

Spin-dynamical theory of ultrafast pulse propagation in active semiconductor microcavities

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Outline

- *Theoretical fundamentals of the dynamical model*
- *Maxwell-pseudospin equations in 2D for a QW-active medium modelled by a degenerate four-level system*
- *Simulation results for 2D Self-Induced Transparency Effects in:*
 - *planar optical waveguides*
 - *semiconductor microcavities*
- *Summary and conclusions*

Group-theoretical formalism of Hioe and Eberly

Description of optical resonance phenomena simplified by the use of the Bloch spin or pseudospin vector (NMR):

$$\frac{d\mathbf{S}}{dt} = \gamma \mathbf{B} \times \mathbf{S}$$

valid for physical systems with equally spaced energy levels

Solution of the problem for a general N-level system with an arbitrary level spacing given by Hioe and Eberly

Atom/molecule /quantum system : E_m – unperturbed energies

$|m\rangle$ - *eigenstates*

$\hat{V}(t)$ - *time-dependent perturbation (not necessarily small)*

System Hamiltonian in the presence of perturbation:

$$\hat{H} = \sum_m E_m \hat{P}_{mn} + \sum_{m,n} V_{mn}(t) \hat{P}_{mn}$$

$$\hat{P}_{mn} = |m\rangle\langle n| \quad \text{Transition-projection operators}$$

$$V_{mn}(t) = \langle m | \hat{V}(t) | n \rangle, \quad m, n = 1, 2, \dots, N \quad N - \text{number of eigenstates of } \hat{H}$$

Group-theoretical formalism of Hioe and Eberly

N^2 many P 's $\rightarrow$ generators of the unitary group $U(N)$

Fundamental commutation relation:

$$[\hat{P}_{kl}, \hat{P}_{mn}] = \hat{P}_{kn} \delta_{lm} - \hat{P}_{ml} \delta_{nk}$$

Schrödinger picture: dynamical evolution of the quantum system described by Liouville equation:

$$i\hbar \frac{\partial \hat{\rho}}{\partial t} = [\hat{H}, \hat{\rho}]$$

Since $\langle \hat{P}_{mn}(t) \rangle = \text{Tr} \{ \hat{P}_{mn} \hat{\rho}(t) \} = \rho_{mn}(t)$ dynamics can be equally well explored by P_{mn} operators in the Heisenberg picture

We use P 's to define another set of N^2-1 operators formed in 3 groups (u, v, w):

$$\hat{u}_{jk} = \hat{P}_{jk} + \hat{P}_{kj} \quad 1 \leq j < k \leq N$$

$$\hat{v}_{jk} = -i(\hat{P}_{jk} - \hat{P}_{kj}) \quad 1 \leq l \leq N-1$$

$$\hat{w}_l = -\left[\frac{2}{l(l+1)} \right]^{1/2} (P_{11} + \dots + P_{ll} - l P_{l+1, l+1}) \quad \text{Traceless operators!!!}$$

Easy to check for $N=2$, (u, v, w) identical with Pauli matrices $\sigma_x, \sigma_y, \sigma_z$

Group-theoretical formalism of Hioe and Eberly

u, v, w designed to generate the algebra $SU(N)$, $\Rightarrow$

$\vec{\lambda} = \{\hat{\lambda}_i\} = (\hat{u}_{12}, \dots, \hat{v}_{12}, \dots, \hat{w}_1, \dots, \hat{w}_{N-1})$ the ordered array of u, v, w 's satisfies:

$$[\hat{\lambda}_j, \hat{\lambda}_k] = 2i f_{jkl} \hat{\lambda}_l$$

f_{jkl} completely antisymmetric tensor of the structure constants of $SU(N)$

Using orthogonality relation $Tr(\hat{\lambda}_j \hat{\lambda}_k) = 2 \delta_{jk}$

$$\hat{\rho}(t) = \frac{1}{N} \hat{I} + \frac{1}{2} \sum_{j=1}^{N^2-1} S_j(t) \hat{\lambda}_j$$

$$\hat{H}(t) = H_o + H_{int}(t) = \frac{\hbar}{2} \left[\frac{2}{N} \left(\sum_{k=1}^N \omega_k \right) \hat{I} + \sum_{j=1}^{N^2-1} \gamma_j(t) \hat{\lambda}_j \right]$$

$$\gamma_j(t) = \frac{1}{\hbar} Tr(\hat{H}(t) \hat{\lambda}_j)$$

torque vector

$$S_j(t) = Tr[\hat{\rho}(t) \hat{\lambda}_j]$$

Pseudospin (coherence) vector

Generalised pseudo spin formalism

Equation of motion for the state vector

By taking the components of Liouville eq. the time evolution of $\hat{\rho}$ can be expressed by the evolution of a real state vector $S = (S_1, S_2, \dots, S_{N^2-1})$:

$$\dot{S}_i = f_{ijk} \gamma_j S_k \quad i, j, k = 1, \dots, N^2 - 1$$

Generalised rotation of S in N^2-1 dimensional space; antisymmetric tensor f_{ijk} guarantees constant length:

$$|\vec{S}| = \text{const.}$$

Time evolution of a 2-level quantum system with electric dipole transitions excited by linearly polarised light ($\Delta J_z = 0$)

For $H_{int} = e\mathbf{E} \cdot \hat{\mathbf{Q}}$ and $N=2$ optical Bloch equation for a 2-level system:

$$\frac{d\mathbf{S}}{dt} = \boldsymbol{\gamma} \times \mathbf{S}$$

$$\begin{aligned}\frac{\partial \mathbf{S}_1}{\partial t} &= \omega_o \mathbf{S}_2 \\ \frac{\partial \mathbf{S}_2}{\partial t} &= -\omega_o \mathbf{S}_1 + 2\Omega_R \mathbf{S}_3 \\ \frac{\partial \mathbf{S}_3}{\partial t} &= -2\Omega_R \mathbf{S}_2\end{aligned}$$

$\mathbf{S} = (\mathbf{S}_1, \mathbf{S}_2, \mathbf{S}_3)$ real 3-vector

$\boldsymbol{\gamma} = (2\Omega_R, 0, \omega_o)$ torque (Bloch) vector for electric dipole transitions ($\Delta J_z = 0$) ; ω_o - atomic transition resonant frequency

Rabi frequency: $\Omega_R = \frac{\wp}{\hbar} E$; $\wp = eq_o$ dipole coupling coefficient

Time evolution of a 2-level quantum system with electric dipole transitions excited by circularly polarised light ($\Delta J_z = \pm 1$)

For electric dipole transitions ($\Delta J_z = \pm 1$) excited by circularly polarised optical wave :

$$\frac{d\mathbf{S}}{dt} = \boldsymbol{\gamma} \times \mathbf{S}$$

$$\boldsymbol{\gamma} = (\Omega_x, \Omega_y, \omega_o)$$

$$\frac{\partial \mathbf{S}_1}{\partial t} = -\omega_o \mathbf{S}_2 - \Omega_y \mathbf{S}_3$$

$$\frac{\partial \mathbf{S}_2}{\partial t} = \omega_o \mathbf{S}_1 + \Omega_x \mathbf{S}_3$$

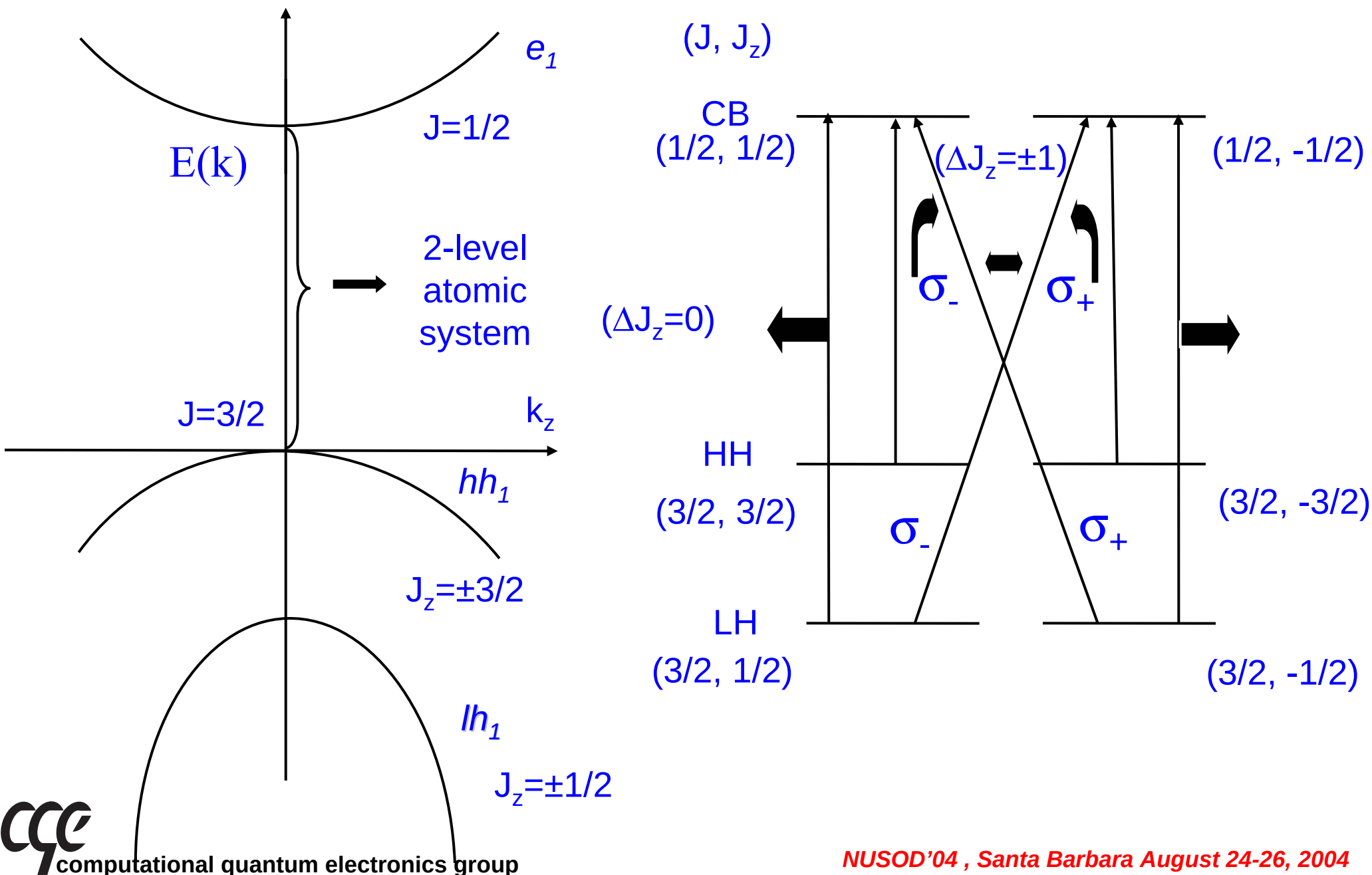
$$\frac{\partial \mathbf{S}_3}{\partial t} = \Omega_y \mathbf{S}_1 - \Omega_x \mathbf{S}_2$$

