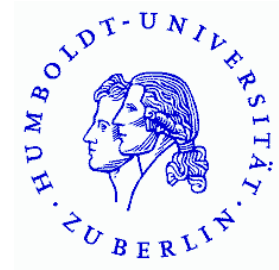


Traveling-Wave Modeling of Semiconductor Lasers and Amplifiers

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Weierstrass Institute for Applied Analysis
and Stochastics, Berlin, Germany



introduction

basic traveling-wave (TW) model

advanced aspects

exemplaric simulations

mode analysis

mode approximations

bifurcation analysis

summary

chapter 1: introduction

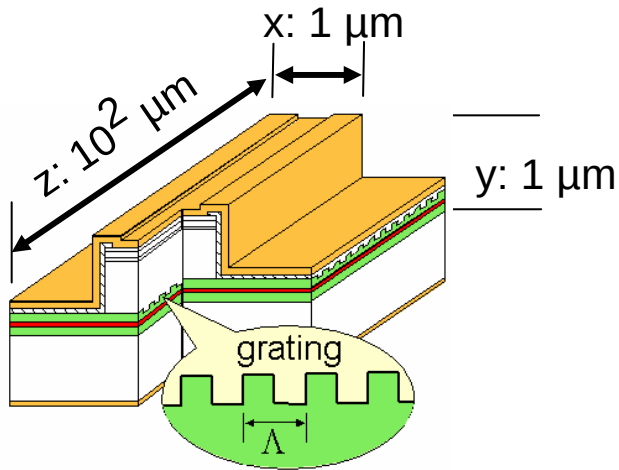
what is traveling wave modeling?

relations to other model classes

advantages

origins

what is traveling wave modeling ?



devices with good waveguides

⇓
transverse-longitudinal
separation possible

$$\mathcal{E}(\vec{r}, t) = \mathcal{E}_{\perp}(x, y) \cdot \mathcal{E}_{\parallel}(z, t)$$

subjects of TW modeling:

optical amplitude $\mathcal{E}_{\parallel}(z, t)$

inversion density $n(z, t)$

relation to other model classes

- **sophisticated:** $\vec{\mathcal{E}}(\vec{r}, t) \vec{\mathcal{B}}(\vec{r}, t) \quad \& \quad n(\vec{r}, t; \nu \vec{k}, \dots)$



Hess O, ... Prog Quant Electr 20, 85 (96)

- **TW models:**

$$\begin{aligned} \vec{\mathcal{E}}(\vec{r}, t), \vec{\mathcal{B}}(\vec{r}, t) &\Rightarrow \mathcal{E}_{\parallel}(z, t) \cdot [\vec{\mathcal{E}}_0(x, y), \vec{\mathcal{B}}_0(x, y)] \\ n(\vec{r}, t) &\Rightarrow n(z, t) = \iint dx dy \, n(\vec{r}, t) |\mathcal{E}_0(x, y)|^2 \end{aligned}$$



- **rate equations:**

$$\begin{aligned} \mathcal{E}_{\parallel}(z, t) &\Rightarrow \mathcal{E}(t) \cdot \mathcal{E}_0(z) \\ n(z, t) &\Rightarrow n(t) = \int dz \, n(z, t) |\mathcal{E}_0(z)|^2 \end{aligned}$$

advantages of TW models

intermediate level of complexity: as fast as rate equations
but more flexible and detailed

time domain model: stationary, small & large signal properties

covers many types of devices:

- semiconductor optical amplifier (SOA)
- standard edge emitting laser
- multisection laser
- ring laser configurations
- coupled laser structures

but not: tapered structures, VCSELs, LEDs, ...

origins of traveling wave modeling

- **Maxwell-Bloch equations** prototype of TW models
Arecchi & Bonifaci, JQE 1,169 (65)
Theory of optical maser amplifiers
- **small-signal laser modeling** TU of Denmark
Tromborg B, ... JQE 23, 1875 (87)
- **large-signal laser modeling** Nottingham: transmission-line model
Lowery AJ, IEE Proc J 134, 281 (87)
Gent: CLADISS
Morthier G, ... JQE 26, 1728 (90)
Cambridge Carroll JE ... JQE 28, 604 (92)
Berlin Bandelow U, ... PTL 5, 1176 (93)

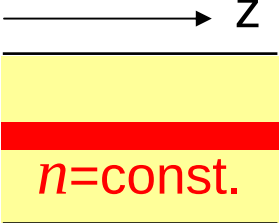
let us understand the equations

- ingredients from waveguiding
- slowly varying amplitudes
- propagation equations
- carrier rate equations

ingredients from waveguiding

relative
 $\downarrow$
 $e^{i(\omega_0 + \omega)t}$
 $\uparrow$
reference

$\Rightarrow$



$\mathcal{E}_{\parallel}(z, t) = e^{i(\omega_0 + \omega)t} e^{-i(k_0 + k)z}$

relative
 $\downarrow$
 $e^{-i(k_0 + k)z}$
 $\uparrow$
reference

supposed
to be known:

$$k(n, \omega) = \beta(n) + \frac{\omega}{v_g} + \mathcal{D}(n, \omega)$$

$\nearrow$
 $k(n, 0)$

$\nwarrow$
fundamental
dispersion

$\nwarrow$
higher order
contributions

basic model:

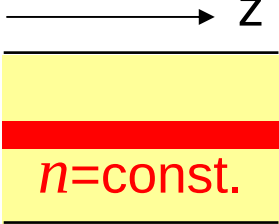
$$\mathcal{D} = 0 \quad \& \quad 2\beta = \delta - i\alpha_b + (i + \alpha_H)g(n)$$

$\uparrow$
losses

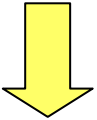
$\nearrow$
Henry
factor

$\uparrow$
gain

slowly varying amplitudes

$A(t)e^{i\omega_0 t}$ $\Rightarrow$ 
 $\mathcal{E}_{\parallel}(z, t) = E_{+}(z, t)e^{i(\omega_0 t - k_0 z)}$

$$E_{+}(z, t) = \int d\omega \tilde{A}(\omega) e^{i(\omega t - k(\omega)z)} \overset{\mathcal{D}=0}{\approx} A(t - z/v_g) e^{-i\beta z}$$

