

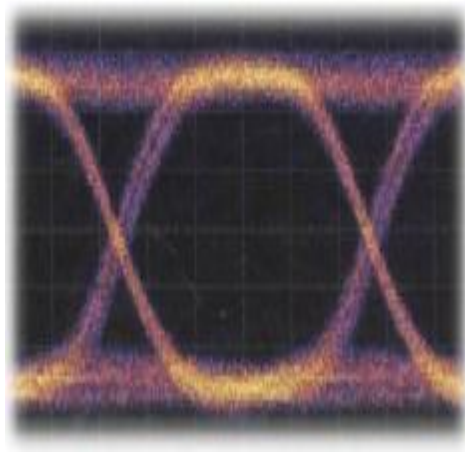
Comprehensive Physics-Based Intensity Noise Simulation of VCSELs

Stefan Odermatt, Bernhard Schmithüsen
and Bernd Witzigmann

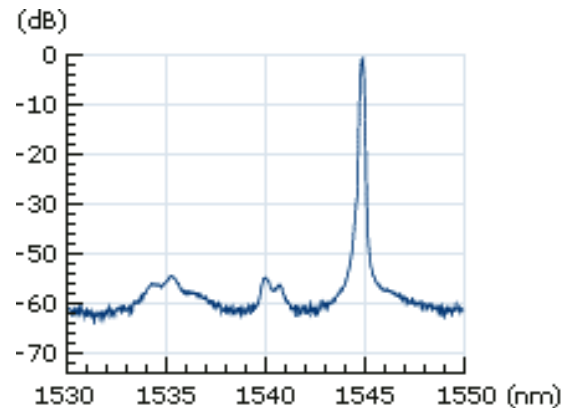
NUSOD 2004, Santa Barbara



Why Noise Simulation?

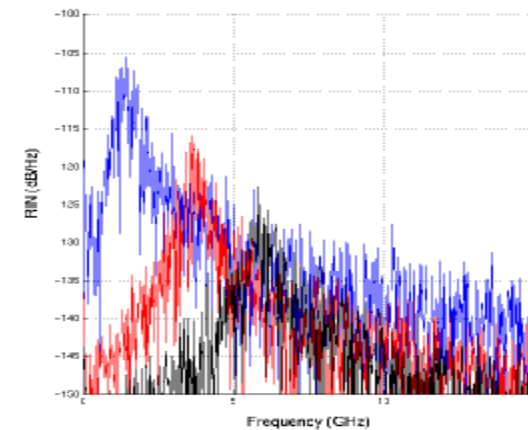


Eye-Diagram



y) Relative intensity
x) Wavelength

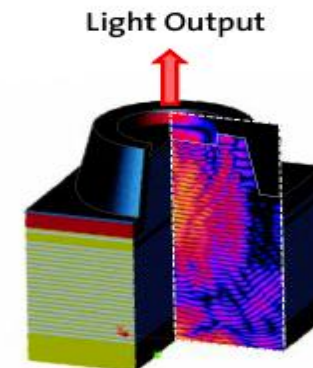
Spectral Linewidth



Power Fluctuations (RIN)

1.) In an **optical communication** system, noise plays a detrimental role. Noise-considerations in design phase **to minimize noise systematically !!**

2.) Take noise into account in **calibration process** to **eliminate “fitting” parameters** in DC characteristics.