torque (Bloch) vector

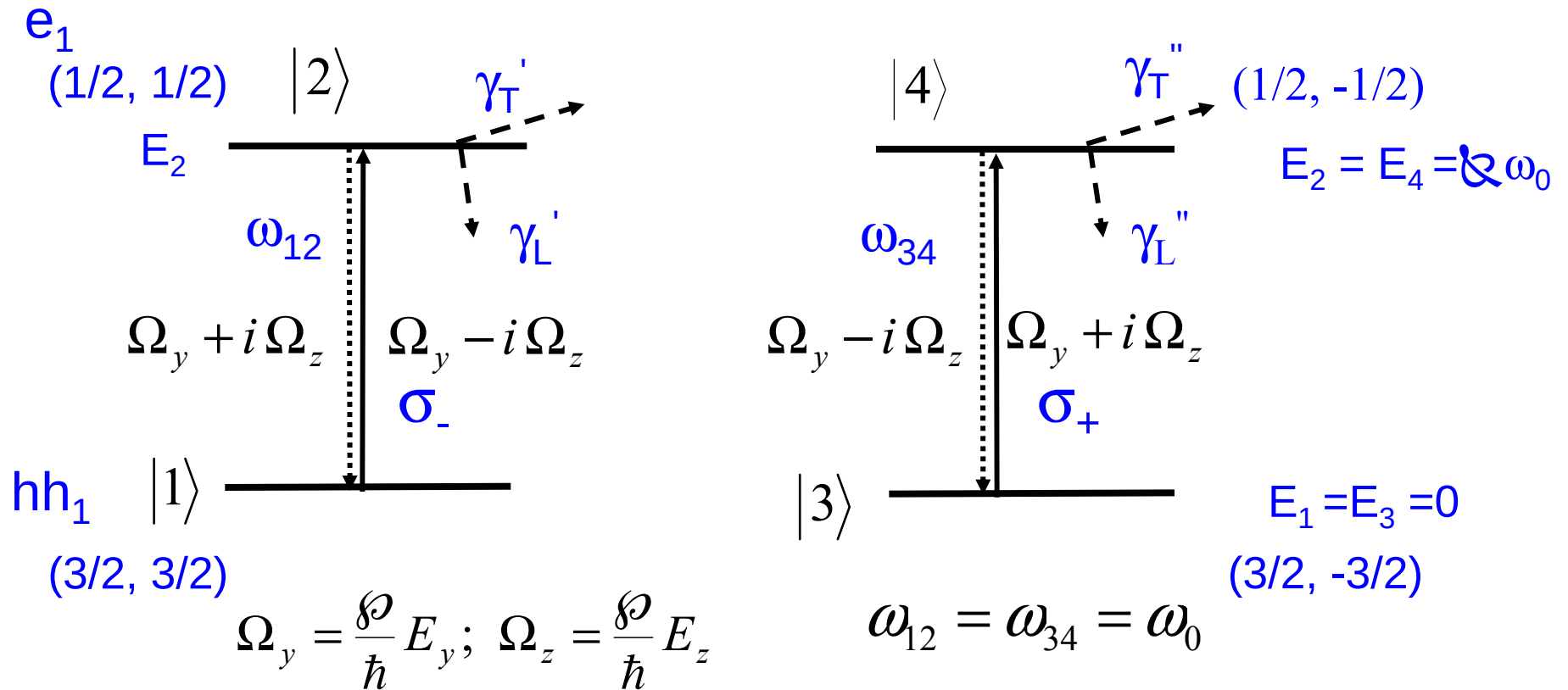
Rabi frequencies:

$$\Omega_x = \frac{\wp}{\hbar} E_x; \quad \Omega_y = \frac{\wp}{\hbar} E_y$$

Schematic diagram of a III-V semiconductor quantum well dipole transitions



Schematic diagram of the selection rules for the transitions from the heavy-hole valence band to CB



System Hamiltonian:

$$\hat{H} = \hbar \begin{pmatrix} 0 & \Omega_y - i\Omega_z & 0 & 0 \\ \Omega_y + i\Omega_z & \omega_0 & 0 & 0 \\ 0 & 0 & 0 & \Omega_y + i\Omega_z \\ 0 & 0 & \Omega_y - i\Omega_z & \omega_0 \end{pmatrix}$$

Generators of $SU(4)$ Lie Algebra

$$\hat{\lambda}_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \hat{\lambda}_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \hat{\lambda}_3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \hat{\lambda}_4 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \quad \hat{\lambda}_5 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\hat{\lambda}_6 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad \hat{\lambda}_7 = \begin{pmatrix} 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \hat{\lambda}_8 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \hat{\lambda}_9 = \begin{pmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \hat{\lambda}_{10} = \begin{pmatrix} 0 & 0 & 0 & i \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}$$

$$\hat{\lambda}_{11} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & i \\ 0 & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{pmatrix} \quad \hat{\lambda}_{12} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \end{pmatrix} \quad \hat{\lambda}_{13} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\hat{\lambda}_{14} = \frac{1}{\sqrt{3}} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \hat{\lambda}_{15} = \frac{1}{\sqrt{6}} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 3 \end{pmatrix}$$

Nonvanishing components of f-tensor

f_{ijk} has 174 nonvanishing elements grouped in 29 permutations of i,j,k

ijk	1,2,9	1,3,8	1,4,11	1,5,10	1,7,13	2,3,7	2,5,12
f_{ijk}	-1/2	-1/2	-1/2	-1/2	1	1/2	-1/2
ijk	2,6,11	2,8,13	2,8,14	3,4,12	3,6,10	3,9,13	3,9,14
f_{ijk}	-1/2	-1/2	$\sqrt{3}/2$	-1/2	-1/2	1/2	$\sqrt{3}/2$
ijk	4,5,7	4,6,9	4,10,13	4,10,14	4,10,15	5,6,8	5,11,13
f_{ijk}	-1/2	-1/2	1/2	$1/2\sqrt{3}$	$\sqrt{2}/\sqrt{3}$	-1/2	-1/2
ijk	5,11,14	5,11,15	6,12,14	6,12,15	7,8,9	7,10,11	8,11,12
f_{ijk}	$1/2\sqrt{3}$	$\sqrt{2}/\sqrt{3}$	$-1/\sqrt{3}$	$\sqrt{2}/\sqrt{3}$	1/2	-1/2	-1/2
ijk	9,10,12						
f_{ijk}	-1/2						

Determination of the torque vector

The torque vector is calculated from:

$$\gamma_j(t) = \frac{1}{\hbar} \text{Tr}(\hat{H}(t) \hat{\lambda}_j)$$

$$\vec{\gamma} = \left(2\Omega_y, 0, 0, 0, 0, 2\Omega_y, -2\Omega_z, 0, 0, 0, 0, 2\Omega_z, \omega_o, -\frac{\omega_o}{\sqrt{3}}, \sqrt{\frac{2}{3}}\omega_o \right)$$

Degenerate 4-level system resonantly coupled to the 2D – TM optical wave:

N=4 $\Rightarrow$ 15 generators of SU(4) group:

$$\dot{S}_i = f_{ijk} \gamma_j S_k \quad f_{ijk} = \frac{1}{4} i [\text{Tr}(\hat{\lambda}_i \hat{\lambda}_k \hat{\lambda}_j) - \text{Tr}(\hat{\lambda}_i \hat{\lambda}_j \hat{\lambda}_k)] \quad i, j, k = 1, 2, \dots, 15$$