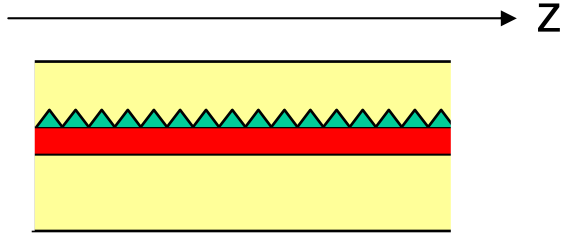
 propagation equation for forward traveling wave

$$\left(\frac{\partial}{\partial t} + v_g \frac{\partial}{\partial z} \right) E_{+}(z, t) = -iv_g \beta E_{+}(z, t) \quad \text{bc: } E_{+}(0, t) = A(t)$$

keeps valid for $\beta = \beta(z, t)$!

backward wave: analogous

propagation equations: distributed feedback



grating with period $\Lambda = \frac{\lambda_{\text{internal}}}{2} \ll L_{\text{laser}}$

reflections at grating couple forward-backward

choosing $k_0 = \pi/\Lambda$ yields

$$i \left(\frac{1}{v_g} \frac{\partial}{\partial t} + \frac{\partial}{\partial z} \right) E_+(z, t) = \beta E_+(z, t) + \kappa_- E_-(z, t)$$

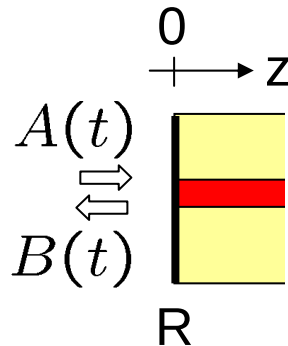
$$i \left(\frac{1}{v_g} \frac{\partial}{\partial t} - \frac{\partial}{\partial z} \right) E_-(z, t) = \beta E_-(z, t) + \kappa_+ E_+(z, t)$$

2 equations coupled by

$$\kappa_{\mp} = \int_{-\Lambda/2}^{\Lambda/2} \frac{dz}{\Lambda} k(n, \omega_0, z) e^{\pm 2\pi i z / \Lambda}$$

boundary conditions

example: device facet



$$E_+(0, t) = tA(t) + r_+E_-(0, t)$$
$$B(t) = r_-A(t) + tE_-(0, t)$$

$$r_{\pm} \sim \sqrt{R} \quad t = \sqrt{1 - R}$$

general junction at z_0 :

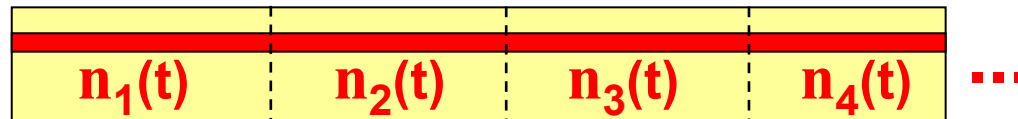
$$\Psi_v^{out}(z_0, t) = \sum_u \mathcal{T}_{vu} \Psi_u^{in}(z_0, t)$$

may connect multiple incoming waveguides u
with multiple outgoing waveguides v

very flexible !

carrier rate equations

averaging in appropriate subsections s:



$$\frac{d}{dt}n_s(t) = \overset{\text{pump rate}}{\mathcal{J}_s(t)} - \overset{\text{spont reco}}{R_s(n)} - v_g \text{Im} \sum_{w=\pm} \left\langle \overset{\text{stimulated recombination}}{E_w^* (2\beta + i\alpha_b) E_w} \right\rangle_s$$

↑
background losses in β

basic model:

$$\mathcal{J}_s(t) = \frac{I}{eV_{AZ}} \qquad R_s(n) = \frac{n}{\tau}$$

summary of basic equations

optical TW equation

$$-i\frac{\partial}{\partial t}\Psi(z, t) = H(\beta)\Psi(z, t)$$

$$z \in \text{device} \quad \Psi = (E_+, E_-)^T$$

$$H(\beta) = -v_g \begin{pmatrix} -i\frac{\partial}{\partial z} + \beta & \kappa_- \\ \kappa_+ & +i\frac{\partial}{\partial z} + \beta \end{pmatrix}$$

waveguide propagation model:

$$2\beta = \delta - i\alpha_b + (i + \alpha_H)g(n)$$

β = propagation constant
relative to k_0
taken at ω_0

carrier rate equations:

$$\frac{d}{dt}n_s(t) = \mathcal{J}_s(t) - R_s(n) - v_g \text{Im} \sum_{w=\pm} \left\langle E_w^* (2\beta + i\alpha_b) E_w \right\rangle_s$$

some important extensions of the basic model

- nonlinear gain saturation
- gain dispersion
- spontaneous emission
- current redistribution
- few words on numerics

nonlinear gain saturation

gain decreases with optical intensity $|E|^2$

possible reasons: redistribution of carriers

- in energy (carrier heating, spectral hole burning) ... Schimpe R, APL 60, 2720 (92)
also tutorial Koch (MA2)
- in transverse space (transverse spatial hole burning)

effective refractive index is much less influenced

model:
$$2\beta = \delta - i\alpha_b + i \frac{g(n)}{1 + \varepsilon |E|^2} + \alpha_H g(n)$$

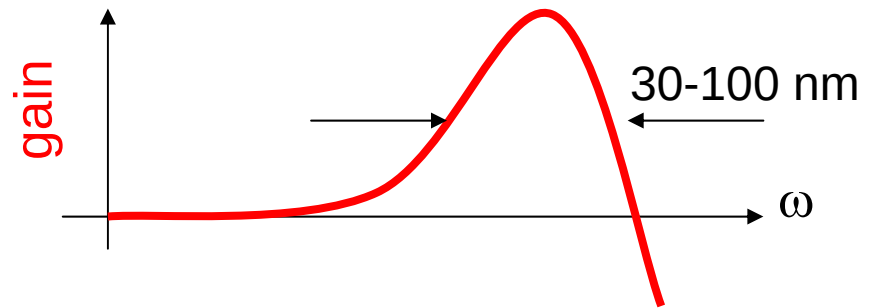
typical order of magnitude: $\varepsilon \approx 1 \text{ W}^{-1} \Rightarrow$ small effect

but important for dynamics,
e.g. damping of relaxation oscillations,
four-wave mixing

