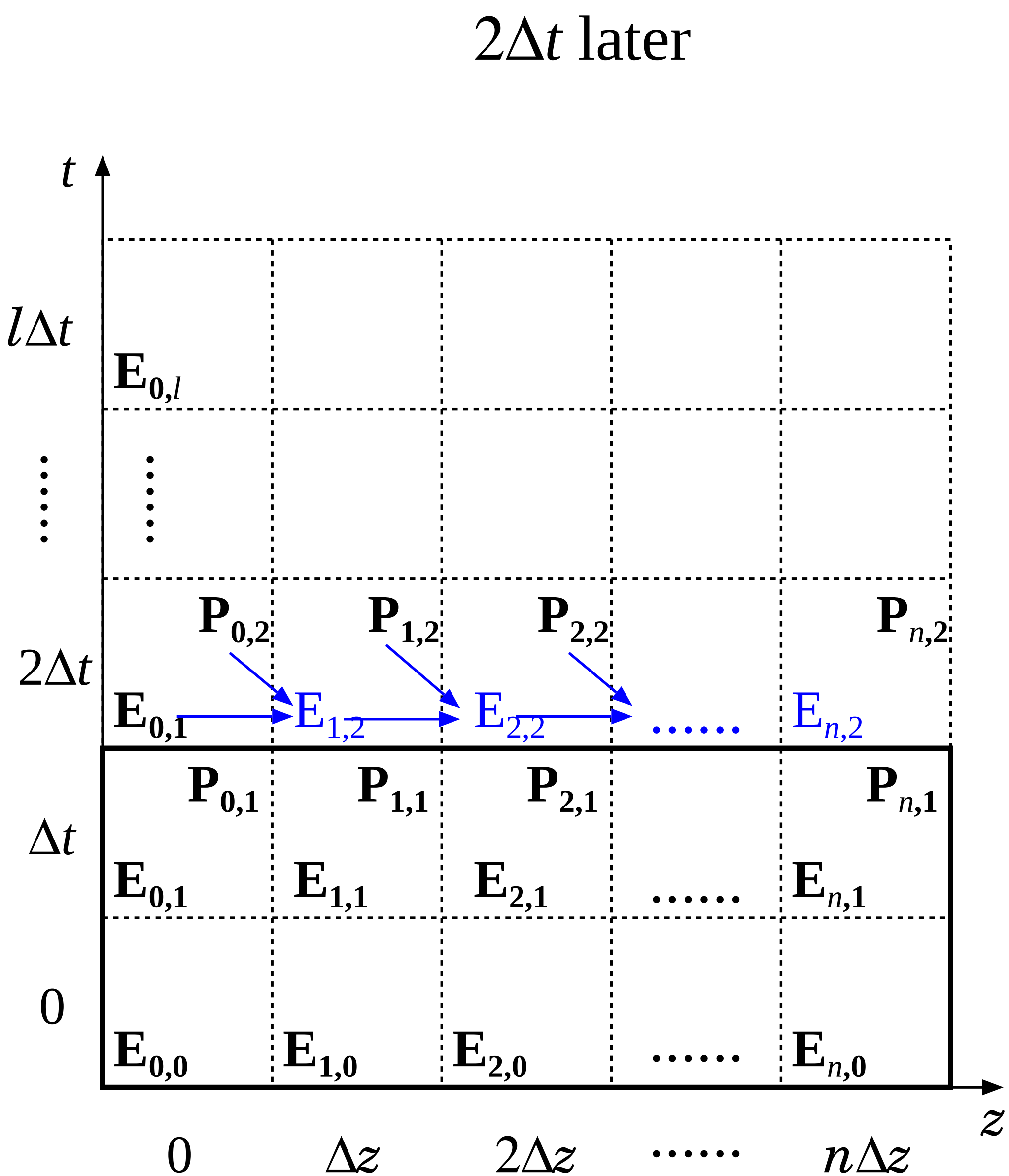
$2\Delta t$ later



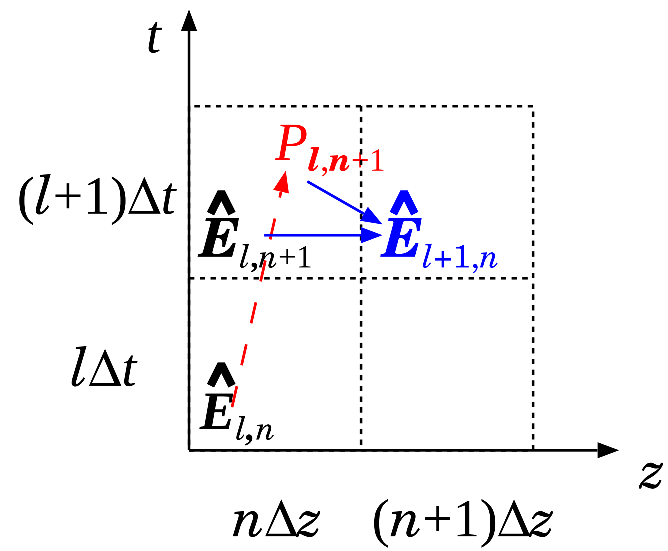
- Calculate **Bloch** equation
- E-field drives Bloch vectors to get polarization



