A new Technique for Simulating Semiconductor Laser Resonators

Britta Heubeck Christoph Pflaum

Department of System Simulation (LSS)
University of Erlangen-Nuremberg, Germany

NUSOD conference, September 1st, 2008

Outline

Outline

Goal

Optimal Laser

Good beam quality ↔ High output power ↔ Stabilized wavelength

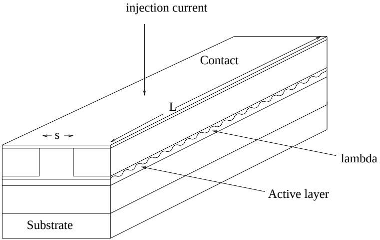
Understanding of the **influence** of different parameters on the dynamics in the laser device

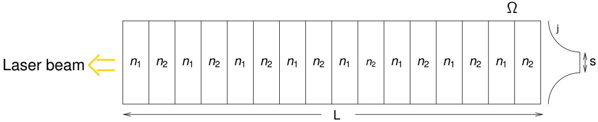
→ **Simulation of the laser device**

SAOT

Distributed Feedback Laser

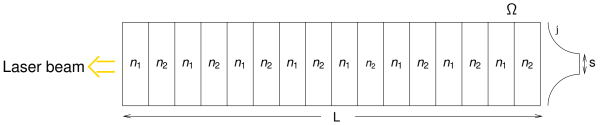
- **Long** resonator with length L
- **Small** stripe width of size s of injection current j
- Layers with different refractive indices (gratings) $\longrightarrow$ **Internal reflections** of the optical wave





Difficulties in the simulation of the optical wave

- Large-scale simulations of the wave equation require a **large number of grid points**
- Internal reflections → **Propagation** in forward and backward direction has to be treated **simultaneously**



Outline

Usual Approaches

Well-known methods for the simulation of optical waves

- Beam Propagation Method
- Finite Difference Time Domain Method
- Finite Element Method (FEM) with standard Finite Elements

But: Internal reflections and large resonators **cannot** be simulated by these methods

$\xrightarrow{1D}$ Transfer Matrix Method (TMM)

But: Tapered lasers need a **2D**-simulation

Idea

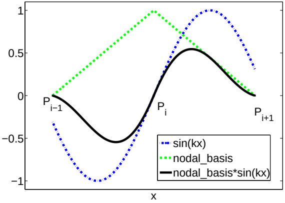
Trigonometric Finite Wave Elements (TFWE)

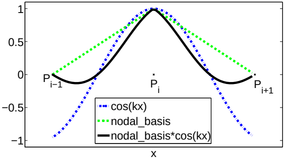
- Special Finite Elements
- Provide the same solution as the **TMM** for 1D Helmholtz equation
- Extendable to **higher dimensions**
- Extendable to **time-dynamic** problems

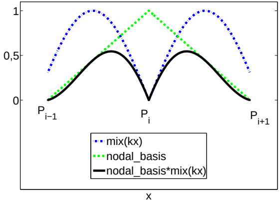
$\rightarrow$ **TFWE method combines advantages of TMM and FEM**

Trigonometric Finite Wave Elements in 1D

1D linear nodal basis functions are multiplied by appropriate **sine** and **cosine** functions





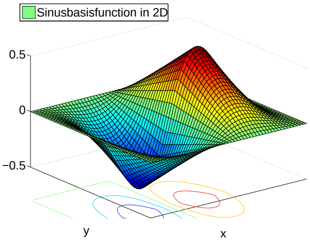


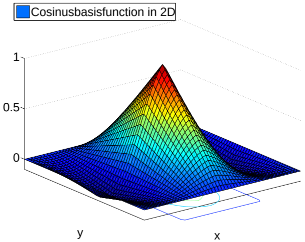
These 3 basis functions per node span the Finite Element space V_h.