Petermann K, chapter 5 in
Laser diode modulation and noise,
Kluwer 1988

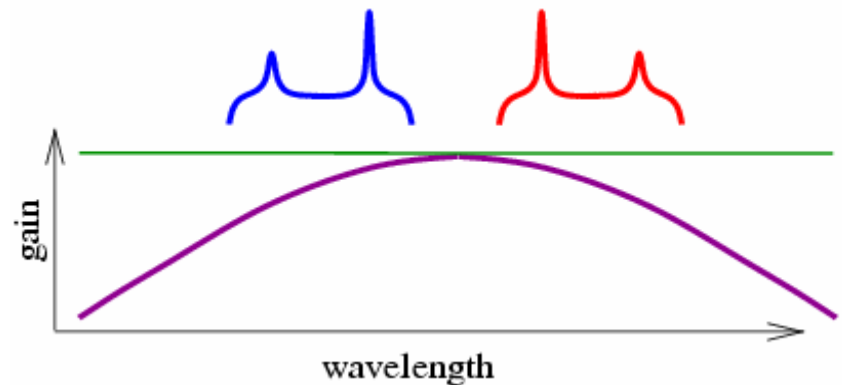
gain dispersion

gain dispersion: $g = g(\omega)$



although gain bandwidth $\gg$ width of laser spectrum,
gain dispersion is important for

- mode selection in FP & index coupled DFB lasers
- avoiding unphysical spurious solutions (high frequency components)



treating gain dispersion in TW models

$$k(n, \omega) = \beta(n) + \frac{\omega}{v_g} + \cancel{\mathcal{D}(n, \omega)}$$

basic model

inclusion of gain dispersion : $\mathcal{D}(\omega) \neq 0$ operator in time domain models

main methods:

- numerical temporal filtering Carroll JE, ... DFB lasers
IEE Publishing (98)
- additional polarization equation
(simplest: Lorentzian line shape) Ning... JQE 33, 1543 (97)
Bandelow... JQE 37, 183 (01)
- spatial digital filtering Kolesik & Moloney
JQE 37, 936 (01)

spontaneous emission

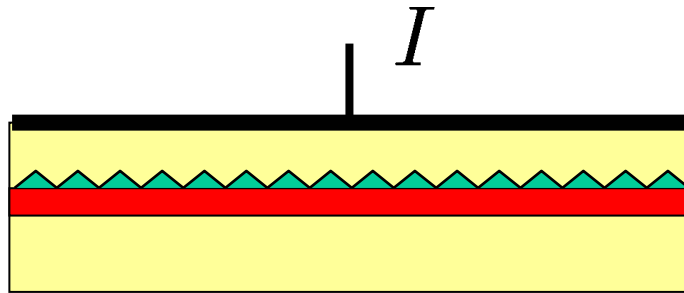
spontaneous emission = quantum-optical effect

semiclassical approximation by spatially uncorrelated white-noise:

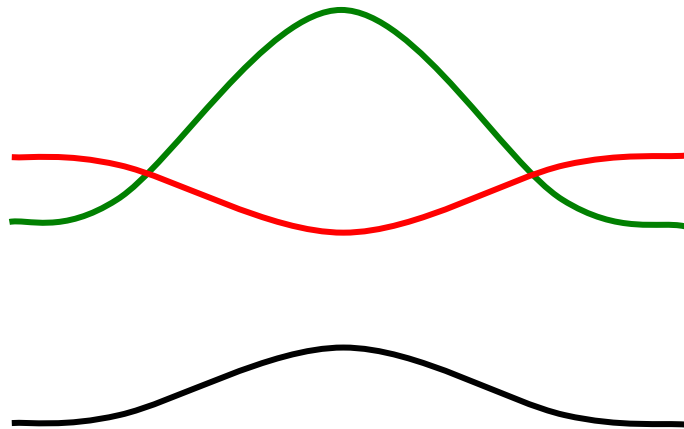
$$-i\frac{\partial}{\partial t}\Psi(z, t) = H(\beta)\Psi(z, t) + F^{\text{sp}}(z, t) \quad \text{Langevin Equ.}$$

more details in: Henry CH & Kazarinov RF
quantum noise in photonics
Rev Mod Phys 68, 801 (96)
Carroll JE, ... DFB lasers
IEE Publishing (98)

current redistribution



metallic contact = equipotential



intensity $|E|^2$ nonuniform

→ stimulated reco nonuniform

→ inversion n nonuniform

= longitudinal spatial hole burning (LSHB)

injection rate J nonuniform

= current redistribution,
counteracts LSHB

model:

$$\mathcal{J}(z, t) = \frac{I}{eV} - \frac{U'_F}{eV R_s} (n(z, t) - \bar{n}(t))$$

Champagne Y... JAP 72, 2110 (92)

Eliseev PG... JSTQE 3, 499 (97)

few words on traveling-wave numerics

finite difference schemes approximating fields

$\Psi(z,t)$ along characteristic lines $v_g t \pm z = \text{const}$:

- basics with numerical filtering:

J. Carroll ... DFB lasers, Chapter 7, IEE & SPIE 1998

- with polarization equations:

M. Radziunas ... Chapter 5 in "Optoelectronic Devices ...",
J. Piprek ed., Springer, 2005

further approaches:

- transfer matrix method

Zhang LM, JQE 28, 604 (92)

Marcenac DD, PhD Thesis, Cambridge 93

- split step method

Kim BS ... JQE 36, 787 (00)

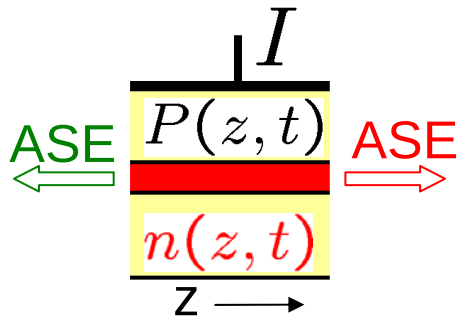
- transmission line approach

Lowery AJ, IEE Proc J 134, 281 (87)