S – 15-dimensional real coherence vector;

Pseudospin equations for degenerate 4-level system in 2D

$$\frac{\partial \mathbf{S}}{\partial t} = M \mathbf{S} - \sigma (\mathbf{S} - \mathbf{S}_E) \quad \sigma = \text{diag}(1/T_1, 1/T_2, \dots, 1/T_{15})$$

$$M = \begin{pmatrix} \underline{P}_{6 \times 6} & \underline{Q}_{6 \times 6} & \underline{R}_{6 \times 3} \\ -\underline{Q}_{6 \times 6}^T & \underline{P}_{6 \times 6} & \underline{S}_{6 \times 3} \\ -\underline{R}_{3 \times 6}^T & -\underline{S}_{3 \times 6}^T & \underline{0}_{3 \times 3} \end{pmatrix}$$

15×15 antisymmetric matrix
(26 independent components)

$$\hat{P} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \Omega_z & 0 & \Omega_z & 0 \\ 0 & -\Omega_z & 0 & \Omega_z & 0 & 0 \\ 0 & 0 & -\Omega_z & 0 & -\Omega_z & 0 \\ 0 & -\Omega_z & 0 & \Omega_z & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\hat{R} = \begin{pmatrix} -2\Omega_z & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -\frac{2\Omega_z}{\sqrt{3}} & 2\sqrt{\frac{2}{3}}\Omega_z \end{pmatrix}$$

$$\hat{S} = \begin{pmatrix} -2\Omega_y & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \frac{2\Omega_y}{\sqrt{3}} & -2\sqrt{\frac{2}{3}}\Omega_y \end{pmatrix}$$

$$\hat{Q} = \begin{pmatrix} -\omega_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \omega_0 & \Omega_y & 0 & -\Omega_y & 0 \\ 0 & \Omega_y & 0 & -\Omega_y & 0 & 0 \\ 0 & 0 & -\Omega_y & -\omega_0 & -\Omega_y & 0 \\ 0 & -\Omega_y & 0 & \Omega_y & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\omega_0 \end{pmatrix}$$

Degenerate four-level quantum system

Relationship between the real-vector components and the density matrix components for N=4

$$S_1 = \hat{\rho}_{12} + \hat{\rho}_{21}$$

$$S_7 = -i(\hat{\rho}_{12} - \hat{\rho}_{21}) = i(\hat{\rho}_{21} - \hat{\rho}_{12})$$

$$S_2 = \hat{\rho}_{23} + \hat{\rho}_{32}$$

$$S_8 = -i(\hat{\rho}_{23} - \hat{\rho}_{32}) = i(\hat{\rho}_{32} - \hat{\rho}_{23})$$

$$S_3 = \hat{\rho}_{13} + \hat{\rho}_{31}$$

$$S_9 = -i(\hat{\rho}_{13} - \hat{\rho}_{31}) = i(\hat{\rho}_{31} - \hat{\rho}_{13})$$

$$S_4 = \hat{\rho}_{14} + \hat{\rho}_{41}$$

$$S_{10} = -i(\hat{\rho}_{14} - \hat{\rho}_{41}) = i(\hat{\rho}_{41} - \hat{\rho}_{14})$$

$$S_5 = \hat{\rho}_{24} + \hat{\rho}_{42}$$

$$S_{11} = -i(\hat{\rho}_{24} - \hat{\rho}_{42}) = i(\hat{\rho}_{42} - \hat{\rho}_{24})$$

$$S_6 = \hat{\rho}_{34} + \hat{\rho}_{43}$$

$$S_{12} = -i(\hat{\rho}_{34} - \hat{\rho}_{43}) = i(\hat{\rho}_{43} - \hat{\rho}_{34})$$

$$S_{13} = \hat{\rho}_{22} - \hat{\rho}_{11}$$

$$S_{14} = -\frac{1}{\sqrt{3}}(\hat{\rho}_{11} + \hat{\rho}_{22} - 2\hat{\rho}_{33})$$

$$S_{15} = -\frac{1}{\sqrt{6}}(\hat{\rho}_{11} + \hat{\rho}_{22} + \hat{\rho}_{33} - 3\hat{\rho}_{44})$$

Normalisation of S:

$$|\mathbf{S}| = \sqrt{\frac{3}{2}}$$

Occupation probability conservation: $\hat{\rho}_{11} + \hat{\rho}_{22} + \hat{\rho}_{33} + \hat{\rho}_{44} = 1$

$$\Rightarrow \mathbf{S}_E = (0, 0, 0, \dots, S_{13e}, S_{14e}, S_{15e}) \quad S_{13e} = \pm 1; \quad S_{14e} = \pm \frac{1}{\sqrt{3}}; \quad S_{15e} = \pm \frac{1}{\sqrt{6}}$$

2D polarisation. Splitting of displacement operator

$$\hat{Q} = q_0 \left\{ \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \vec{e}_y - \begin{pmatrix} 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \vec{e}_z + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \vec{e}_y + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \end{pmatrix} \vec{e}_z \right\} =$$

$$= q_0 \left[(\hat{\lambda}_1 + \hat{\lambda}_6) \vec{e}_y + (\hat{\lambda}_{12} - \hat{\lambda}_7) \vec{e}_z \right]$$

$$\vec{E} = E_y \vec{e}_y + E_z \vec{e}_z \quad \Omega_y = \frac{\wp}{\hbar} E_y; \quad \Omega_z = \frac{\wp}{\hbar} E_z$$

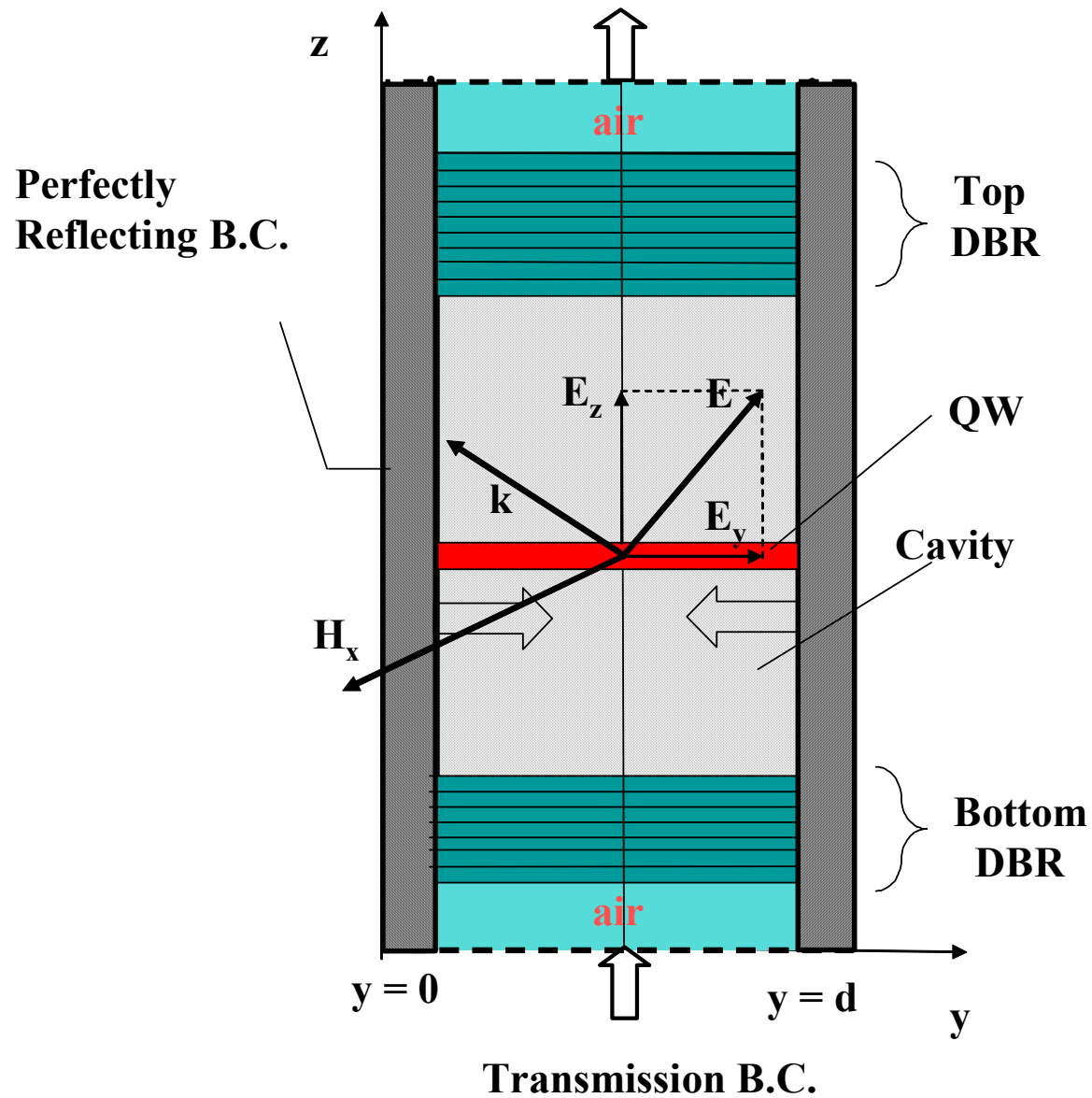
Interaction Hamiltonian:

$$\hat{H}_{\text{int}}(t) = e \vec{E} \cdot \hat{Q} = \hbar \begin{pmatrix} 0 & \Omega_y - i \Omega_z & 0 & 0 \\ \Omega_y + i \Omega_z & 0 & 0 & 0 \\ 0 & 0 & 0 & \Omega_y + i \Omega_z \\ 0 & 0 & \Omega_y - i \Omega_z & 0 \end{pmatrix}$$