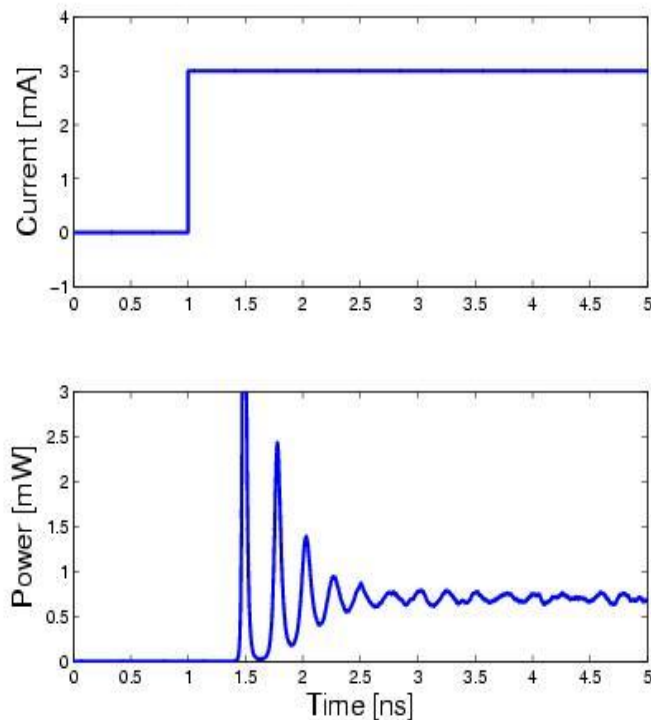


Outline

- Introduction
 - Intensity Noise
- Conventional Treatment of Noise
 - Small-Signal Noise Modelling
 - Numerical Integration
- Physics-Based Noise Modelling
 - Multidimensional Laser Equations
 - Physics-Based Noise Calculation
- Simulation Results
 - Comparison to Measurements
- Conclusions
- Outlook

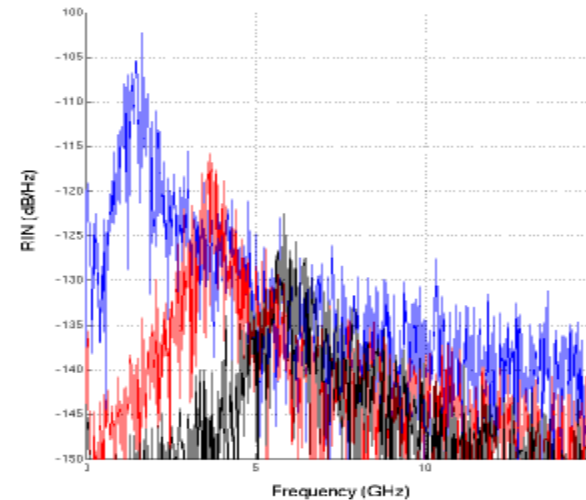
Relative Intensity Noise (RIN)

RIN is a measure of how strong optical output power is fluctuating.



Autocorrelation of Photon Number

$$\frac{RIN(\omega)}{\Delta f} = \frac{2S_{SS}(\omega)}{S^2}$$



Conventional Noise Simulation

Rate Equations with
Langevin Forces:

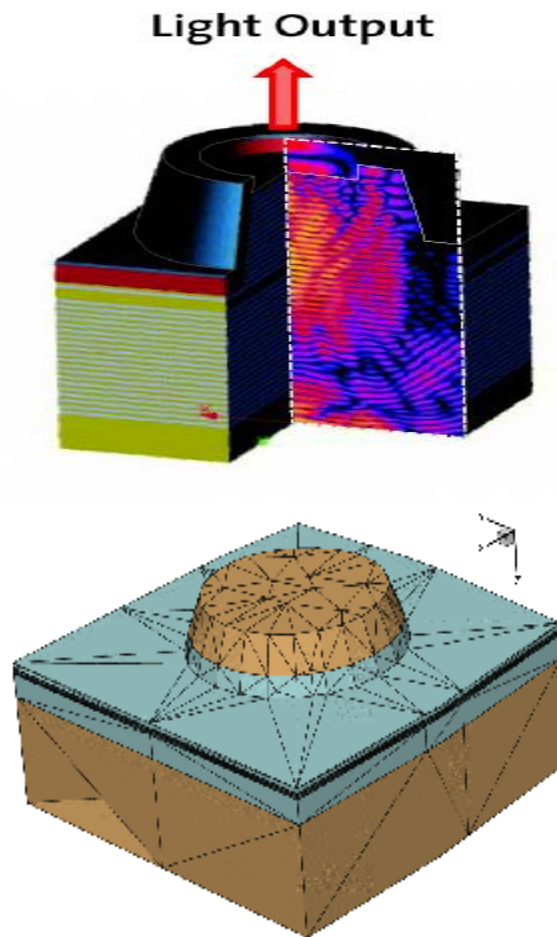
$$\frac{dN}{dt} = \frac{\eta_i I}{q} - GS - R^{sp} + F_N(t)$$
$$\frac{dS}{dt} = (\Gamma G - L)S + \beta \Gamma R^{sp} + F_S(t)$$