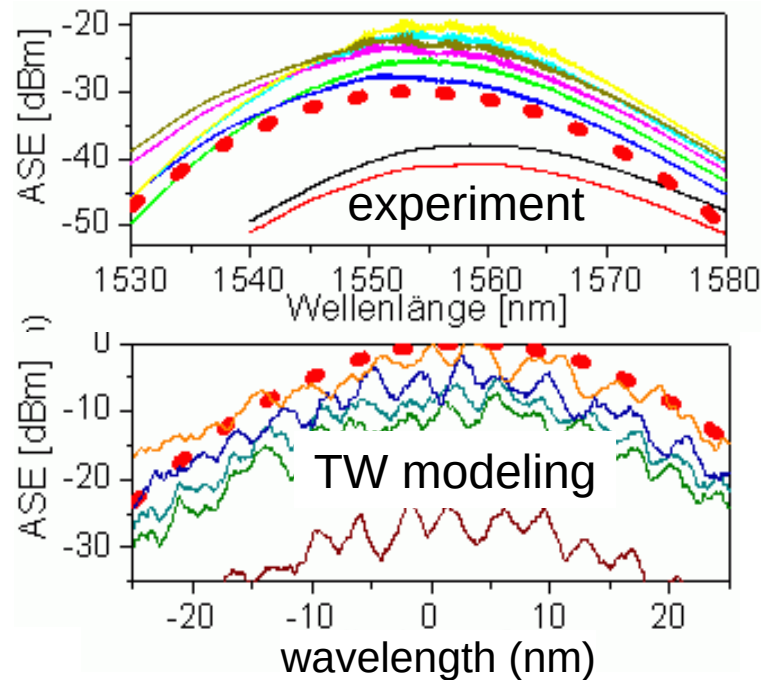
TW modeling on work - settle back and enjoy!

- semiconductor optical amplifier (SOA)
- simple Fabry-Perot (FP) laser
- simple index-coupled distributed feedback (DFB) laser
- small signal modulation response
- large signal modulation
- mode locked laser
- self-pulsating multi-section DFB laser
- locking to external bit streams
- ring laser

semiconductor optical amplifier (SOA): idle running



- ideal SOA: zero facet reflectivities
- idle running: without optical injection
- only radiation source: spontaneous emission
- device emits amplified spontaneous emission (ASE)

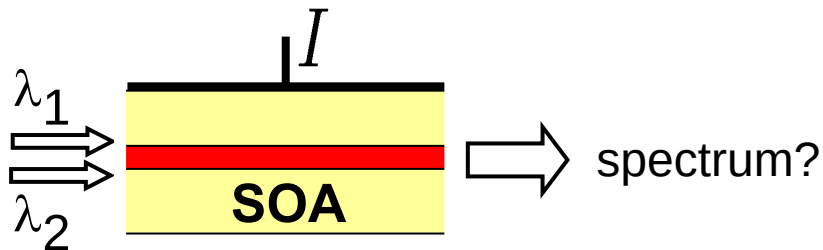


components & measurements: HHI Berlin

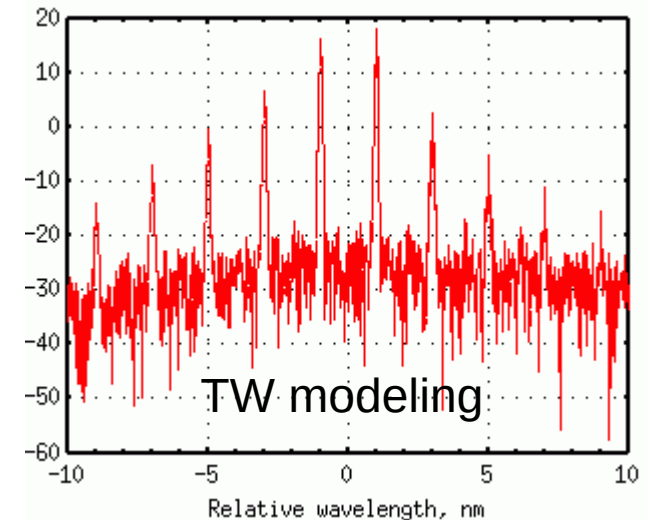
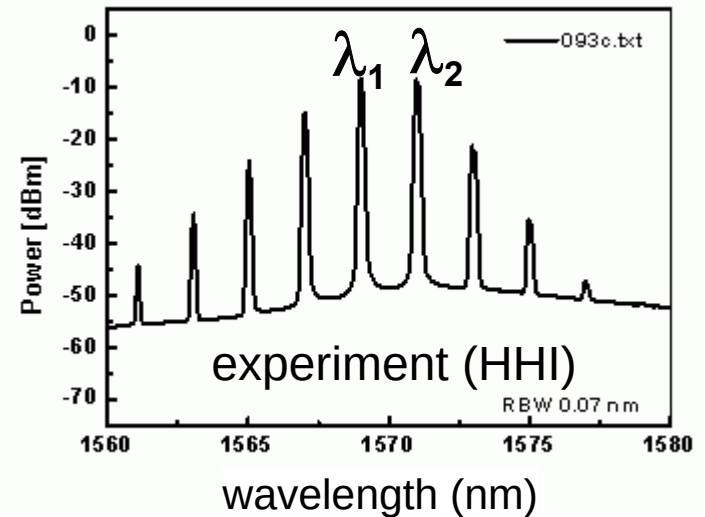
TW model well describes
ASE spectra of idle SOA

SOA: four wave mixing (FWM)

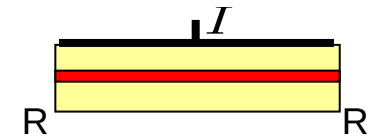
experiment: two waves are injected with different wavelengths