Advancing the Maxwell-Bloch equations

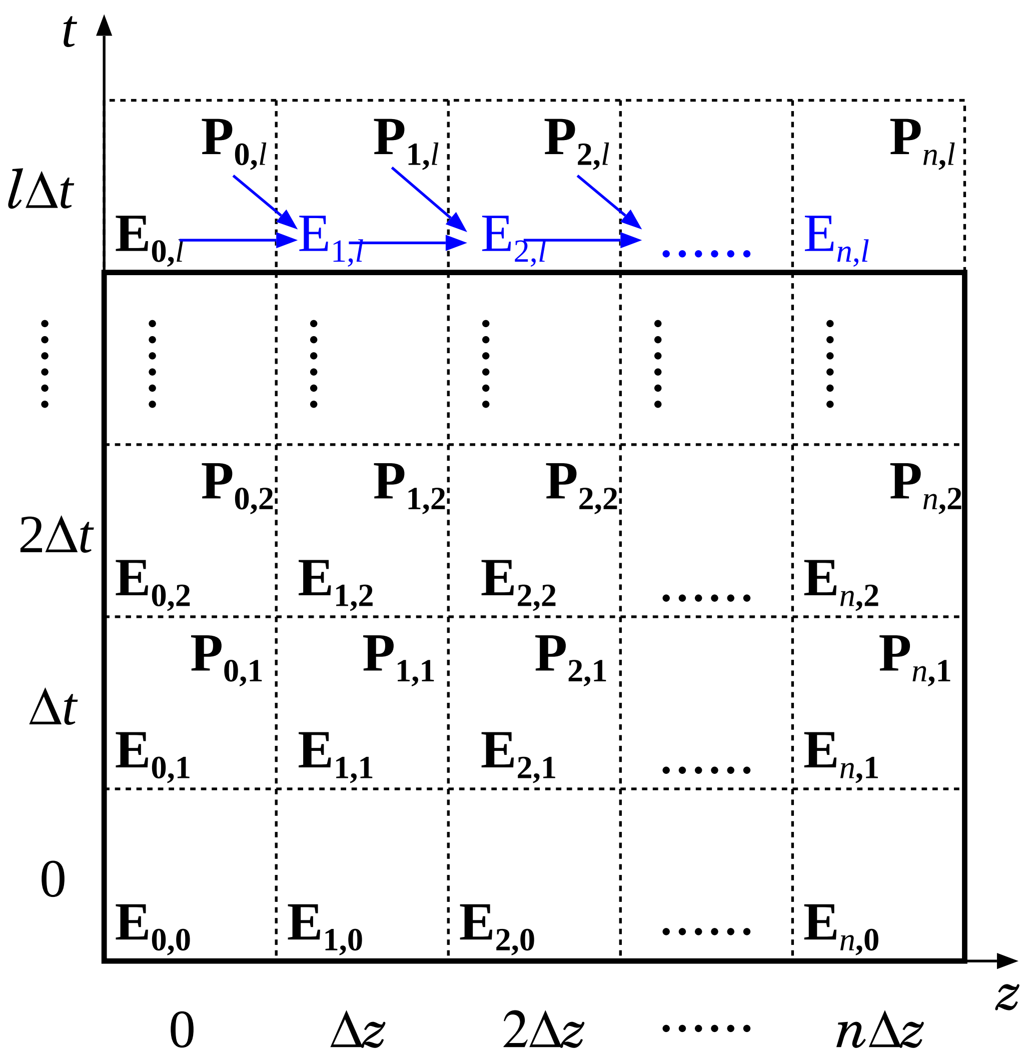


- Calculate **Maxwell's** equation
- Contribute polarization to propagate E-field