Two Classes:

1. **Small Signal Analysis** with Langevin Correlation Functions to obtain **Photon Number Autocorrelation** (analytical expressions possible).
2. Direct **Numerical Integration** of the Rate Equation with Langevin Forces. **Photon Number Autocorrelation** through Fourier transformation.

Problems: Phenomenological Parameters?
Multi-Dimensional Noise Effects?
Compatibility with Device Simulators (like DESSIS)?

Physics-Based Modelling



What is Physics-Based Modelling?

Multidimensional (Geometry, Materials)

Electro-Opto-Thermal Modelling

No Phenomenological Parameters

Goal: **“Predictive” Simulation**, i.e.

if L-I, V-I, λ -I, Modulation Response
and Noise fits with the same Material
Parameters, Physics should be correct.

Physics-Based Noise Modelling

Laser Equations

Photon Rate:

$$\partial_t S_\nu = (G_\nu - L_\nu) S_\nu + T_\nu^{sp} + F_{S_\nu}$$

+ Poisson + Continuity Equations

$$\begin{aligned}\nabla \cdot \epsilon \nabla V &= -q \left(p^{tot} - n^{tot} + N_D^+ - N_A^- \right) \\ \pm \nabla \cdot \mathbf{J}_c^{3D} &= q \left(C_{cap,c} + \partial_t c^{3D} \right) + F_{c^{3D}}(t) \\ \pm \nabla \cdot \mathbf{J}_c^{2D} &= q \left(R^{st} + R^{sp} + R^{nr} - C_{cap,c} + \partial_t c^{2D} \right) + F_{c^{2D}}(t)\end{aligned}$$

+ Thermodynamic Equation for Lattice-Temperature

+ Helmholtz-Equation for Optical Field.

Physics-Based Noise Modelling

Laser Equations

Vectorial Helmholtz Equation:

$$\left(\nabla^2 + [k_\omega(x)^2 + ik_\omega(x)a_\omega(x)] \right) \hat{A}_\omega(x) = -\frac{4\pi}{c} \hat{j}_\omega(x)$$

Approximations:

$$\begin{aligned} k_\omega(x)^2 &\approx k(x)^2 + \frac{dk(x)^2}{d\omega}(\omega - \omega_s) \\ a_\omega(x) &\approx a(x) \\ \hat{A}(x, t) &= \sum_n \hat{A}_n \equiv \sum_n D_n \hat{b}_n \Psi_n(x) \end{aligned}$$

$$\left(\nabla^2 + [k(x)^2 + ik(x)a(x)] \right) \Psi_n(x) = \left(-\frac{dk(x)^2}{d\omega}(\tilde{\omega}_n - \omega_s) \right) \Psi_n(x)$$

Eigenvalue expansion:

$$\tilde{\omega}_n \equiv \omega_n + i \frac{\Delta G}{2}$$

$$\frac{d}{dt} \hat{b}_n = \left[-i(\omega_n - \omega_s) + \frac{\Delta G_n}{2} \right] \hat{b}_n + \hat{F}_n(t)$$

$$\frac{\Delta G_n}{2} = \frac{\int \Re(k(x)a(x)) |\Psi_n(x)|^2 dx}{\int \frac{dk^2(x)}{d\omega} |\Psi_n(x)|^2 dx}$$