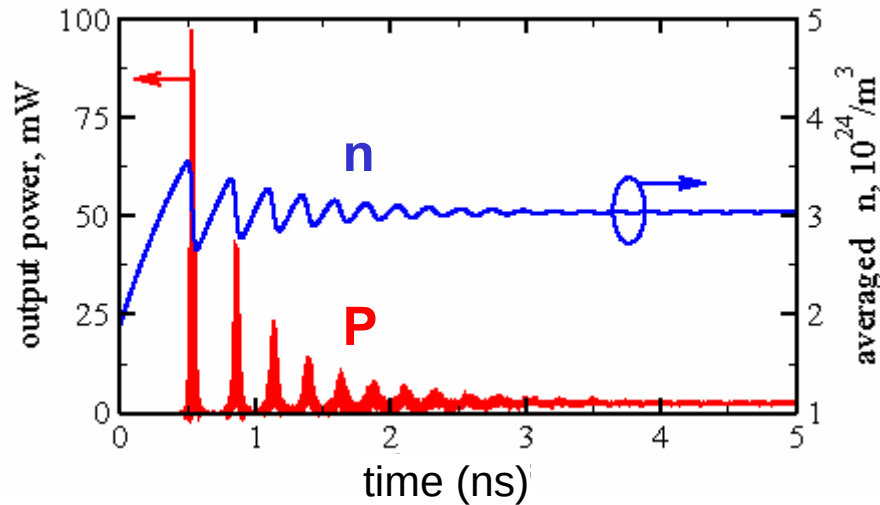
TW model inherently contains the generation of satellite waves by four wave mixing