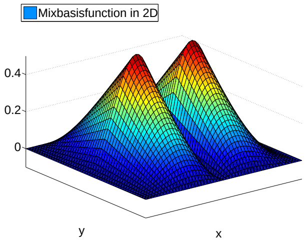
k: wave number

Trigonometric Finite Wave Elements in 2D

Construct TFWE in **2D** by a tensor product of 1D TFWE in propagation direction and linear nodal basis functions in perpendicular direction







Oscillation Assumption

Oscillation Assumption

Let $u \in H^2(\Omega) \cap \mathcal{C}(\Omega)$ **oscillate** with an approximate local wave number k. This means, that $u = u^+ \exp(\mathrm{i}kx) + u^- \exp(-\mathrm{i}kx)$, where $u^+ \exp(\mathrm{i}kx) \in H^2(\tilde{\Omega}_h)$, $u^- \exp(-\mathrm{i}kx) \in H^2(\tilde{\Omega}_h)$, $\|u^+_{xx}\|_{L^2(\tilde{\Omega}_h)} \ll \|u_{xx}\|_{L^2(\Omega)}$, and $\|u^-_{xx}\|_{L^2(\tilde{\Omega}_h)} \ll \|u_{xx}\|_{L^2(\Omega)}$.

1D:

1D: $\tilde{\Omega}_h$: Ω without grid points, $u_{xx} := \frac{d^2u}{dx^2}$

$\tilde{\Omega}_h$: Ω without grid lines, $u_{xx} := \frac{\partial^2 u}{\partial x^2}$

Approximation property of TFWE

Oscillation Assumption

Let $u \in H^2(\Omega) \cap \mathcal{C}(\Omega)$ **oscillate** with local wave number k.

Theorem

Let $u \in H^2(\Omega)$ satisfy the Oscillation Assumption. Then,

$$\|u - I_h^{osc}(u)\|_{H^1(\Omega)} \leq C\boldsymbol{h}(k_{max}\boldsymbol{h} + 1)\Big(\|u^+\|_{H^2(\tilde{\Omega}_h)} + \|u^-\|_{H^2(\tilde{\Omega}_h)}\Big)$$

where C can be chosen **independently** of $\boldsymbol{h}$ and k_{max}, if $\boldsymbol{h}$, $|k|_{H^{1,\infty}(\tilde{\Omega}_h)}$, and $|k|_{H^{2,\infty}(\tilde{\Omega}_h)}$ are bounded from above.

1D: $k_{max} := \max_{1 \leq j \leq N} |k_j|$
k_j: discretized wave numbers
N: number of grid points
$\boldsymbol{h} := h$

2D: $k_{max} := \max_{1 \leq j \leq N_y} \max_{1 \leq i \leq N_x} |k_{ij}|$
k_{ij}: discretized wave numbers
N_x, N_y: number of grid points
$\boldsymbol{h} := \max\{h_x, h_y\}$

$I_h^{osc} : H^2(\Omega) \longrightarrow V_h$: interpolation operator, V_h: TFWE space

Approximation property of standard Finite Elements

Oscillation Assumption

Let $u \in H^2(\Omega) \cap \mathcal{C}(\Omega)$ **oscillate** with local wave number k.

Theorem

Let $u \in H^2(\Omega)$ satisfy the Oscillation Assumption. Then, we have

$$\|u - I_h(u)\|_{H^1(\Omega)} \leq C\boldsymbol{h}(k_{max} + 1)^2 \left(\|u^+\|_{H^2(\tilde{\Omega}_h)} + \|u^-\|_{H^2(\tilde{\Omega}_h)}\right)$$

where C can be chosen **independently** of $\boldsymbol{h}$ and k_{max}, if $\boldsymbol{h}$, $|k|_{H^{1,\infty}(\tilde{\Omega}_h)}$, and $|k|_{H^{2,\infty}(\tilde{\Omega}_h)}$ are bounded from above.