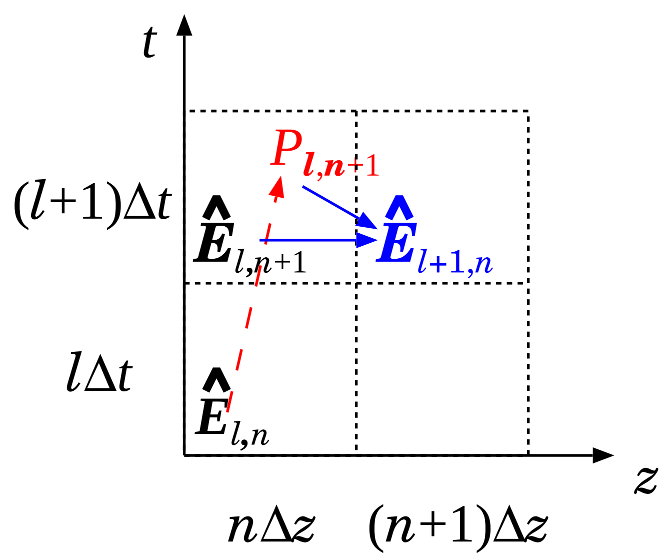


Advancing the Maxwell-Bloch equations

$m\Delta t$ later

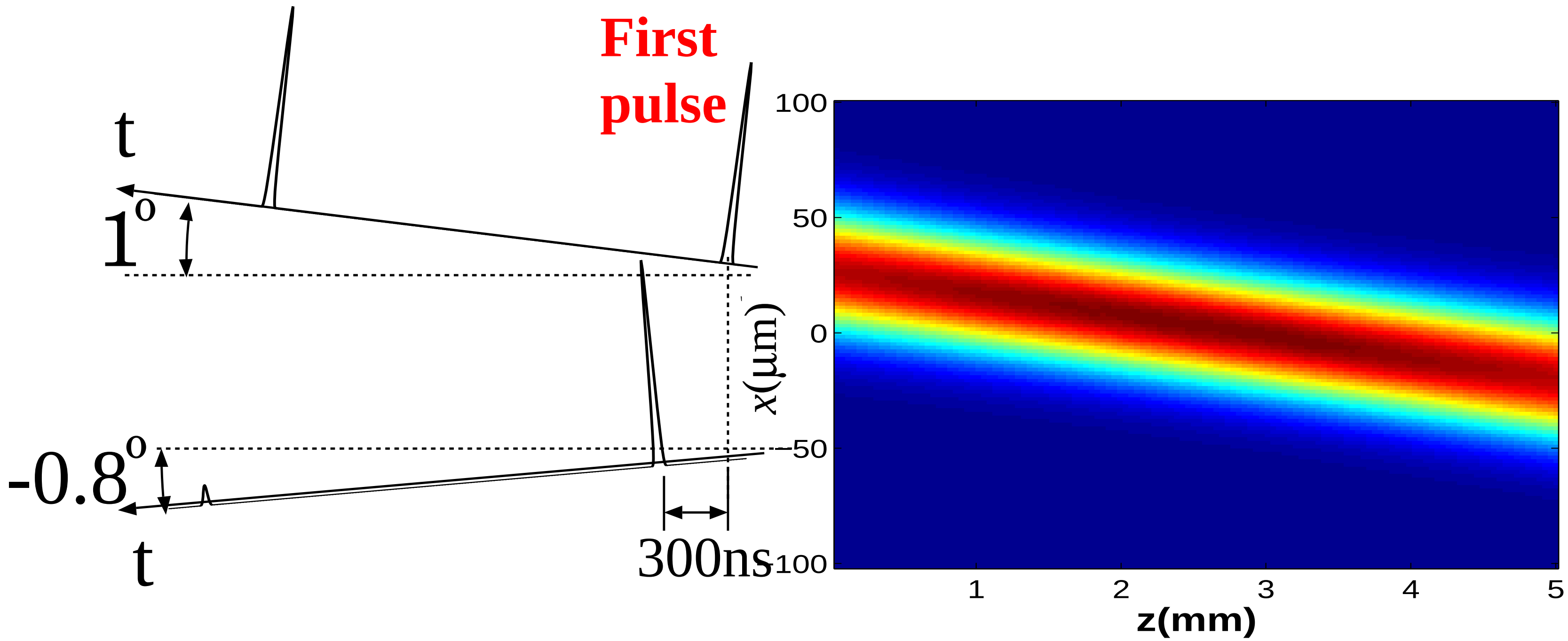


- Calculate **Maxwell's** equation
- Contribute polarization to propagate E-field



Maxwell-Bloch simulation setup

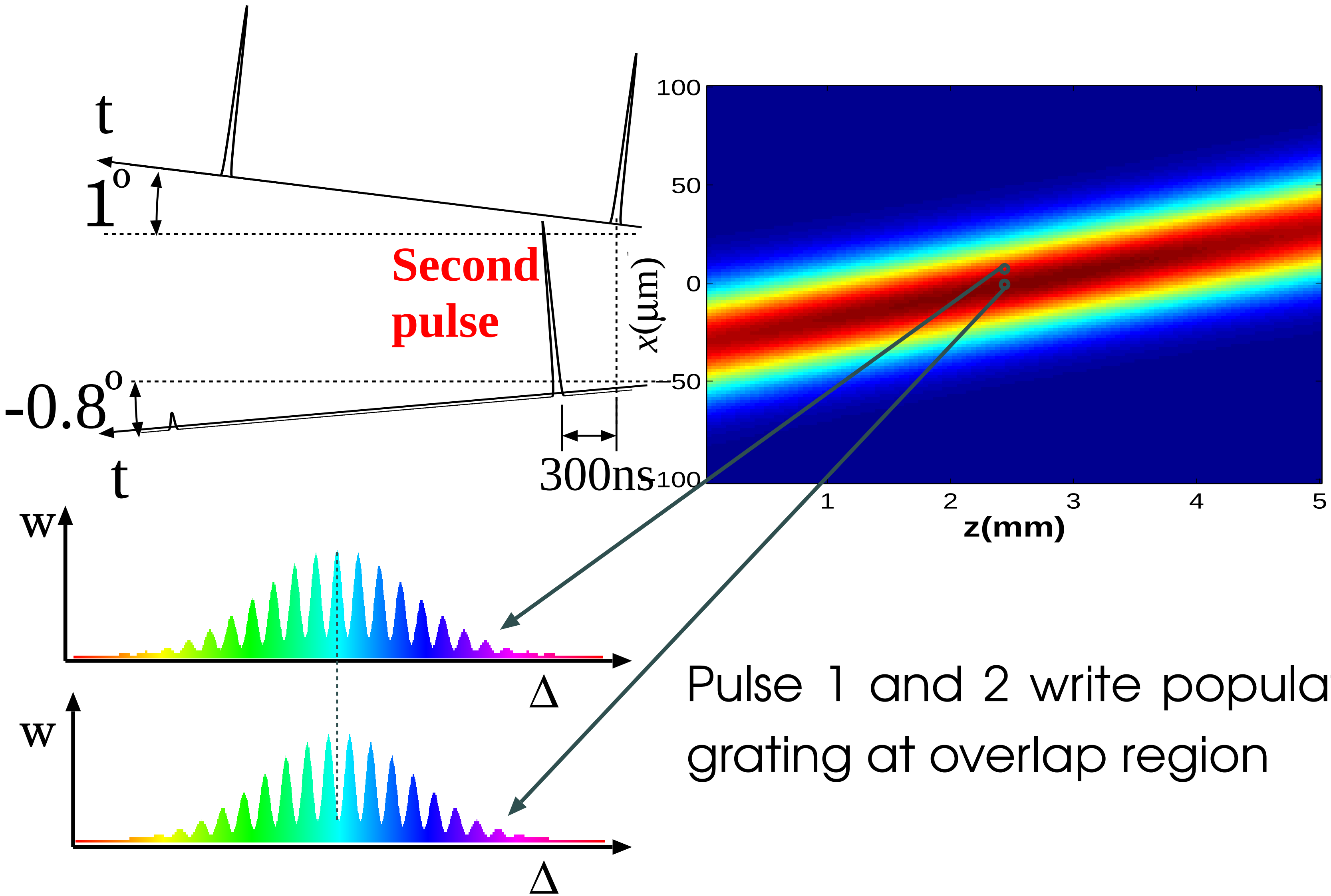
200μm$\times 5$mm, 2.4μsec, 250 spectral lines, 100MHz of BW