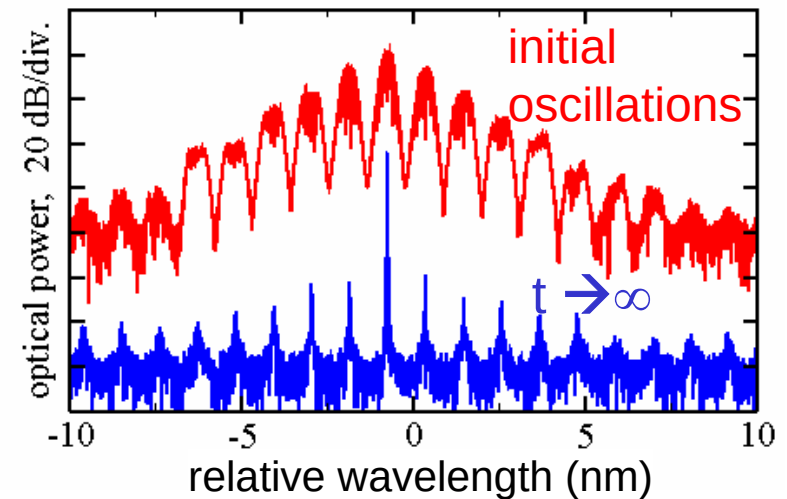
simple Fabry-Perot (FP) laser



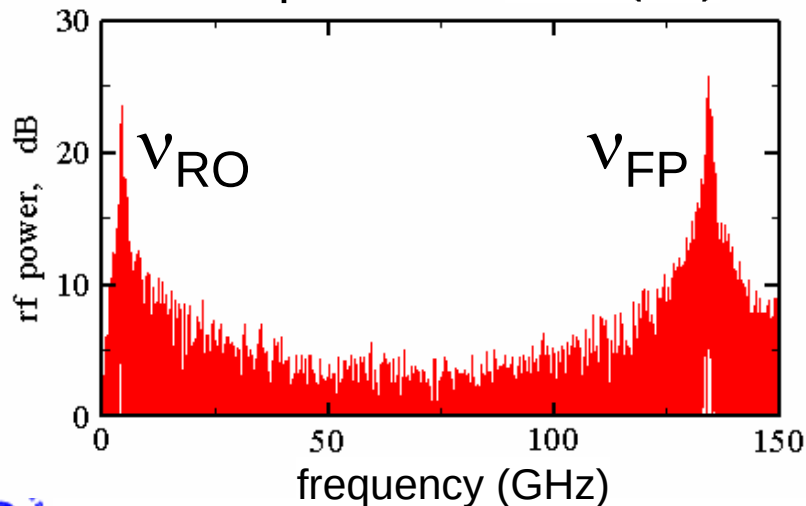
turn-on transient



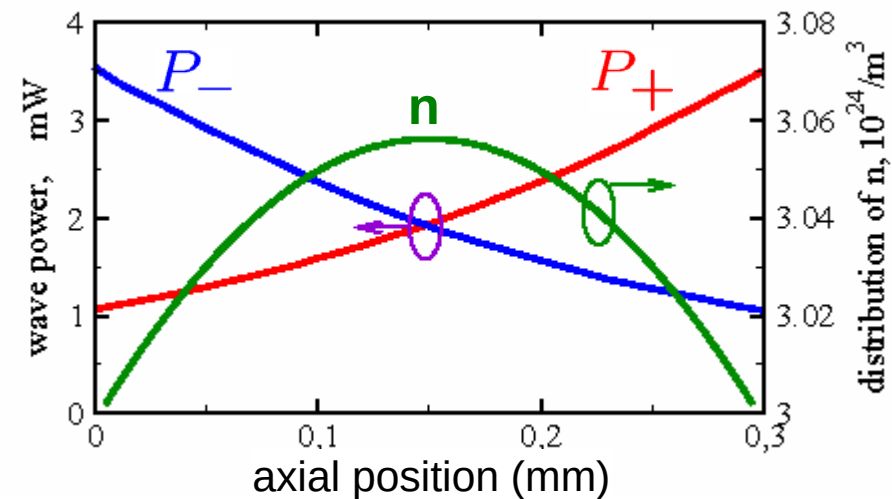
optical spectra $\sim |E(\omega)|^2$



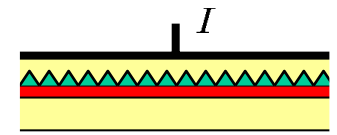
RF spectrum $\sim P(\Omega)$



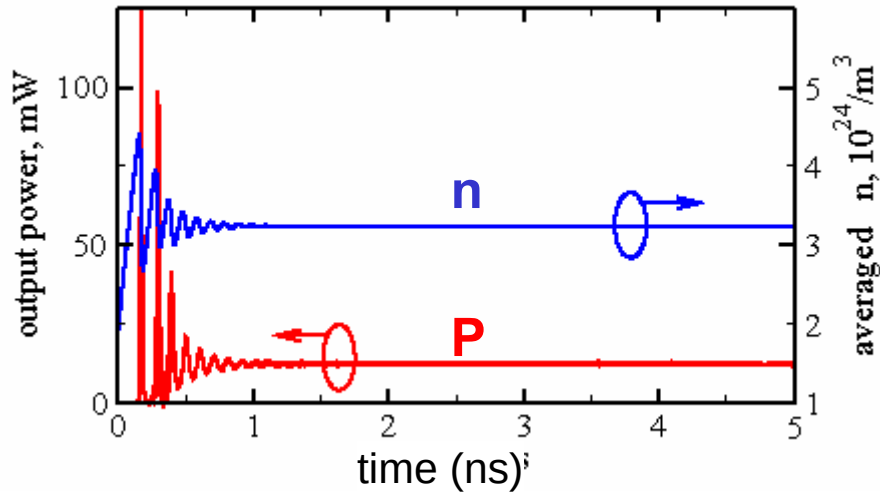
axial distributions



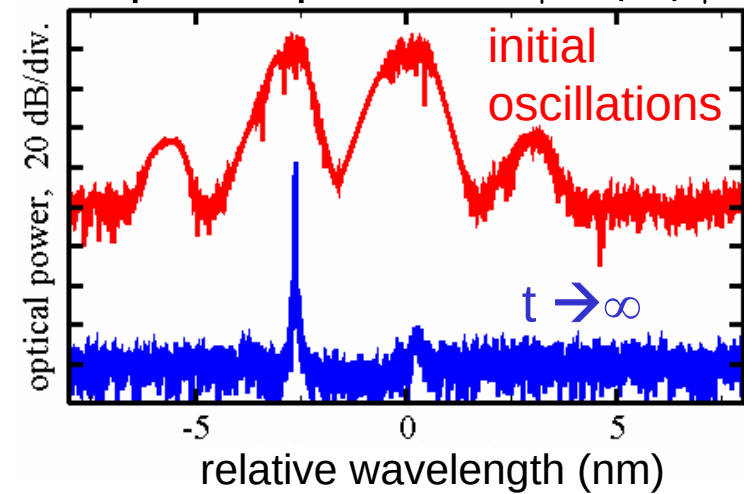
index-coupled DFB laser



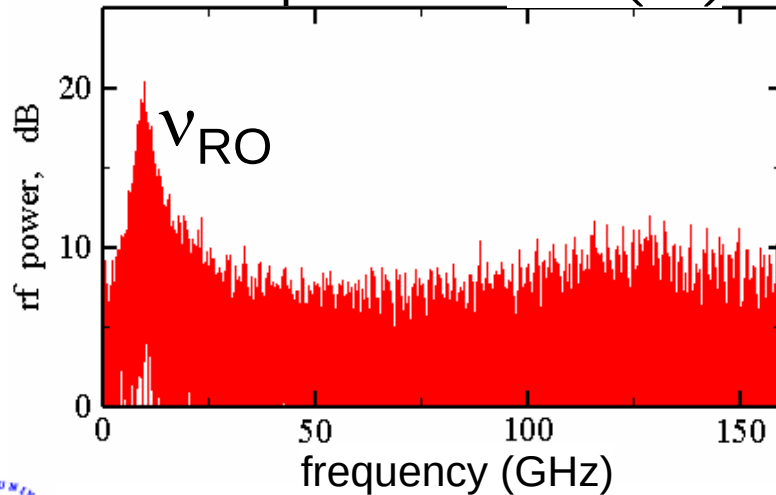
turn-on transient



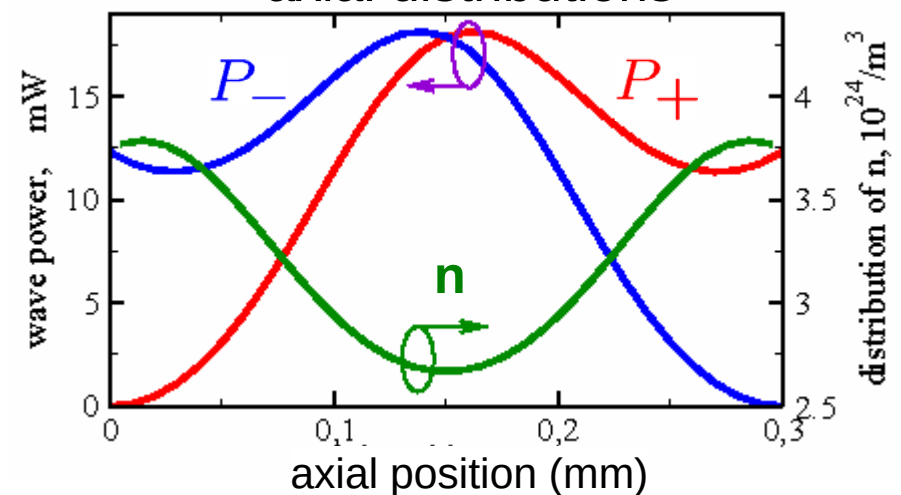
optical spectra $\sim |E(\omega)|^2$



RF spectrum $\sim P(\Omega)$



axial distributions



small-signal modulation (DFB laser)