1D: $I_h : H^2(\Omega) \longrightarrow V_h^{lin}$: linear interpolation operator
V_h^{lin}: linear FE space

2D: $I_h : H^2(\Omega) \longrightarrow V_h^{bilin}$: bilinear interpolation operator
V_h^{bilin}: bilinear FE space

Conclusion

Conclusion

Increasing wave number k_{max}, assuming $\boldsymbol{h}k_{max} < C$:

- The upper bound for the approximation error for standard FE **increases** with k_{max}^2
- The upper bound for the approximation error for TFWE keeps **constant**

Remark

The assumption $\boldsymbol{h}k_{max} < C$ is true in most applications, e.g. in computational optics one has $\boldsymbol{h} < \tilde{C}\lambda = \tilde{C}\frac{2\pi}{k}$.

λ: wavelength of the considered optical wave

$C, \tilde{C} > 0$: constants independent of $\boldsymbol{h}$ and k

SAOT

Outline

1. Challenge
2. Simulation Technique
3. Simulation Results

System of Coupled Partial Differential Equations

- Behavior of wave $\tilde{E}$ is described by the wave equation; assuming $\tilde{E}(x,t) = \mathbf{E}(x,t)\exp(i\omega t)$ leads to

$$2\mathrm{i}\frac{\bar{k}(\mathbf{n_A})}{v_g}\frac{\partial \mathbf{E}}{\partial t} = \triangle \mathbf{E} + k^2(\mathbf{n_A})\mathbf{E}$$

- Drift-diffusion equations determine the time-dynamic behavior of the carrier densities $\mathbf{n_A}$ and $\mathbf{n_B}$

$$\frac{\partial \mathbf{n_A}}{\partial t} = \nabla(D_A \nabla \mathbf{n_A}) + C_1 \mathbf{n_B} - C_2 \mathbf{n_A} - r_{rec,A}(|\mathbf{E}|^2, \mathbf{n_A}) \quad \text{and}$$

$$\frac{\partial \mathbf{n_B}}{\partial t} = \nabla(D_B \nabla \mathbf{n_B}) + \tilde{C}_0 j - \tilde{C}_1 \mathbf{n_B} + \tilde{C}_2 \mathbf{n_A} - r_{rec,B}$$

D_A, D_B: ambipolar diffusion constants, j: current density, v_g: group velocity, $r_{rec,A}$,$r_{rec,B}$: recombination densities

System of Coupled Partial Differential Equations

$$
\begin{aligned}
r_{rec,A} &= A_A \mathbf{n_A} + B_A \mathbf{n_A}^2 + C_A \mathbf{n_A}^3 + r_{stim}, \\
r_{rec,B} &= A_B \mathbf{n_B} + B_B \mathbf{n_B}^2, \\
r_{stim} &= \frac{c}{n_g} g_{nonlin} n, \\
g_{nonlin} &= \begin{cases} \frac{g_0}{1+\varepsilon n} ln(\frac{\mathbf{n_A}}{n_{tr}}) & \text{if } |\mathbf{n_A}| \geq n_{tr} \\ \frac{g_0}{1+\varepsilon n} (\frac{\mathbf{n_A}}{n_{tr}} - 1) & \text{if } |\mathbf{n_A}| < n_{tr} \end{cases} \\
k &= \frac{\omega n'}{c} + \alpha_H \frac{g_{nonlin}}{2} + \mathrm{i} \frac{g_{nonlin} - \alpha_0}{2} \\
n &= \frac{\epsilon_0 n' n_g}{2\hbar\omega} |\tilde{E}|^2 = \frac{\epsilon_0 n' n_g}{2\hbar\omega} |\mathbf{E}|^2.
\end{aligned}
$$

A_A, B_A, C_A, A_B, B_B: recombination coefficients, n_{tr}: transparency carrier density, α_H: Henry factor

g_0: differential gain, ε: gain compression factor, α_0: absorption

Absorbing Boundary Condition (PML)