Pulse 1 comes in at 1°

Maxwell-Bloch simulation setup

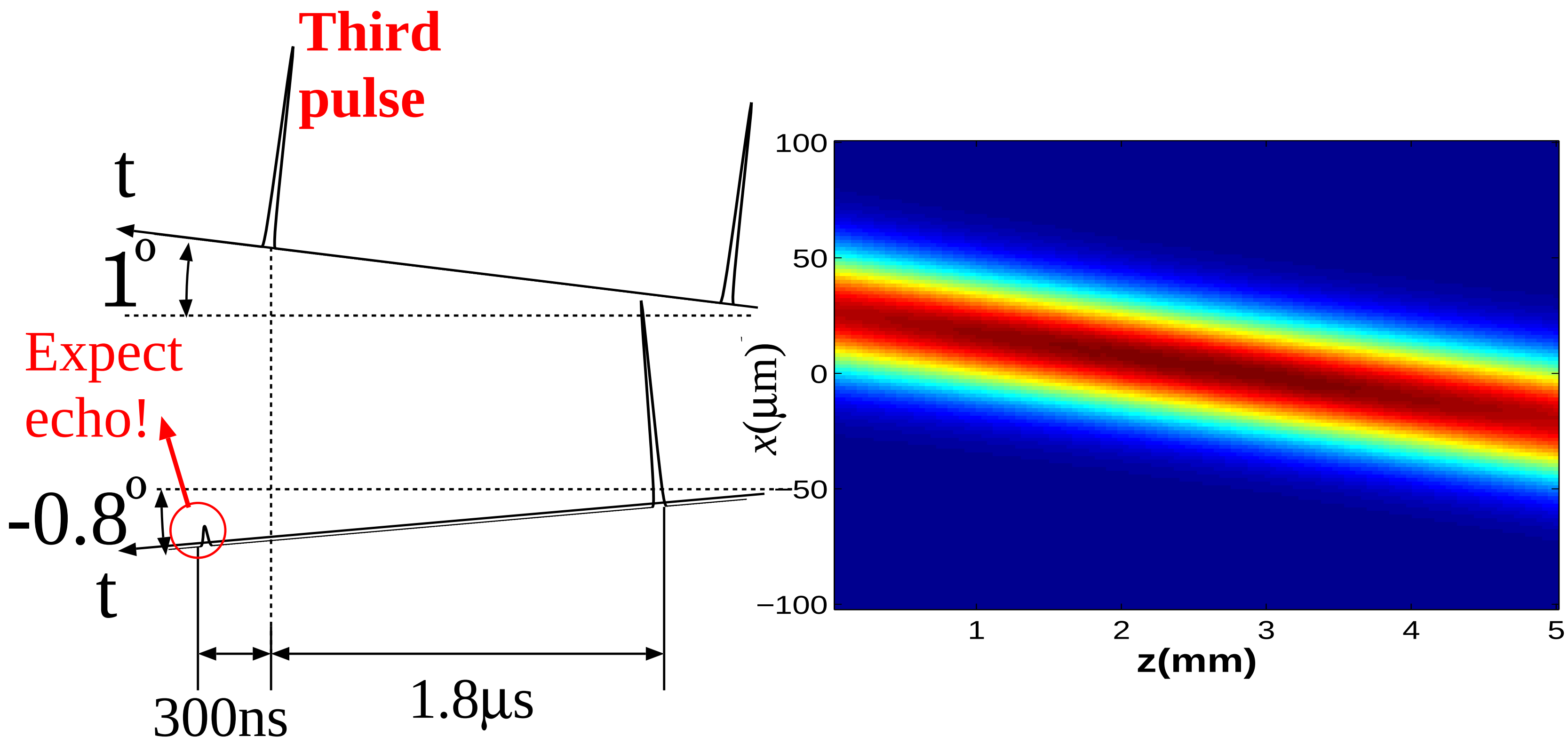
200μm$\times 5$mm, 2.4μsec, 250 spectral lines, 100MHz of BW