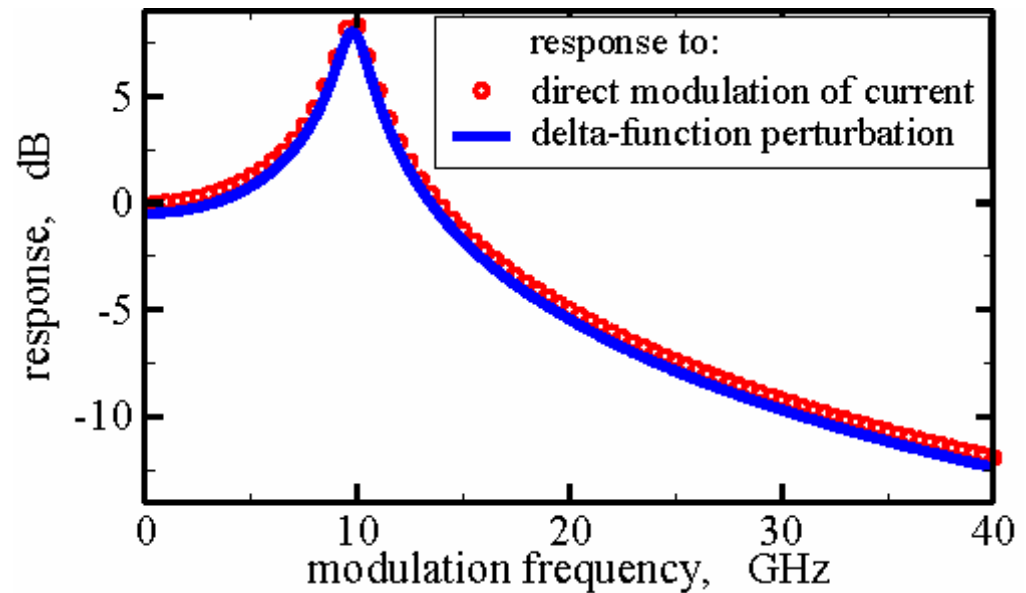
$$I = I_0 + i(t) \quad |i(t)| \ll I_0$$

$$\text{modulation response (MR)} = \frac{P(\Omega)}{P(0)} \quad \text{for } i(t) \sim \cos(\Omega t)$$

two calculation methods:

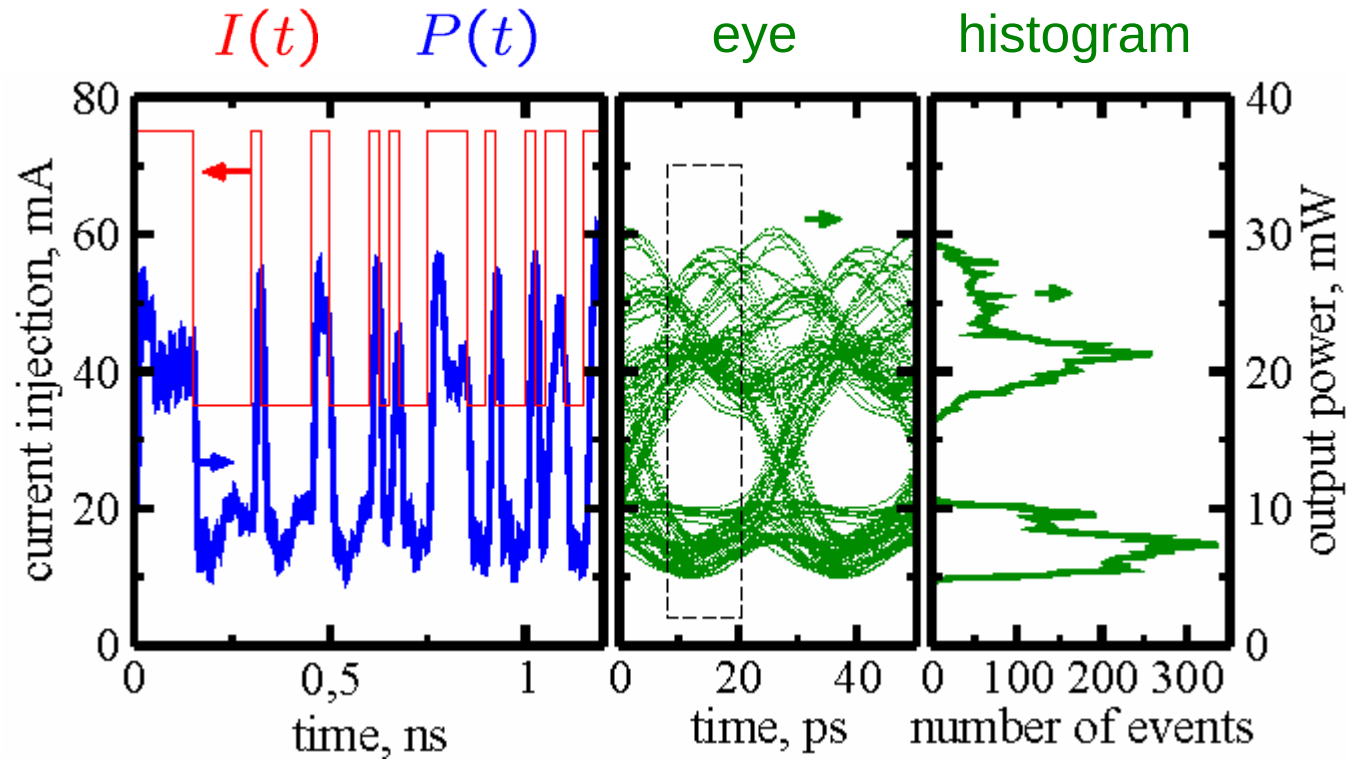
- simulate direct modulation.
cumbersome: needs one transient for each Ω
- apply a short current spike
 $i(t) \sim \delta(t)$

and take the RF spectrum
of the power transient.