Unperturbed (zero-field) Hamiltonian of a degenerate 4-level system:

$$\Rightarrow \hat{H} = \hat{H}_0 + \hat{H}_{\text{int}}(t) = \hbar \begin{pmatrix} 0 & \Omega_y - i \Omega_z & 0 & 0 \\ \Omega_y + i \Omega_z & \omega_0 & 0 & 0 \\ 0 & 0 & 0 & \Omega_y + i \Omega_z \\ 0 & 0 & \Omega_y - i \Omega_z & \omega_0 \end{pmatrix}$$

2D geometry of the simulation domain



Coupling of the quantum pseudospin equations to semiclassical Maxwell's equations in 2D

Relationship between the macroscopic polarisation and the real coherence vector

Macroscopic polarisation: $\mathbf{P} = -e N_a \text{Tr}(\hat{\rho} \hat{\mathbf{Q}})$

$$\hat{\rho}(t) = \frac{1}{N} \hat{I} + \frac{1}{2} \sum_{j=1}^{15} S_j(t) \hat{\lambda}_j$$

$$\begin{aligned} P_y &= -e N_a \text{Tr}(\hat{\rho} \hat{Q}_y) = -\wp N_a \text{Tr}(\hat{\rho} (\hat{\lambda}_1 + \hat{\lambda}_6)) & \Leftrightarrow & P_y = -\wp N_a (S_1 + S_6) \\ P_z &= -e N_a \text{Tr}(\hat{\rho} \hat{Q}_z) = -\wp N_a \text{Tr}(\hat{\rho} (\hat{\lambda}_{12} - \hat{\lambda}_7)) & & P_z = -\wp N_a (S_{12} - S_7) \end{aligned}$$

Coupled Maxwell-pseudospin equations in 2D (quasi 3D)

Maxwell's equations for TM-wave:

$$\begin{aligned}\frac{\partial H_x}{\partial t} &= -\frac{1}{\mu} \frac{\partial E_z}{\partial y} + \frac{1}{\mu} \frac{\partial E_y}{\partial z} \\ \frac{\partial E_y}{\partial t} &= \frac{1}{\epsilon} \frac{\partial H_x}{\partial z} - \frac{1}{\epsilon} \frac{\partial P_y}{\partial t} \\ \frac{\partial E_z}{\partial t} &= -\frac{1}{\epsilon} \frac{\partial H_x}{\partial y} - \frac{1}{\epsilon} \frac{\partial P_z}{\partial t}\end{aligned}$$

Pseudospin equations:

$$\frac{\partial \mathbf{S}}{\partial t} = \mathbf{M} \mathbf{S} - \sigma (\mathbf{S} - \mathbf{S}_E)$$

$$\mathbf{M} = \begin{pmatrix} \underline{P}_{6 \times 6} & \underline{Q}_{6 \times 6} & \underline{R}_{6 \times 3} \\ -\underline{Q}_{6 \times 6}^T & \underline{P}_{6 \times 6} & \underline{S}_{6 \times 3} \\ -\underline{R}_{3 \times 6}^T & -\underline{S}_{3 \times 6}^T & \underline{0}_{3 \times 3} \end{pmatrix}$$

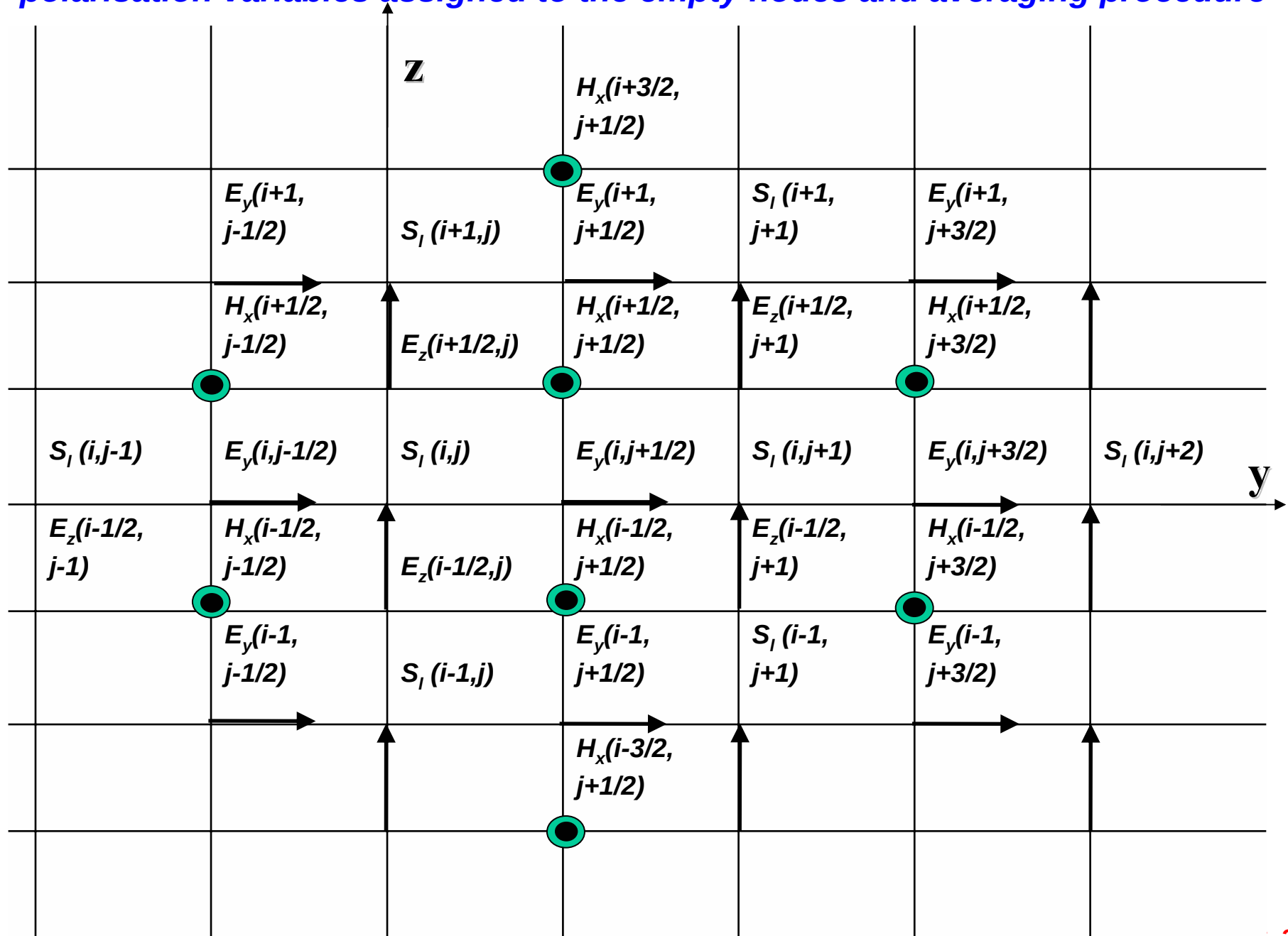
$$\sigma = \text{diag}(1/T_1, 1/T_2, \dots, 1/T_{15})$$



$$\begin{aligned}P_y &= -\wp N_a (S_1 + S_6) \\ P_z &= -\wp N_a (S_{12} - S_7)\end{aligned}$$

Numerical implementation in 2D (TM wave)

- Discretisation of the system using finite differences on a 2D-Yee grid with polarisation variables assigned to the empty nodes and averaging procedure



Numerical implementation in 2D

- FDTD time-stepping algorithm with predictor-corrector iterative scheme implemented
- Engquist-Majda (one-way wave equation) transmission boundary conditions for the input and output facet based on Mur discretisation
- Perfectly reflecting boundary conditions for the side boundaries
- Source field: the fundamental guided modes of a planar parallel mirror light waveguide

TEM (TM_0)