Modal Net Gain
(= (G-L)/2)

$$\frac{d}{dt} S_n = \Delta G_n S_n + T_n^{sp} + F_{S_n}$$

Photon Rate Equation

Physics-Based Noise Modelling

Noise Calculation

$$S_{\partial\alpha,\partial\beta}(x;x';\omega) = \sum_{\gamma} \sum_{\delta} \int \int \hat{G}_{\gamma}^{\alpha}(x;x_1;\omega) K_{S_{\gamma},S_{\delta}}(x_1;x_2;\omega) \hat{G}_{\delta}^{\beta*}(x';x_1;\omega) dx_1 dx_2$$

$$\alpha, \beta, \gamma, \delta = V, n^{3D}, p^{3D}, n^{2D}, p^{2D}, S_{\nu}$$

Bonani, Noise in Semiconductor Devices, Springer, 2001.

$K_{S_{\gamma},S_{\delta}}(x, x_1, \omega)$ Correlation between variables γ and δ

$G_{\gamma}^{\alpha}(x, x_1, \omega)$ Response at variable α at x due to excitation of γ at x_1

Autocorrelation of Photon Number $S_{S_{\nu},S_{\nu}}(\omega)$:

$$S_{S_{\nu},S_{\nu}}(\omega) = \sum_{\gamma} \sum_{\delta} \int \hat{G}_{\gamma}^{S_{\nu}}(x_1;\omega) K_{S_{\gamma},S_{\delta}}(x_1;\omega) \hat{G}_{\delta}^{S_{\nu}*}(x_1;\omega) dx_1$$

Physics-Based Noise Modelling

Noise Correlation Sources

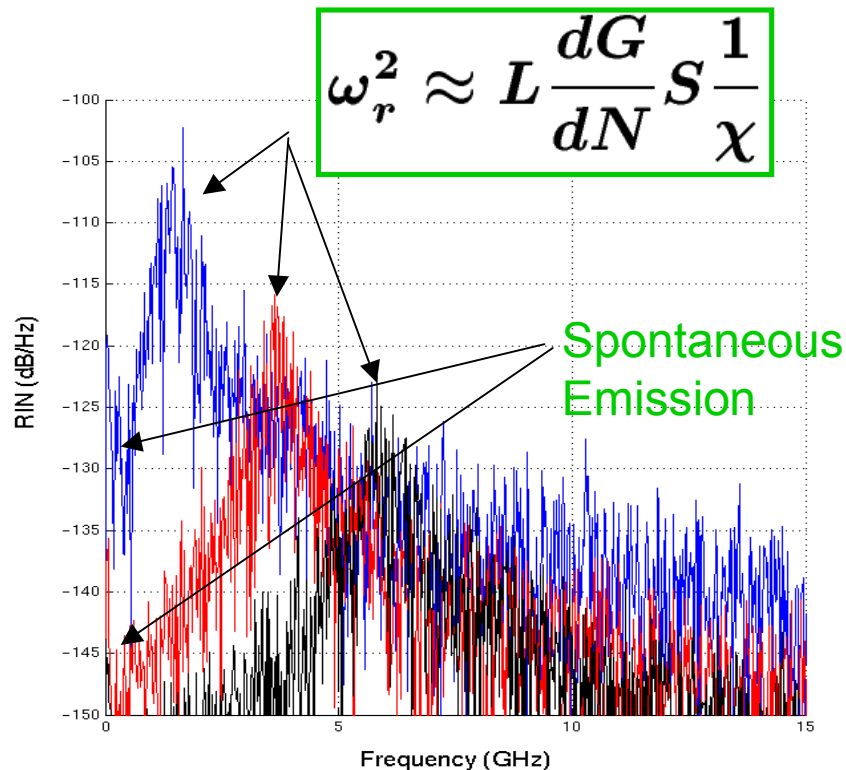
Noise Correlation Sources $K_{S_\gamma S_\delta}(x, x_1, \omega)$ can be obtained either from the **equation of motion** or by evaluating the **second moments of the Fokker-Planck equation** leading to