- Wave is considered on domain $\Omega =]0, L[\times] - \frac{W}{2}, \frac{W}{2}[$
- Wave is emitted on left hand side (at $x = 0$) $\longrightarrow$ **Absorbing boundary condition** $ik\mathbf{E}(0, y) = \frac{\partial \mathbf{E}}{\partial x}(0, y)$
- Absorbing boundary is simulated by **PML** $\longrightarrow$
  - Domain Ω is increased to $\tilde{\Omega} =] - \delta, L[\times] - \frac{W}{2}, \frac{W}{2}[$
  - Schrödinger equation is transformed to

$$2\mathrm{i}\frac{\tilde{\bar{k}}}{v_g}\frac{\partial \mathbf{E}}{\partial t} = \frac{\partial}{\partial x}\frac{k}{\tilde{k}}\frac{\partial E}{\partial x} + \frac{\tilde{k}}{k}\frac{\partial^2 E}{\partial y^2} + k\tilde{k}\mathbf{E}$$

$\tilde{k} := k - \mathrm{i}\sigma(x) := k_0\tilde{n} - \mathrm{i}\sigma(x)$

$\tilde{\bar{k}} := \bar{k} - \mathrm{i}\sigma(x) := k_0\bar{n} - \mathrm{i}\sigma(x)$

$\sigma(x) := \sigma_c \frac{x^2}{\delta^2}$ for $x < 0$, $\sigma(x) := 0$ for $x \geq 0$

$\sigma_c := \frac{3}{2\delta}\log(1/R_0)$

$R_0 := 10^{-4}$ theoretical reflexion coefficient

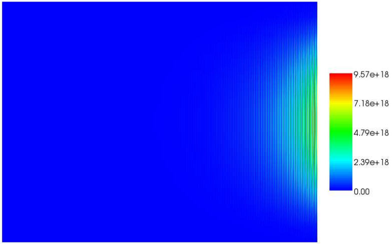
Discretization

- **Spatial** discretization of the wave equation in propagation direction: at least one grid point per layer for resolving internal reflections
- **Temporal** discretization: wave with frequency $\omega \approx 1.935 PHz \longrightarrow$ time step size $\tau \approx 0.3 ps \longrightarrow \omega\tau \approx 600$.

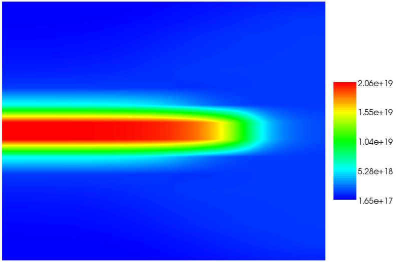
In the following examples: 840 layers, 841 grid points in propagation direction, 16/31 grid points in perpendicular direction, total simulation time: $4ns$, current: $480mA$, stripe width: $2.5\mu m$.

Photon and Carrier density ($\alpha_H = 0.0$)

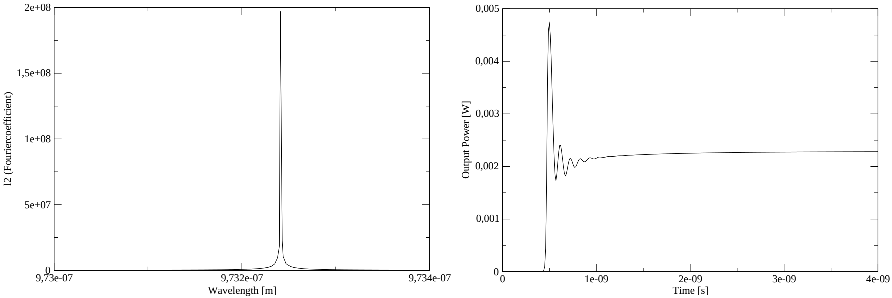
Photon density n



Carrier density n_A

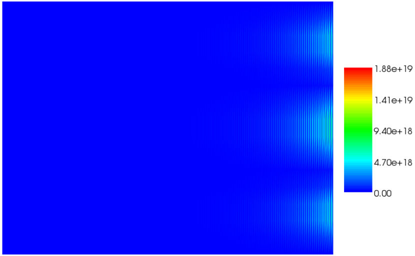


Frequency Spectrum and Output Power ($\alpha_H = 0.0$)