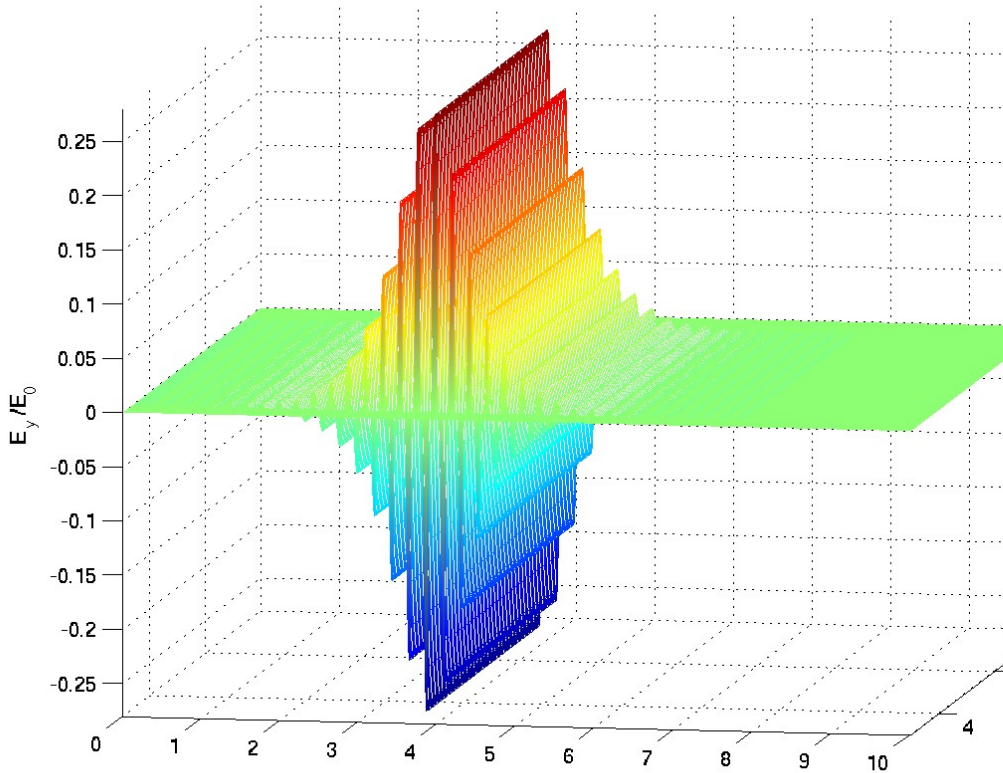
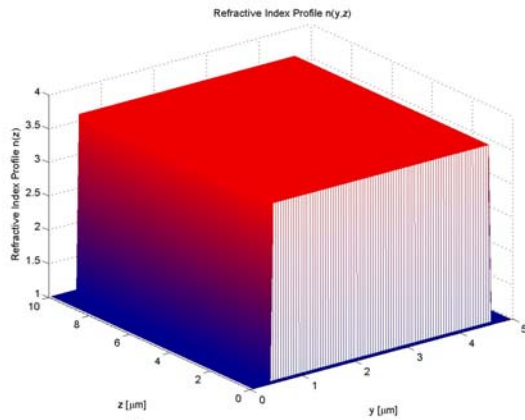
$$\begin{cases} E_y(z=0, y, t) = E_o \operatorname{sech}(10\Gamma) \sin(\omega_o t) \\ E_z(z=0, y, t) = 0 \\ \Gamma = [t - (T_p / 2)] / (T_p / 2) \end{cases}$$

TM_1

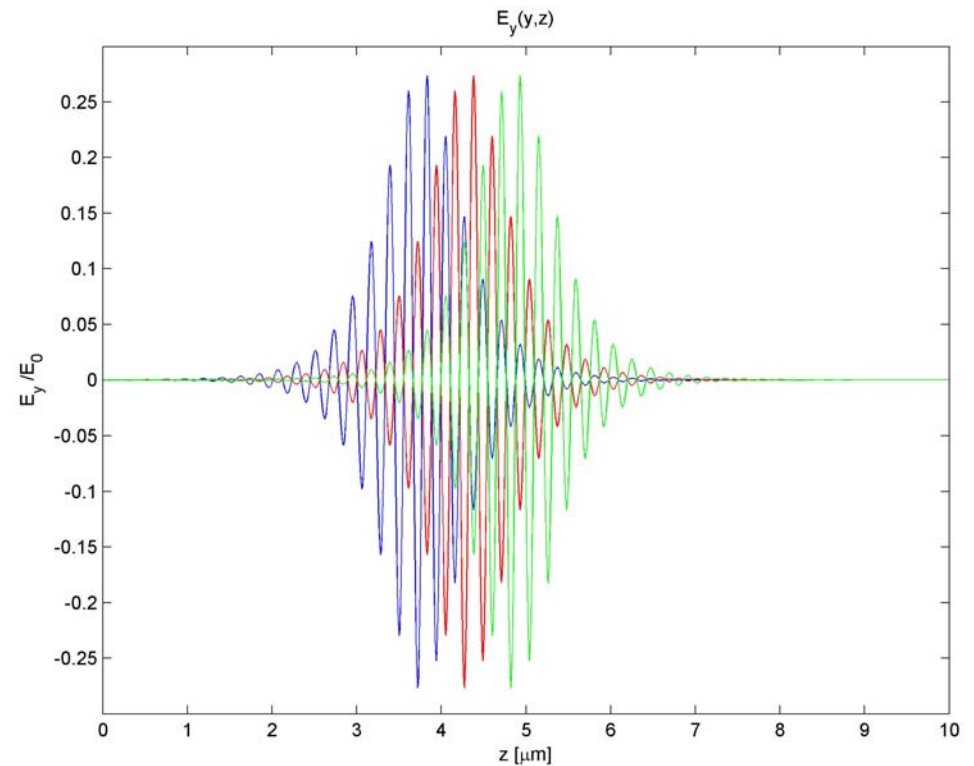
$$\begin{cases} E_y(z=0, y, t) = E_o c_1 \cos\left(\frac{\pi y}{d}\right) \operatorname{sech}(10\Gamma) \sin(\omega_o t) \\ E_z(z=0, y, t) = -E_o c_2 \sin\left(\frac{\pi y}{d}\right) \operatorname{sech}(10\Gamma) \cos(\omega_o t) \end{cases}$$
$$c_1 = \frac{\sqrt{\frac{\omega_o^2 n^2}{c_o^2} - \left(\frac{\pi}{d}\right)^2}}{\omega_o \epsilon_o n^2}; \quad c_2 = \frac{\pi/d}{\omega_o \epsilon_o n^2}$$

Simulation results

Active Optical Waveguides



2π sech pulse, $E_0=0.79045 \times 10^{11}$ [V/m]
 $\tau=100$ fs, $\lambda=800$ nm