$$\begin{aligned}
 K_{S_{S_\nu}, S_{S_\nu}} &= 2T_\nu^{sp} S_\nu \\
 K_{S_{S_\nu}, S_{n2D}}(x) &= -2R_\nu^{sp}(x) S_\nu \\
 K_{S_{S_\nu}, S_{p2D}}(x) &= -2R_\nu^{sp}(x) S_\nu \\
 K_{S_{n2D}, S_{n2D}}(x) &= 2R^{sp}(x) S_\nu \\
 K_{S_{p2D}, S_{p2D}}(x) &= 2R^{sp}(x) S_\nu
 \end{aligned}$$

Henry, *Quantum noise in photonics*, Rev. Mod. Physics, 68, 3, 801-853, 1996.

Physics-Based Noise Modelling

Gain and Spontaneous Emission



Gain, Loss, Spontaneous Emission and Carrier Transport have to be completely correct to match the curves!

Therefore we can use **GEBAS-ISE** simulator:

Bandstructure: **8 band kp**

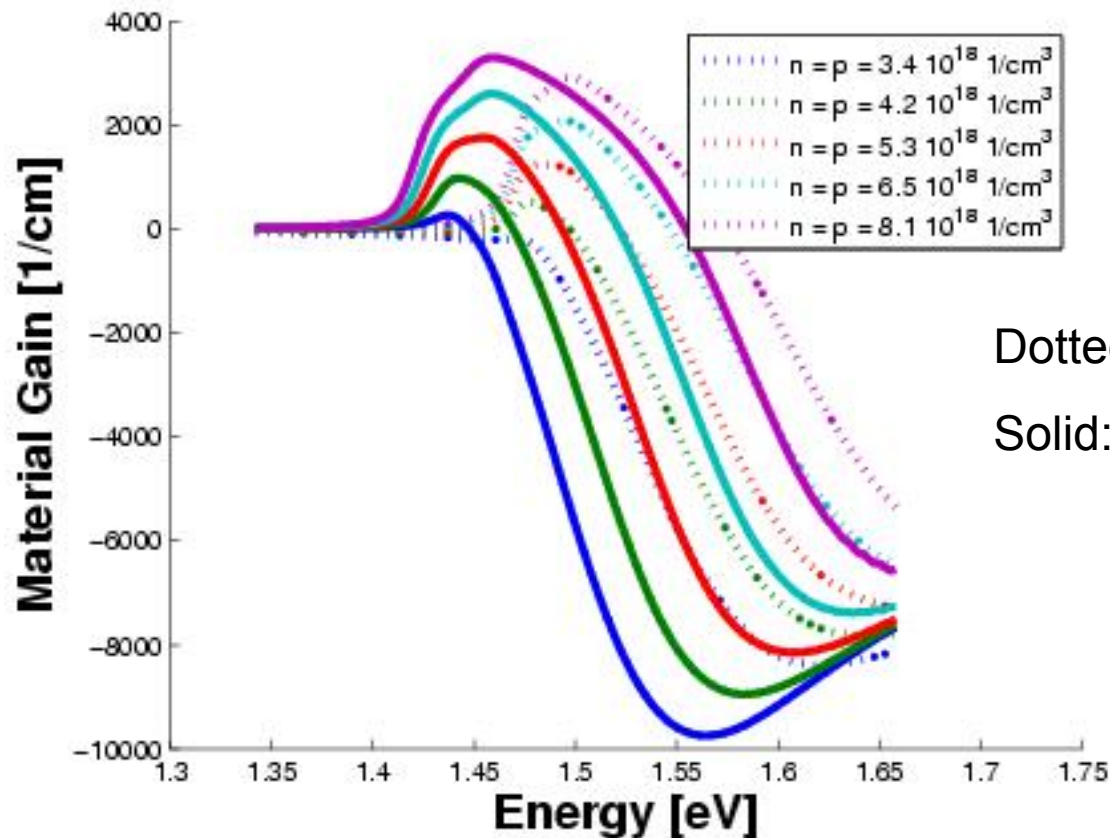
Material Gain: **From Microscopic Polarization**