Pulse 1 and 2 write population grating at overlap region

Maxwell-Bloch simulation setup

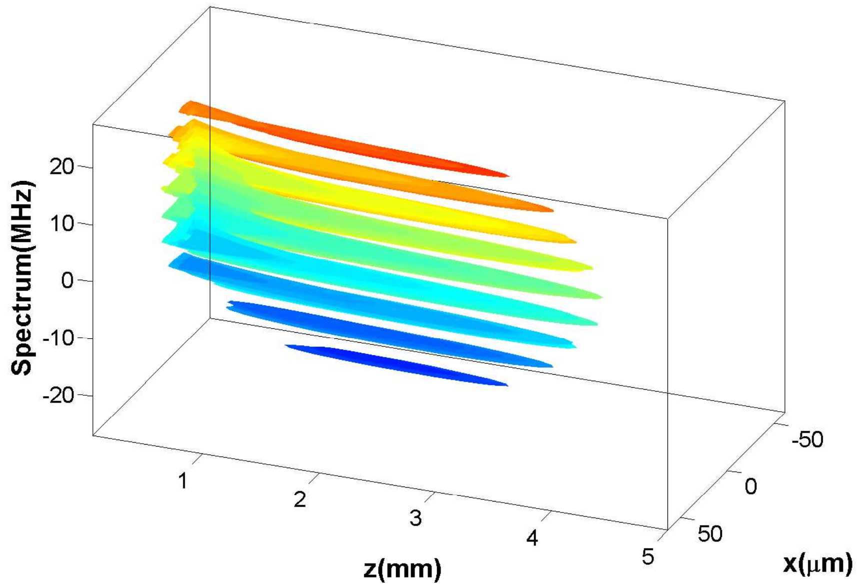
200μm$\times 5$mm, 2.4μsec, 250 spectral lines, 100MHz of BW

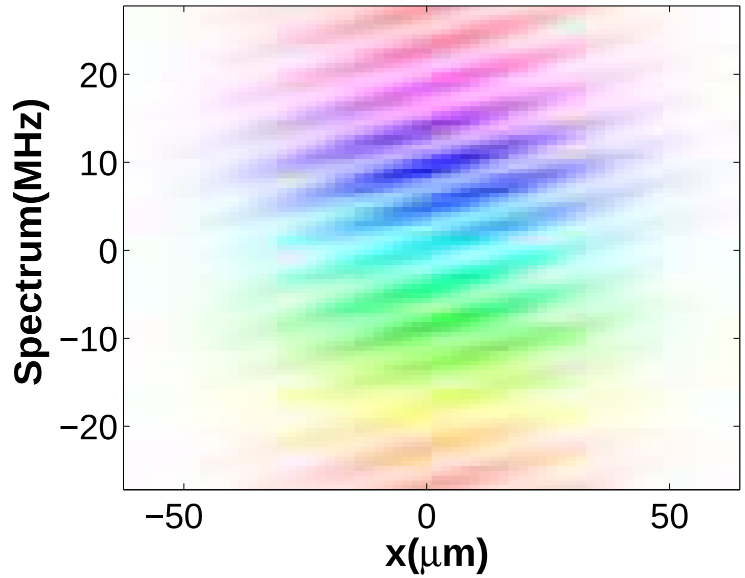


- Pulse 3 comes in at pulse 1 direction
- Population gating diffracts pulse 3 both in space and time
- Echo is expected 300ns later in direction of pulse 2