Simulation times $t_1= 98.3$ fs

$t_2=100.5$ fs

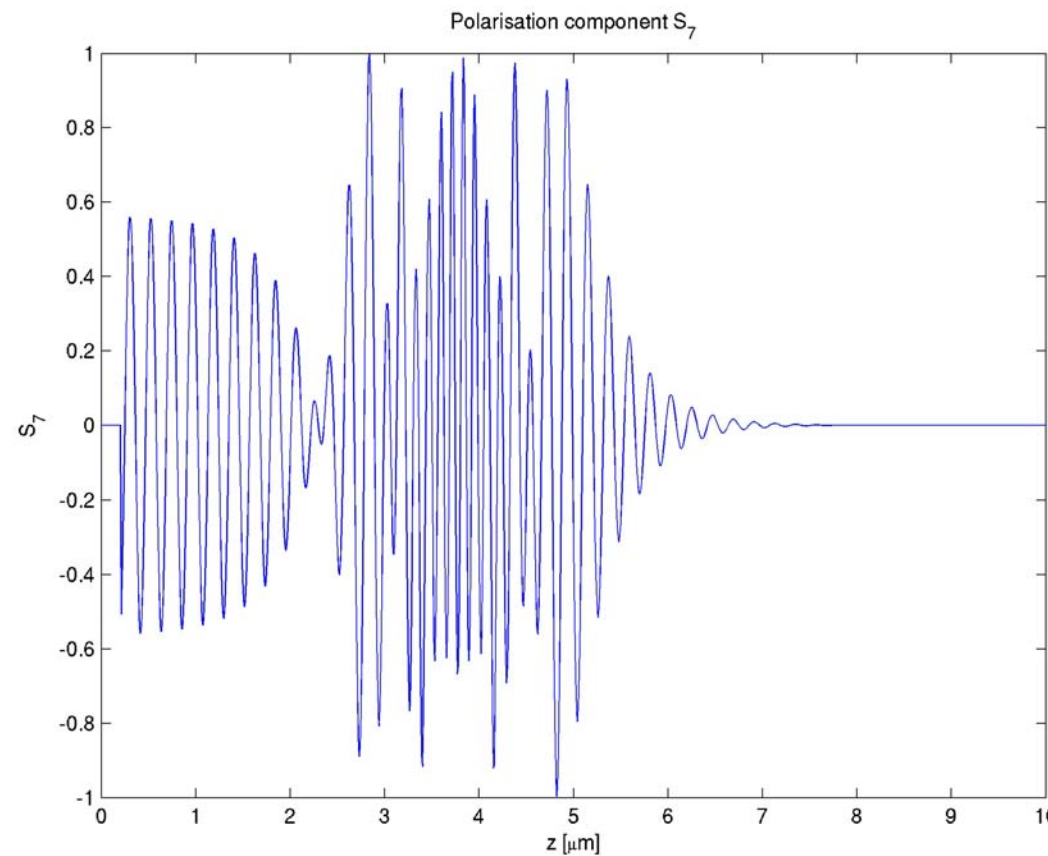
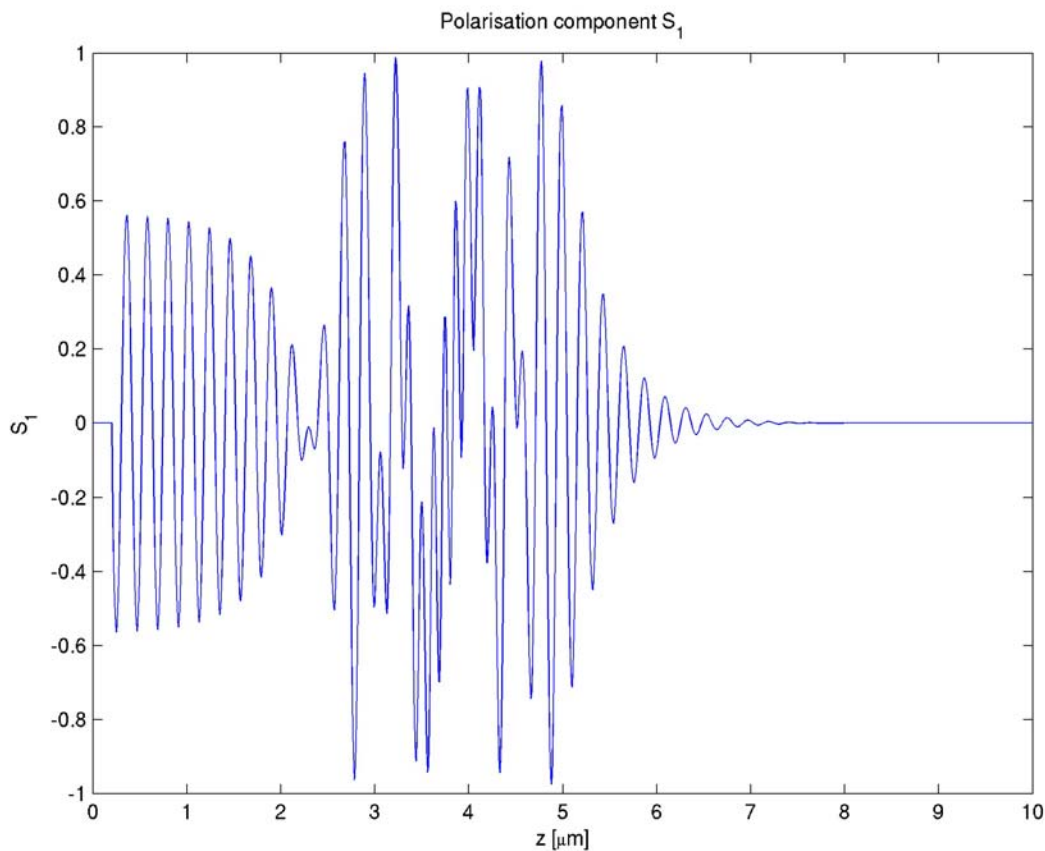
$t_3=107.2$ fs

NUSOD'04 , Santa Barbara August 24-26, 2004

Simulation results

Active Optical Waveguides

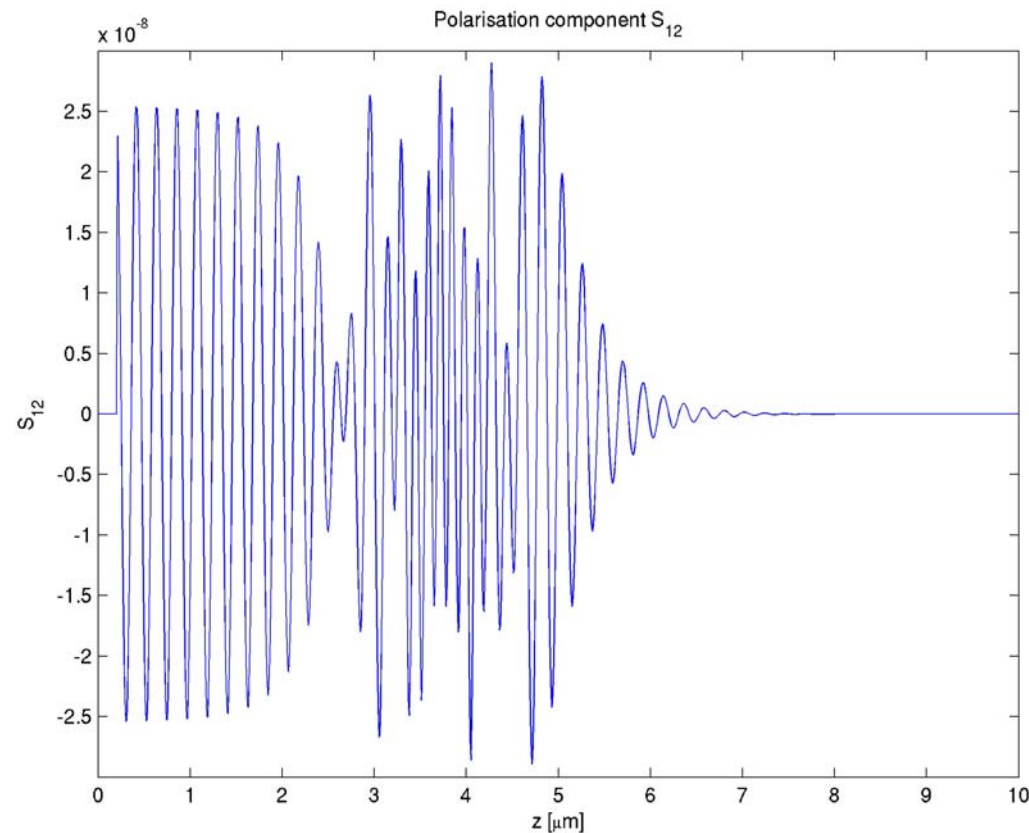
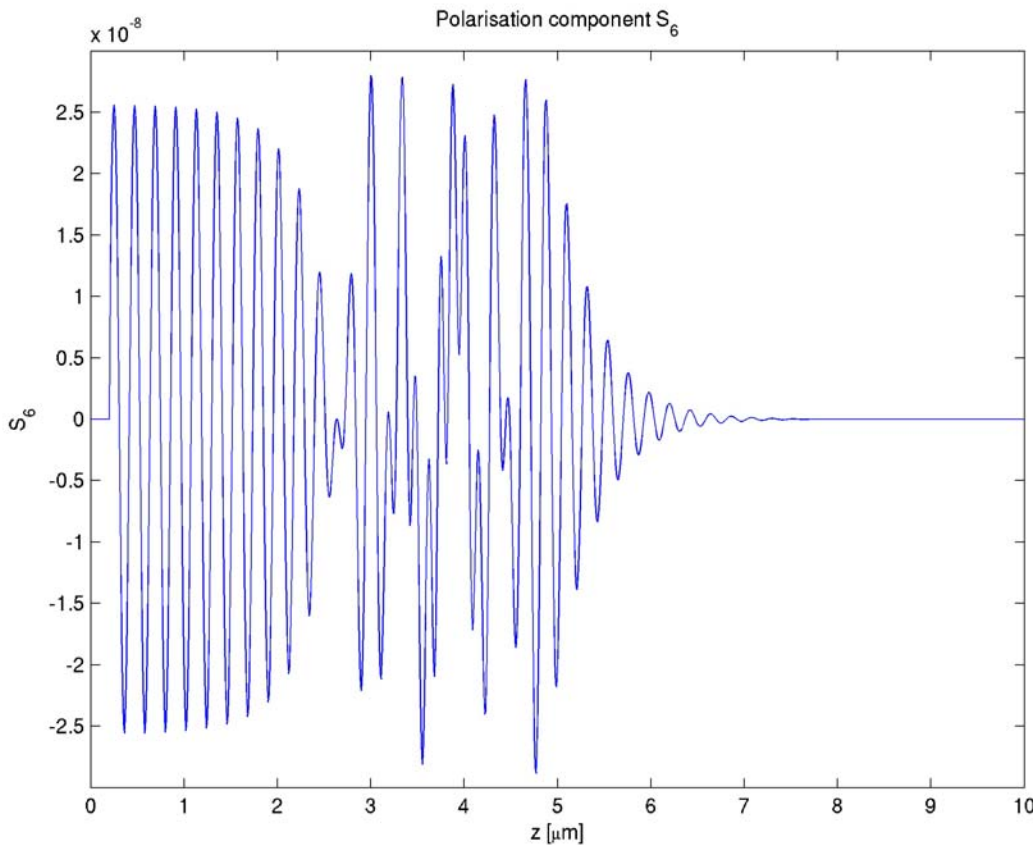
Polarisation components S_1 and S_7 : $|1\rangle \rightarrow |2\rangle$