$$g(x, \omega) = \frac{\omega}{\hbar c \epsilon_0 n_{opt} \pi L_z} \sum_{m,n} \sum_{\sigma} \int_0^{\infty} dk k |\mu_k^{n,m}|^2 \frac{(f_{ek}^n + f_{hk}^m - 1)}{i(\omega_k' - \omega) + \gamma_{kk}} Q_k^{n,m}(\omega)$$

See: Chow and Koch, *Semiconductor Laser Fundamentals*, Springer, 1999.

Physics-Based Noise Modelling

Material Gain

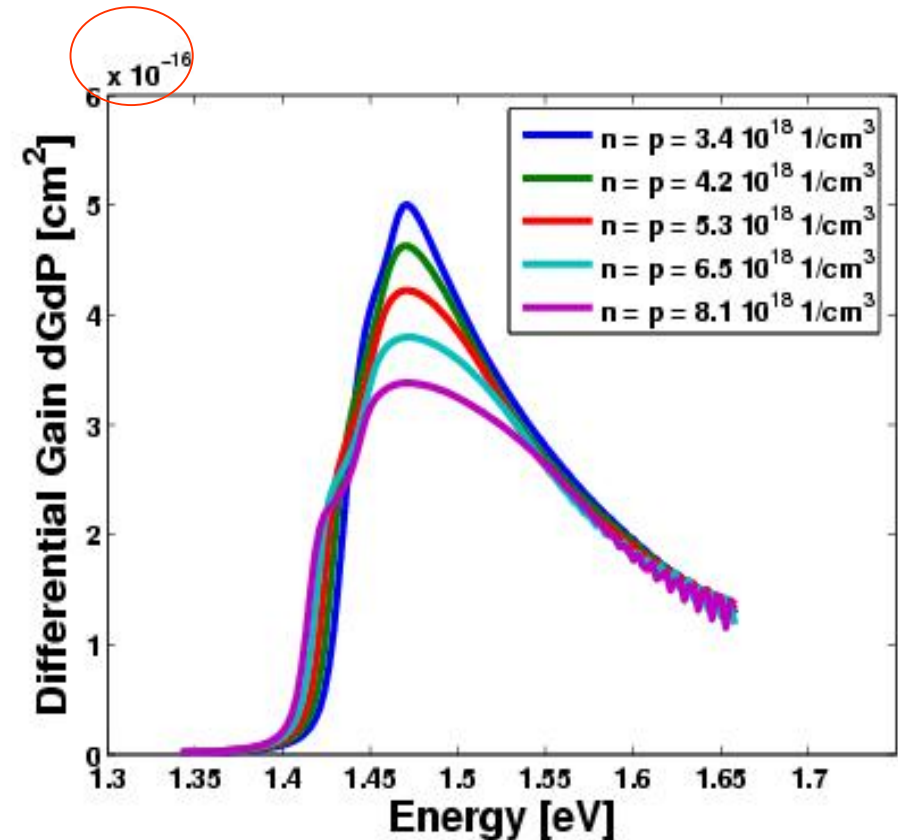
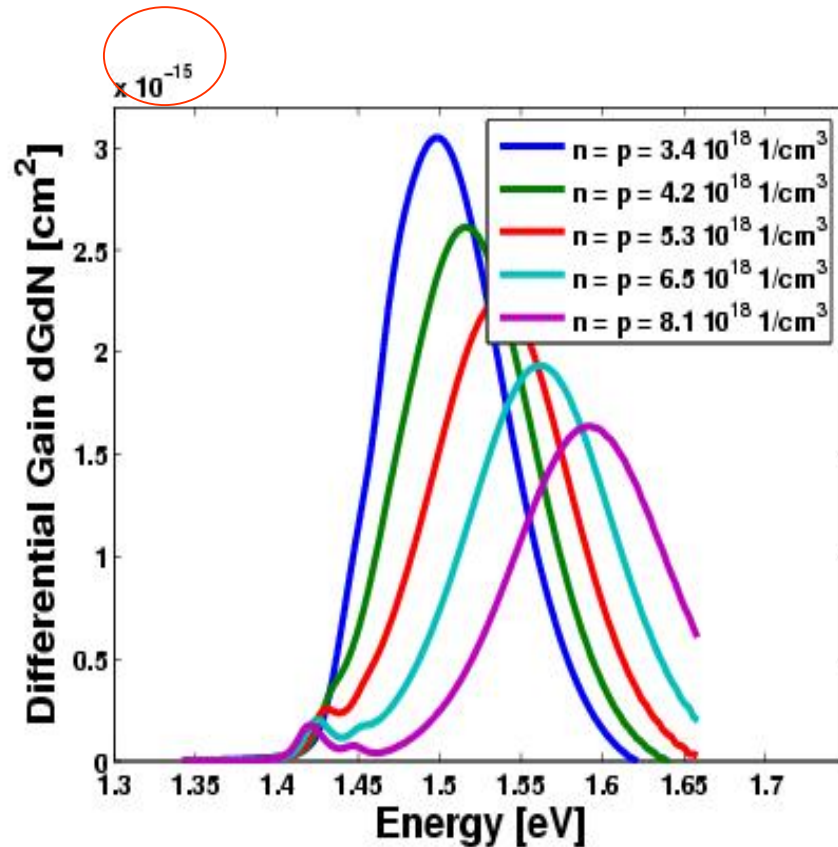