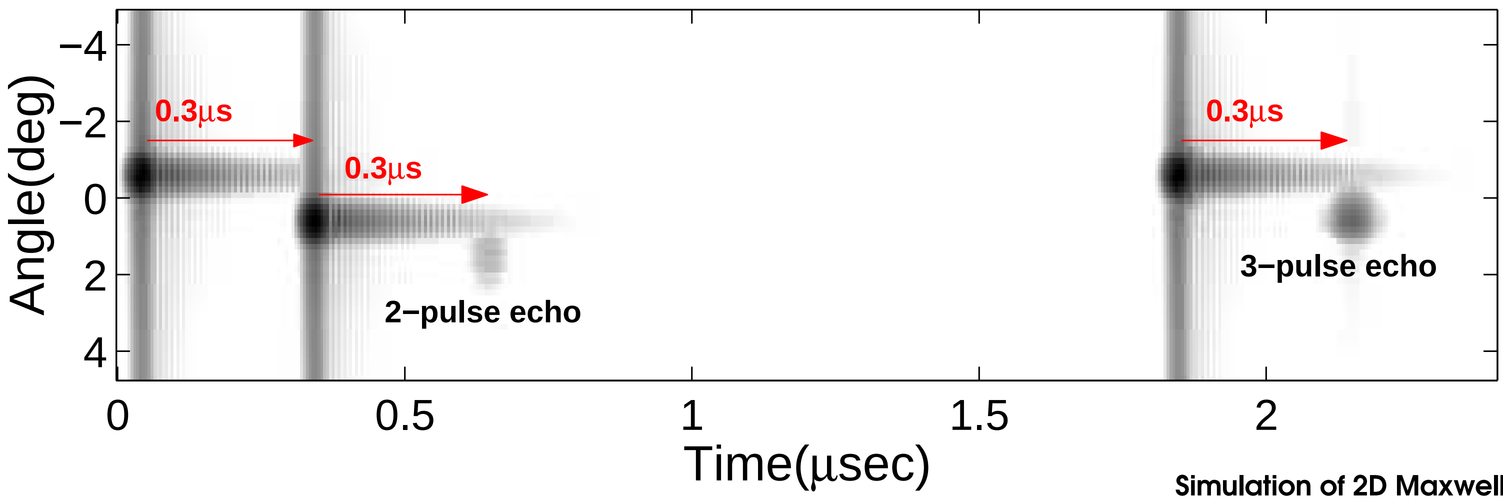
Angled beam photon echo

Volume population grating



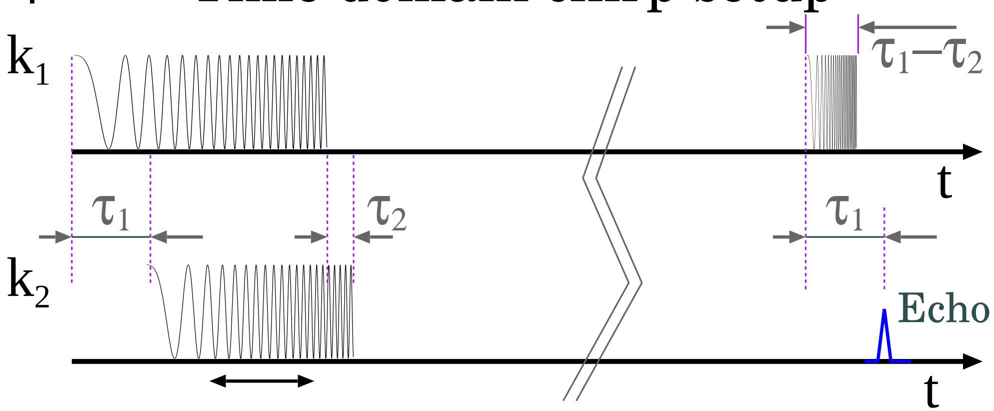


Time-angle waterfall display

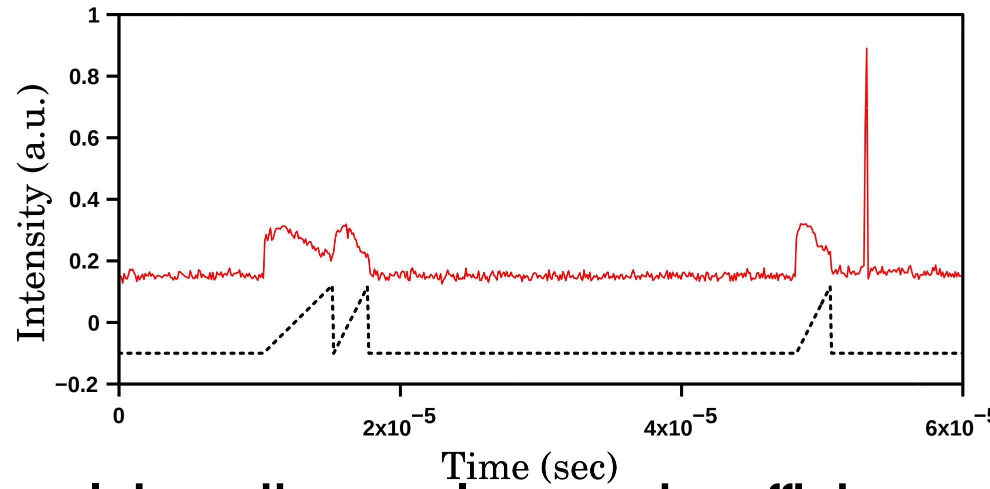


Chirped angled beam photon echoes

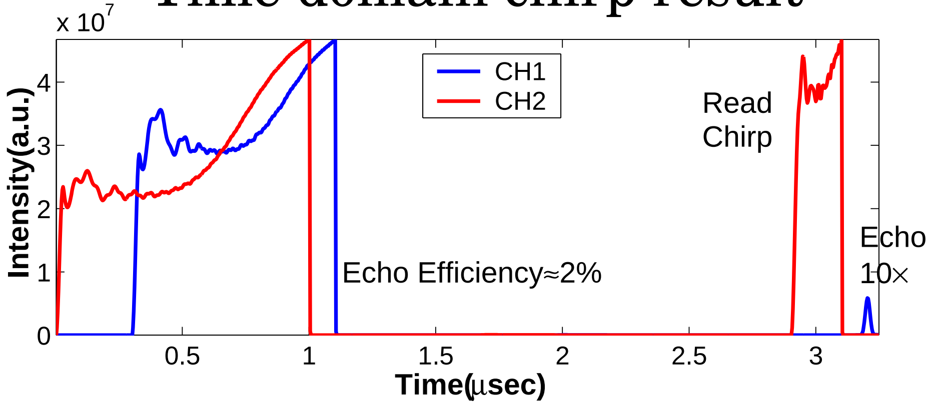
Time domain chirp setup



Chirp photon echo experimental traces

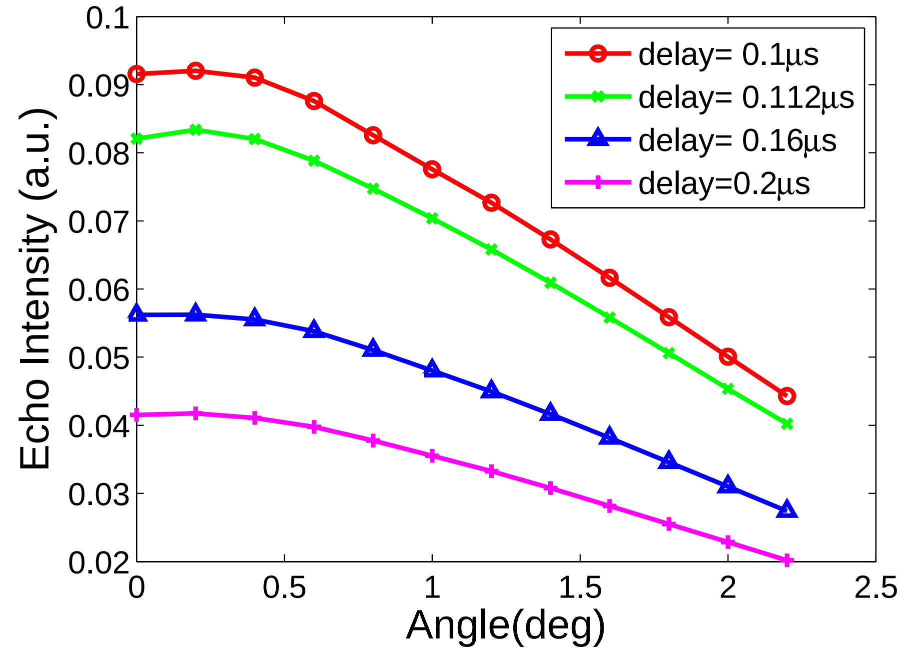


Time domain chirp result

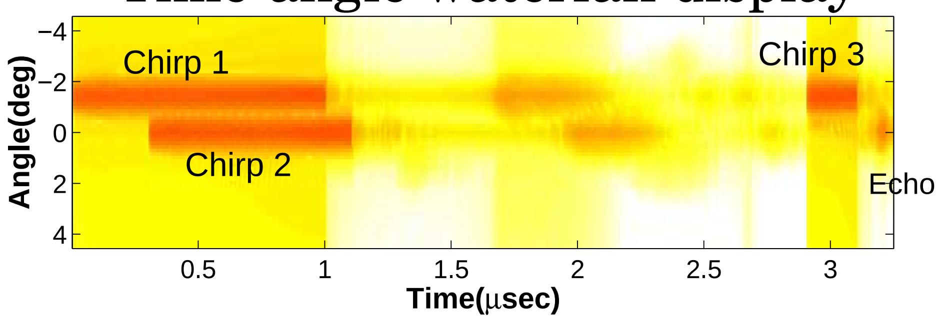