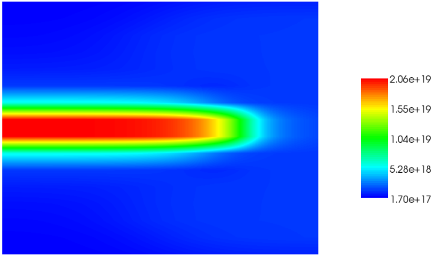


Photon and Carrier density ($\alpha_H = -0.5$)

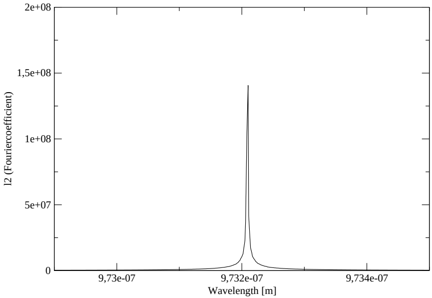
Photon density n

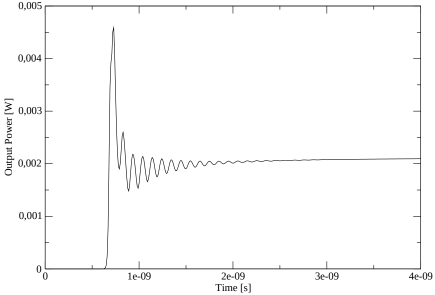


Carrier density n_A



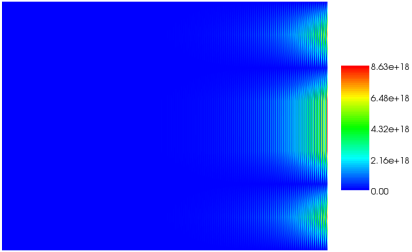
Frequency Spectrum and Output Power ($\alpha_H = -0.5$)



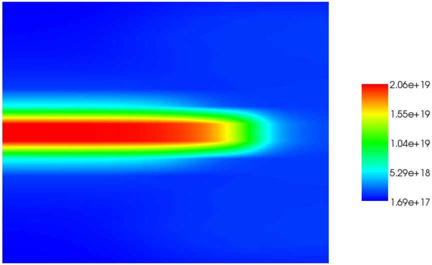


Photon and Carrier density ($\alpha_H = -2.5$)

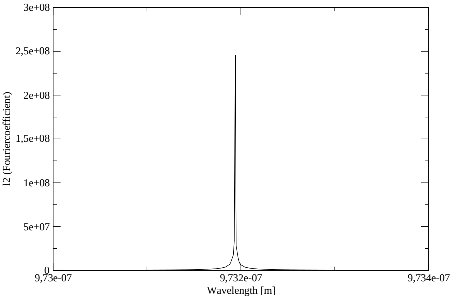
Photon density n

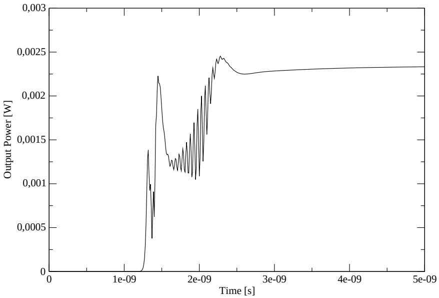


Carrier density n_A



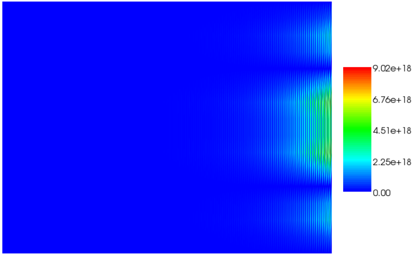
Frequency Spectrum and Output Power ($\alpha_H = -2.5$)



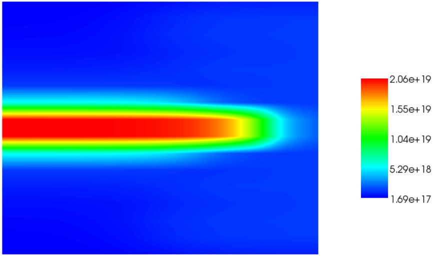


Photon and Carrier density ($\alpha_H = -3.0$)