Simulation results

Active Optical Waveguides

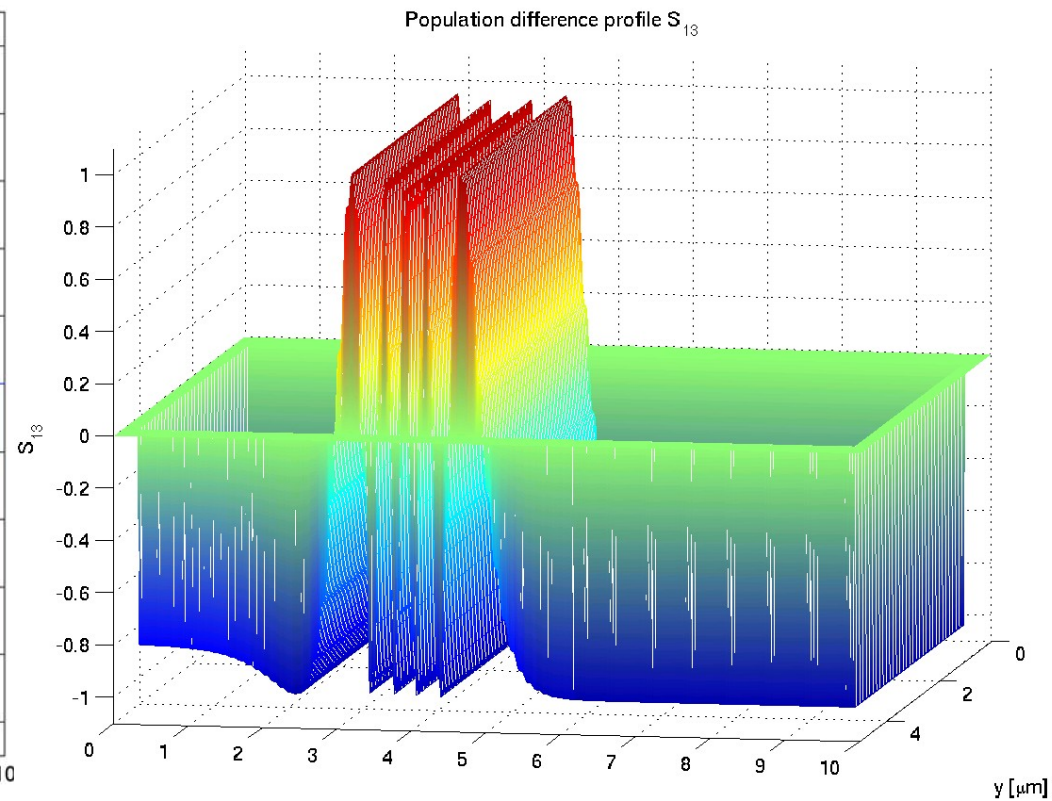
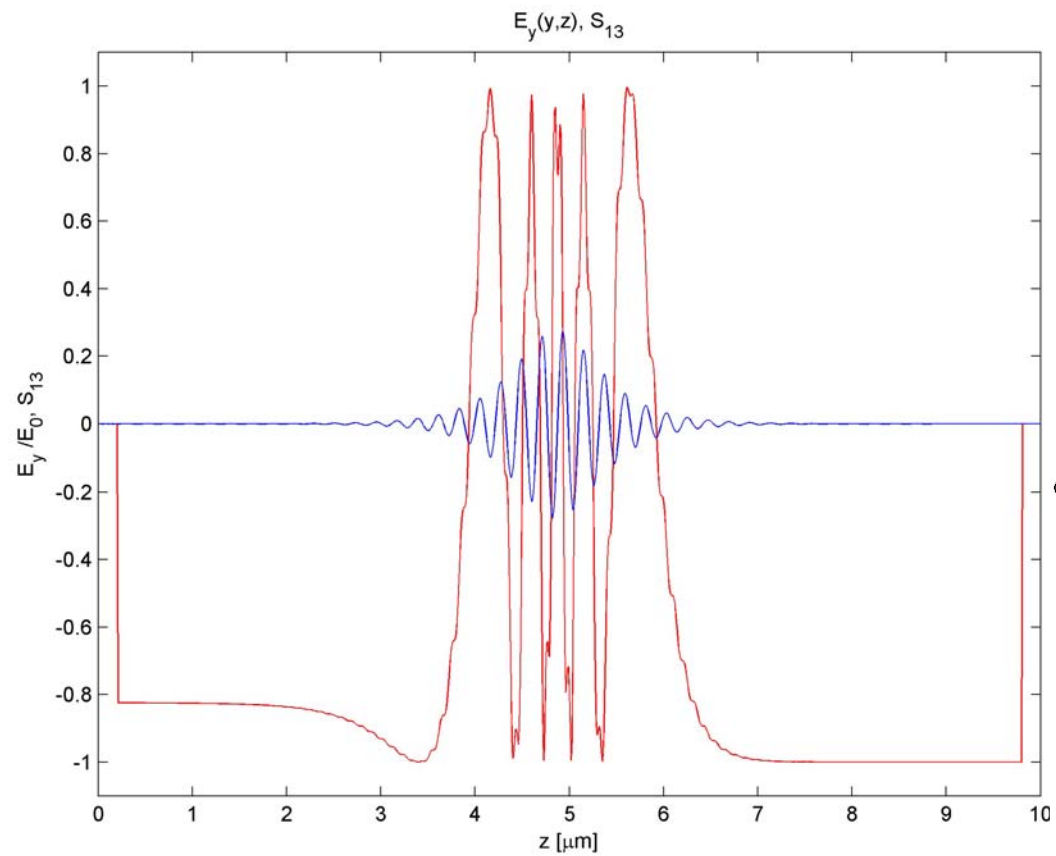
Polarisation components S_6 and S_{12} : $|3\rangle \rightarrow |4\rangle$



Simulation results

Active Optical Waveguides

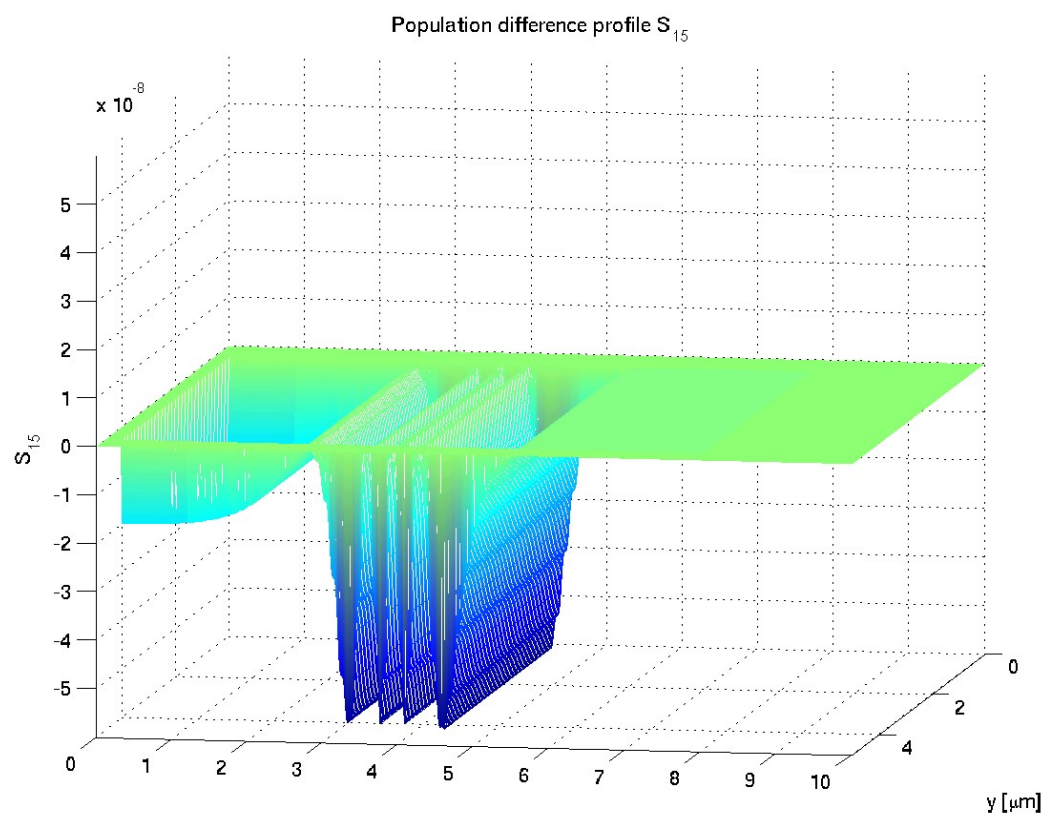
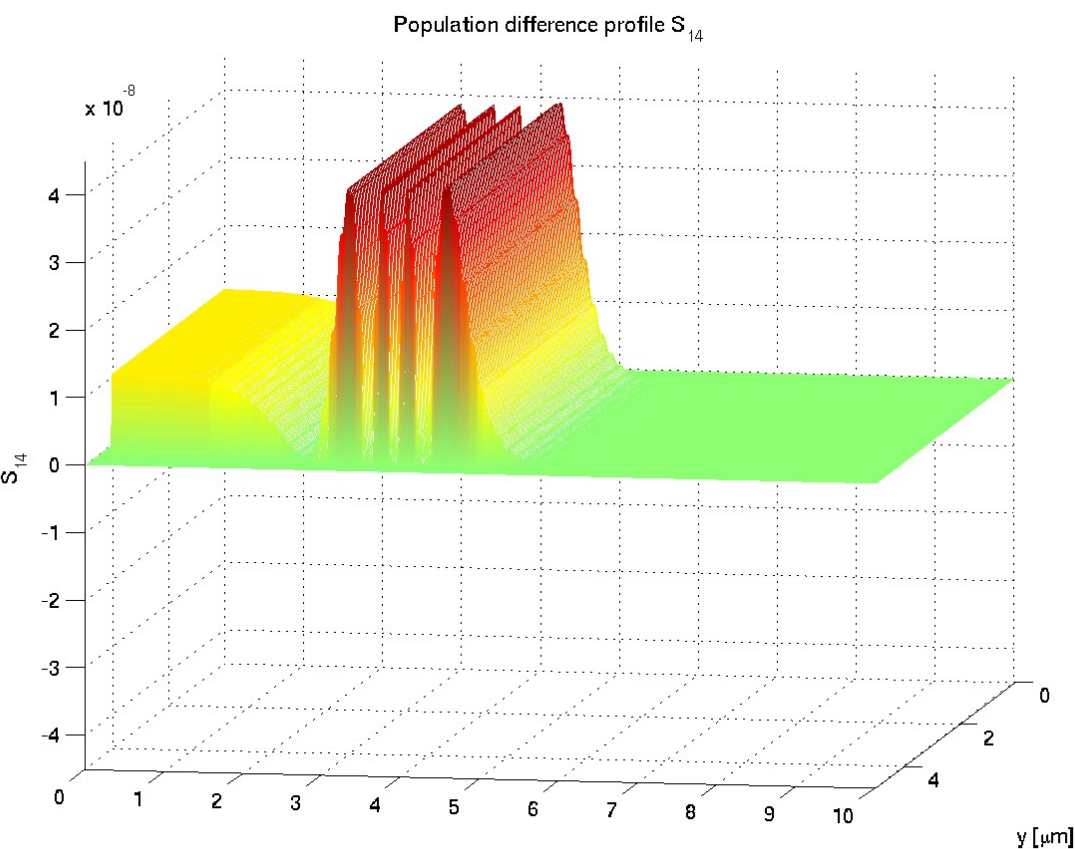
Population term S_{13}