Interaction angle vs. echo efficiency

w_0=60μm, αL=1.0

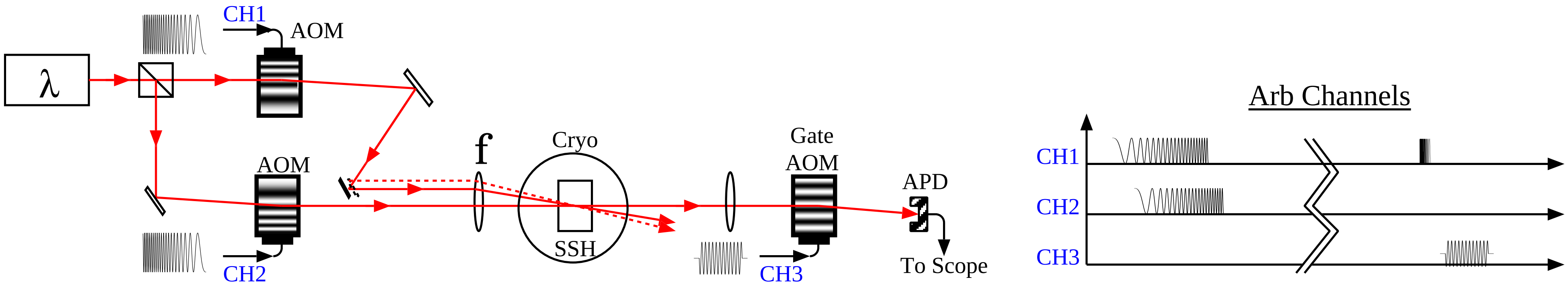


Time-angle waterfall display



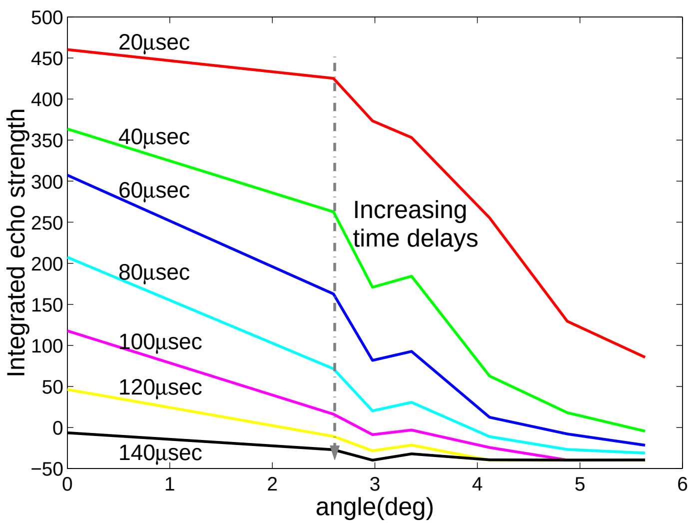
Experimental results

Experimental Setup



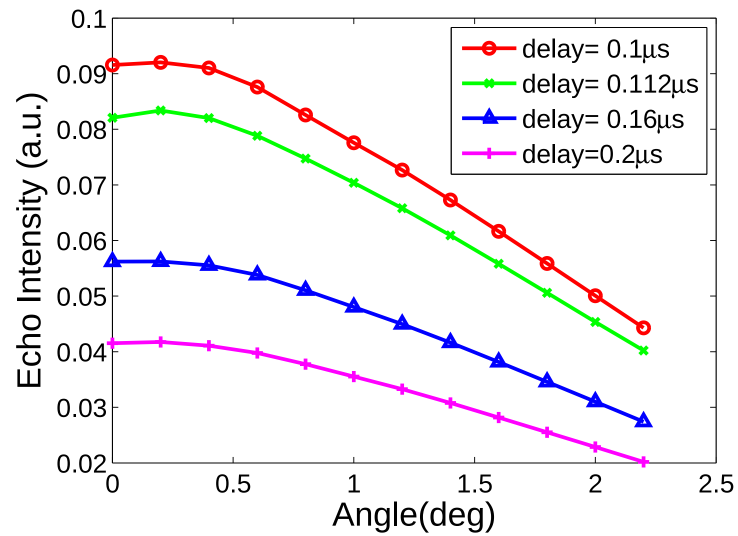
Experimental Results

Spot size: 40μm x 80μm



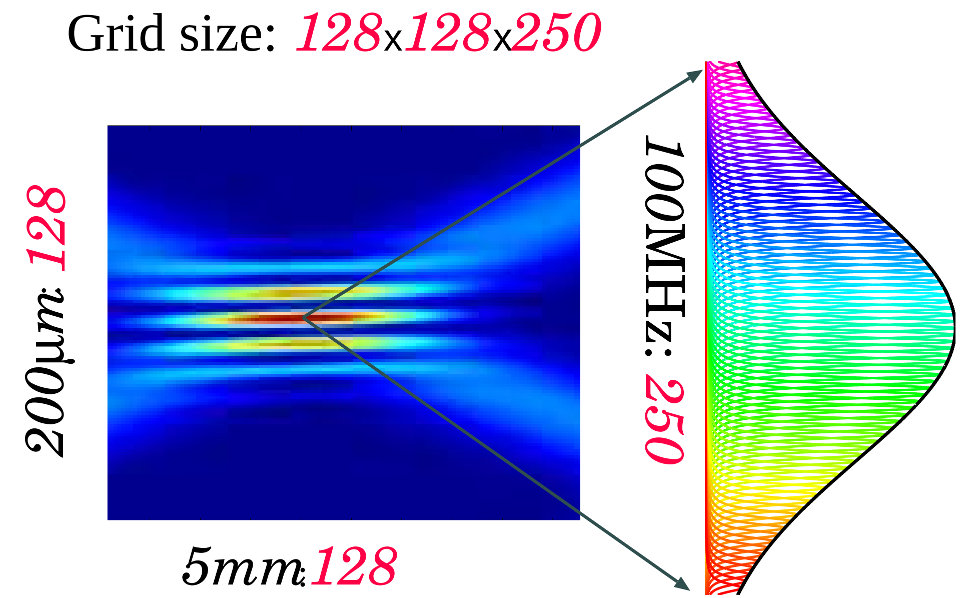
Simulation Results

w_0=60μm, α*L*=1.0



Conclusion

- Spatial-spectral Maxwell-Bloch equations solved using FFT-trapezoidal rule and RK-4



- We are now working towards
  - Increase the number of transverse samples for larger angles
  - Simulate many GHz of bandwidth
  - Simulate realistic T_2 (T_1 effect for accumulation)

Acknowledgement

The authors would like to acknowledge the support from **S. Papport** under the **DARPA AOSP** program. B. Fornberg acknowledges support by **NSF DMS-0309803**. We wish to thank **Tiejun Chang** for valuable discussions.