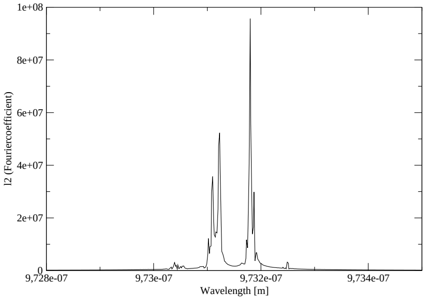
Photon density n

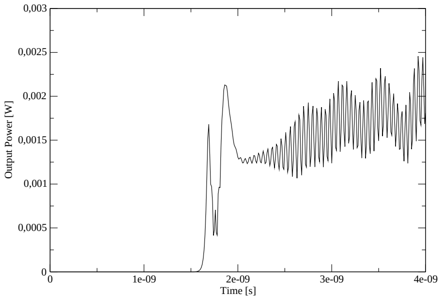


Carrier density n_A



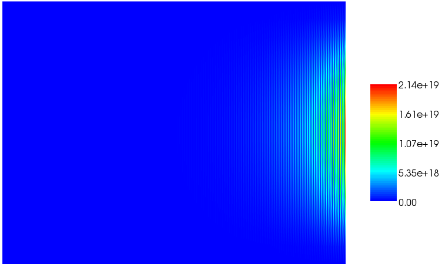
Frequency Spectrum and Output Power ($\alpha_H = -3.0$)



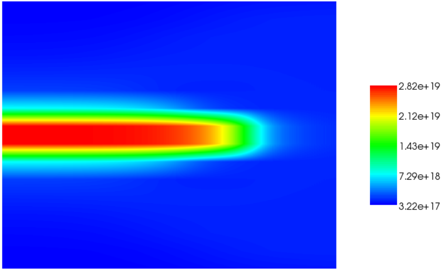


Photon and Carrier density ($\alpha_H = 0.0$, $I = 960mA$)

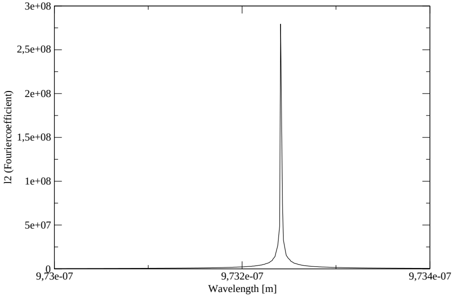
Photon density n

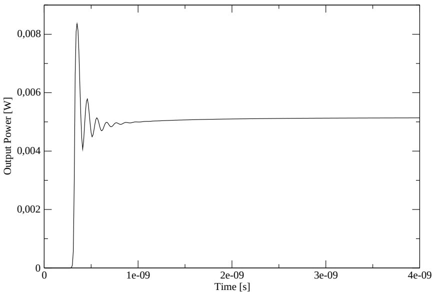


Carrier density n_A



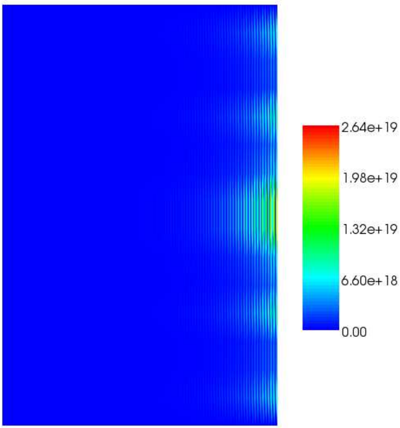
Frequency Spectrum and Output Power ($\alpha_H = 0.0, I = 960mA$)