Dotted: Free Carrier Theory

Solid: Second Born (Full Manybody)

Physics-Based Noise Modelling

Differential Gain



Comparison to Measurement

850 nm VCSEL

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INTERNET
PUBLICATION**

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Conclusion

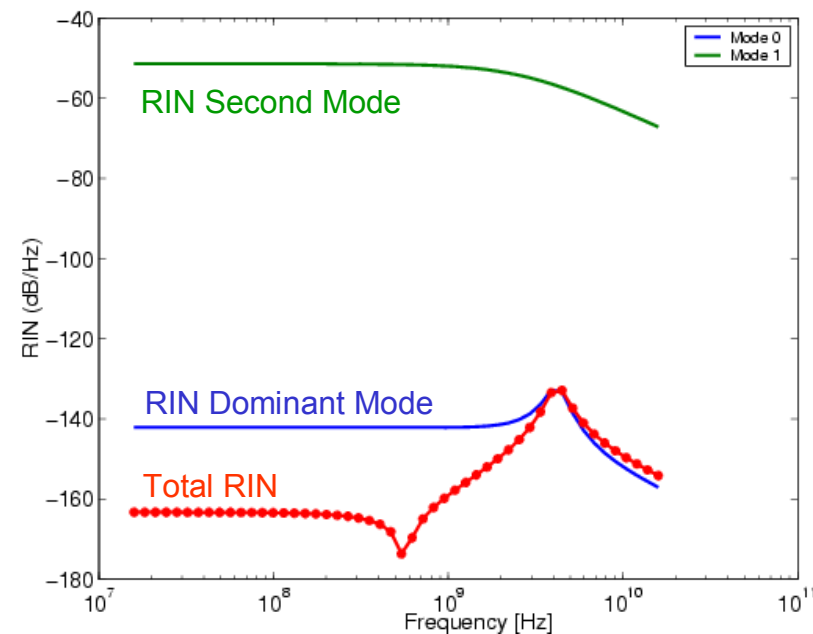
- Conventional Treatment of RIN has been reviewed.
- A new **multi-dimensional noise model** has been presented.
- Noise Sources have been given for physics-based simulations.
- The outstanding importance of **Gain/Spontaneous Emission** has been discussed.
- Comparison with **measurement** shows good agreement.