Simulation results

Active Optical Waveguides

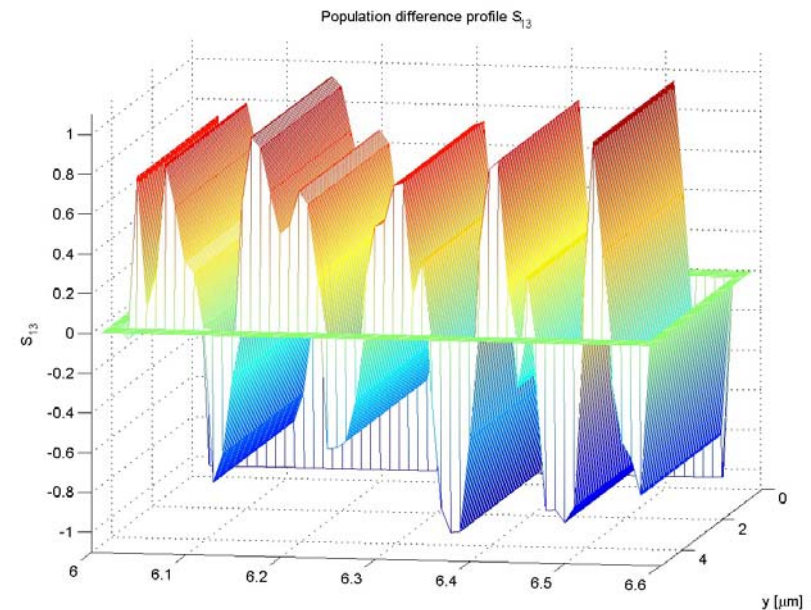
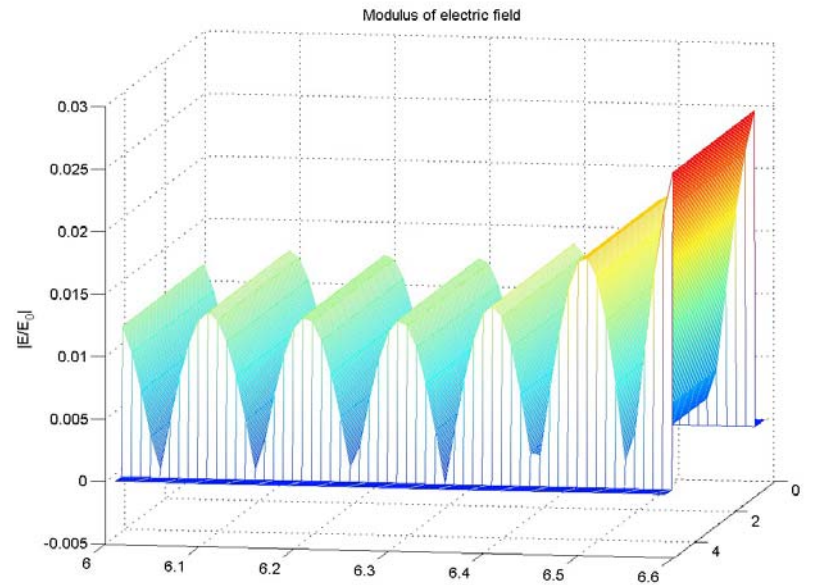
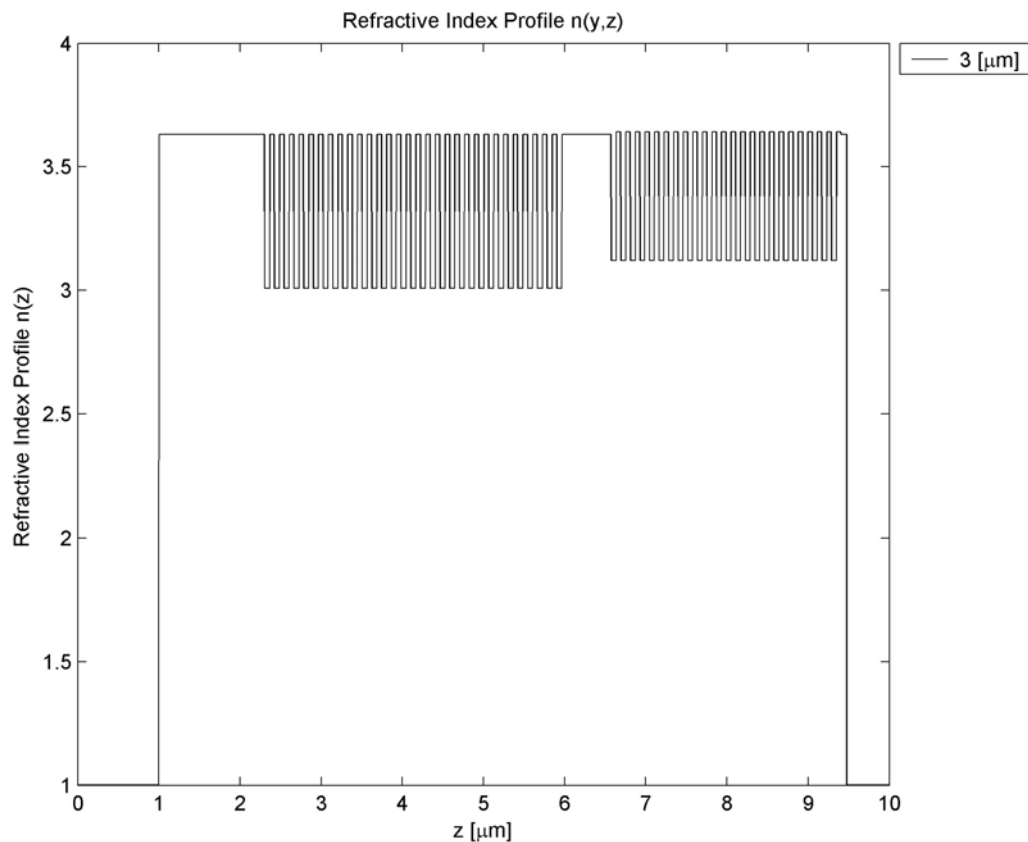
Population Terms S_{14} , S_{15}



Simulation results

Active Semiconductor Microcavities

2π sech pulse, $E_0 = 0.79045 \times 10^{11}$ [V/m]
 $\tau = 100$ fs, $\lambda = 800$ nm, $L_c = 5/2 \lambda/n$
Cavity filled with absorbing medium



Summary

The proposed dynamical model based on the FDTD solution of the coupled curl Maxwell-pseudospin equations:

- *Represents a generalisation to multi dimensions and multi levels of the Maxwell-Bloch equations for a two-level medium*
- *Nonperturbative approach for investigation of the nonlinearities in resonantly absorbing/amplifying media*
- *Generality: without invoking SVEA or RWA approximations*
- *Valid for ultrashort optical pulses with pulse duration $\tau \ll T_1, T_2$ (beyond the limits of the rate equations approximation)*
- *Self-induced transparency solitons and patterns in 2D*
- *Provides a model of the spin dynamics in QWs*
- *Tool for modelling of the fast saturable absorbers and the ultrafast modulation response of a SMC*