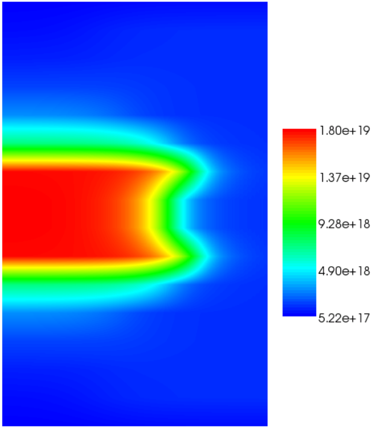


Photon and Carrier density ($\alpha_H = -2.0, s = 25\mu m$)

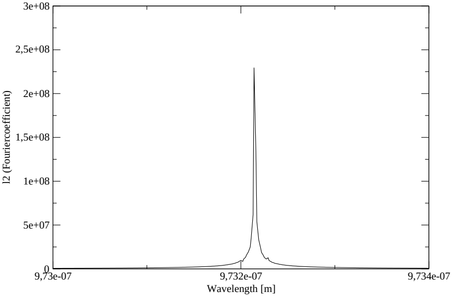
Photon density n

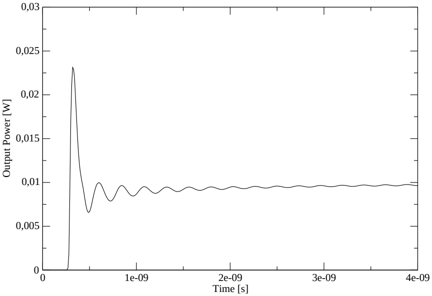


Carrier density n_A



Frequency Spectrum and Output Power ($\alpha_H = -2.0, s = 25\mu m$)

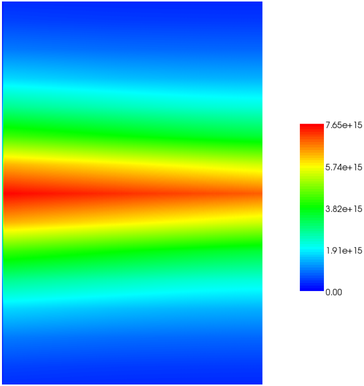




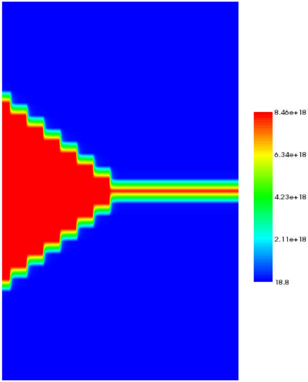
SAOT

Tapering

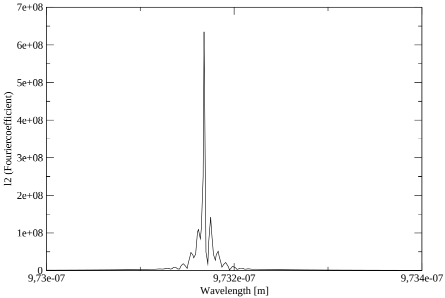
Zoom in photon density n

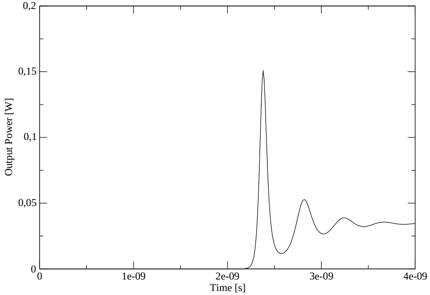


Carrier density n_A



Tapering





SAOT

Summary

TFWE method combines advantages of FEM and TMM

- TFWE can be applied to time-periodic and **time-dynamic** wave problems
- **Internal reflections** can be simulated in 2D
- TFWE lead to better **performance** than standard FE

Simulation can support the tuning of DFB lasers

Influence of stripe width, current, Henry factor,... on the resulting mode, wavelength, and output power can be examined

Outlook

- Comparison: **Experiment** $\leftrightarrow$ **Simulation**
- Simulation of **different** laser devices (disc lasers, three-section lasers,...)
- Extension of the simulation to **3D**
- Introduction of **multigrid** method for solving the wave equation