Outlook

➤ Inclusion of mode partition noise:

Different modes are negatively correlated

→ Total RIN is lower than RIN of dominant mode



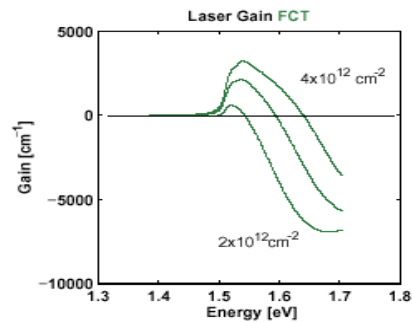
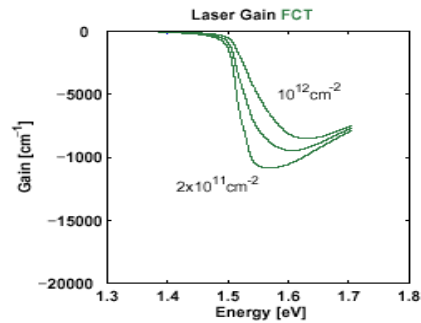
Acknowledgement

Special thanks go to ...

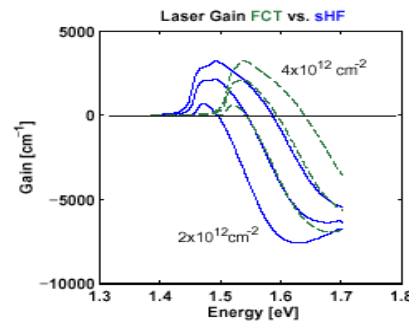
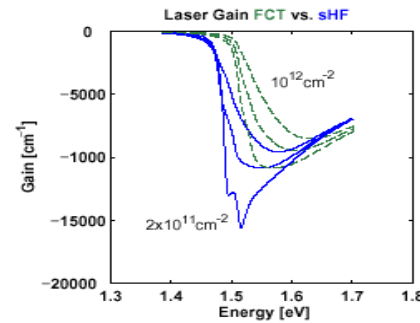
- **AVALON PHOTONICS** for providing the VCSEL devices.
- **Matthias Streiff** for countless discussions about calibration issues.
- **Mathieu Luisier** for stimulating discussions about microscopic physics and gain curves.

Gain Comparison

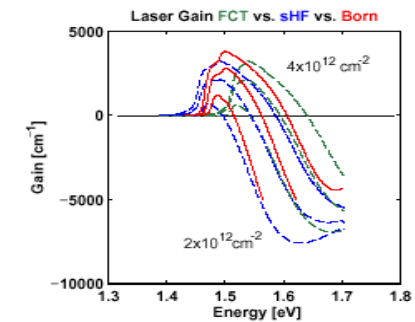
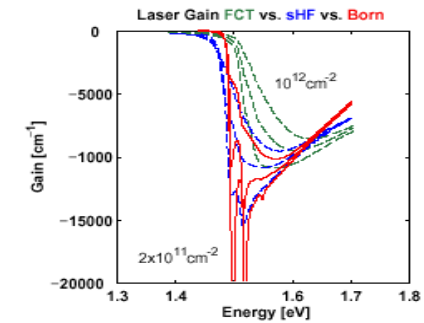
Free Carrier Theory:
neglect interactions



Screened Hartree-Fock:
+ mean-field correction



Born Approximation:
+ particle-particle scattering



Differential Gain