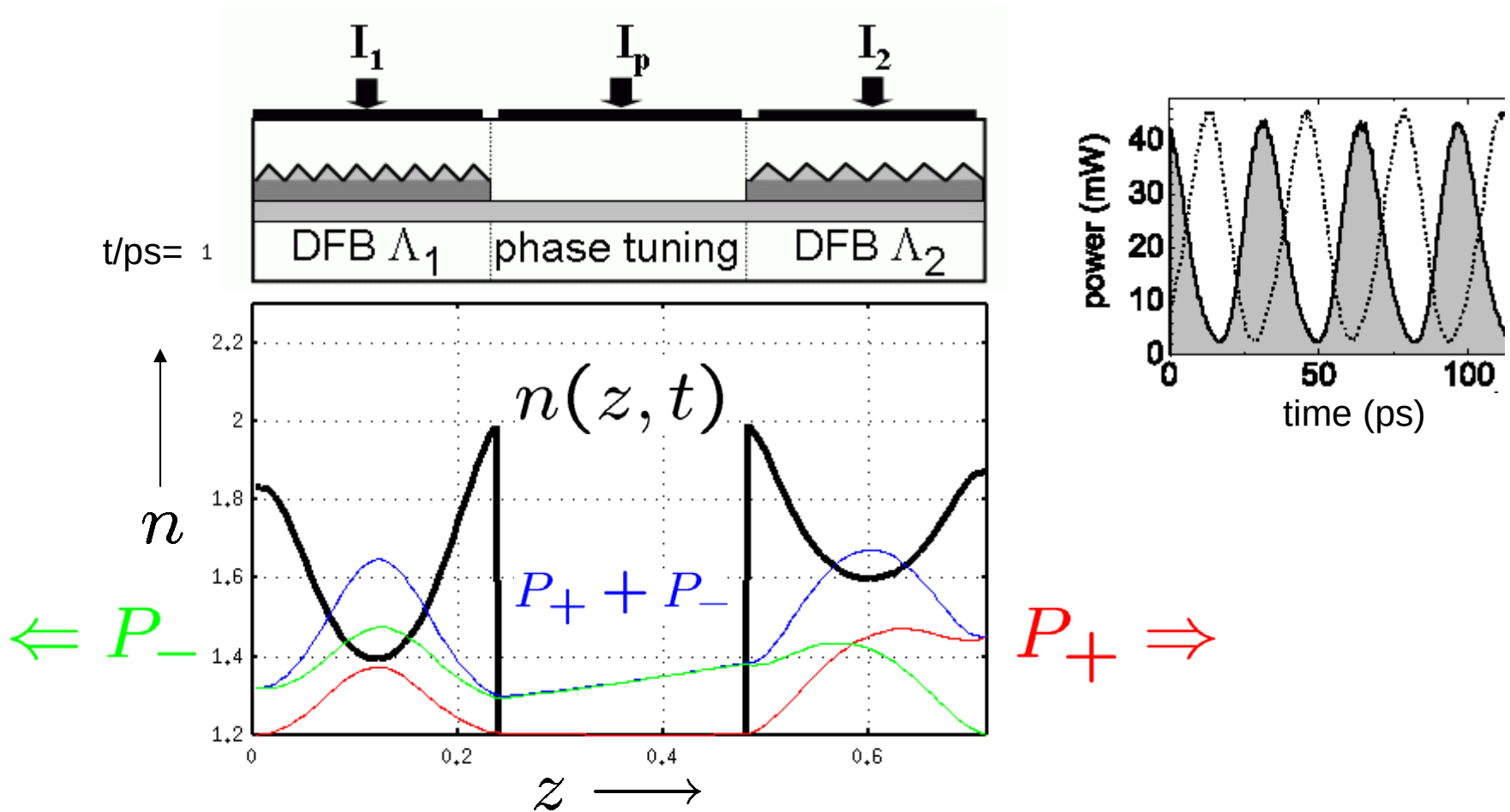
large signal modulation

DFB+EC laser: direct modulation with 40 Gb/s NRZ PRBS

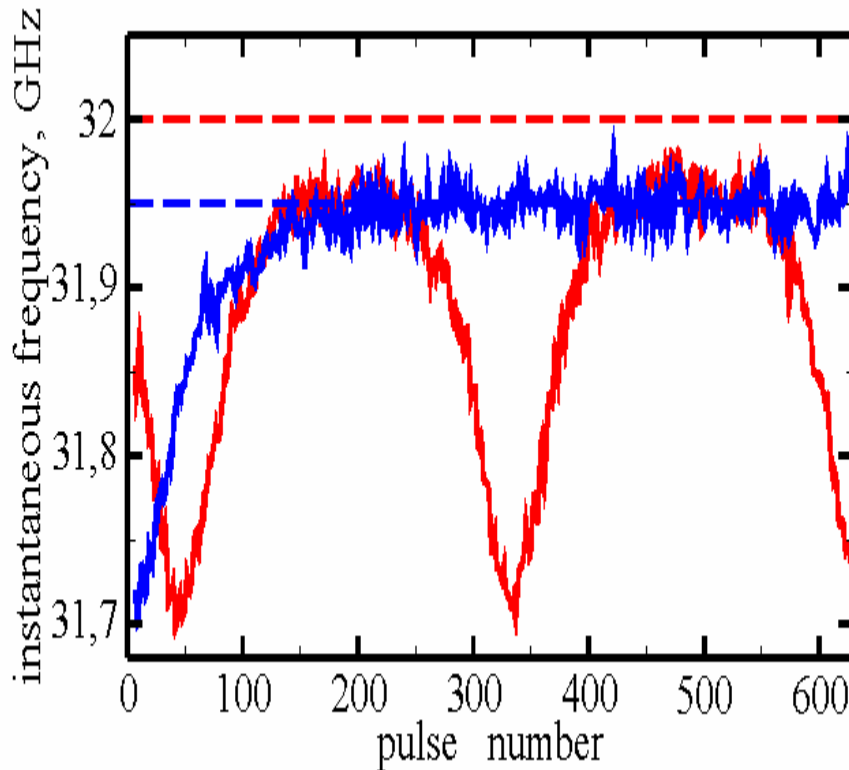
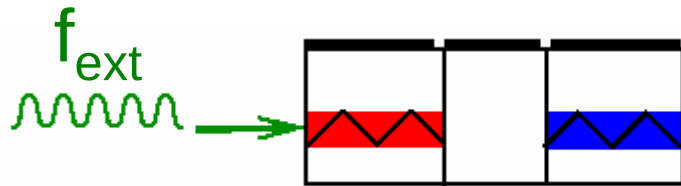


M. Radziunas: talk TuD2 Tuesday afternoon

self-pulsating multisection laser



locking of self-pulsations to external signals



dashed: injected frequencies f_{ext}

blue:

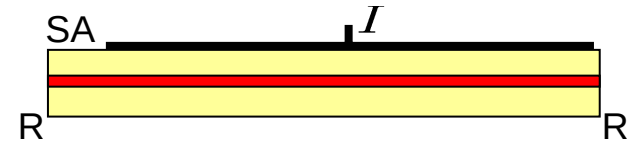
the pulsation locks to the external frequency

red:

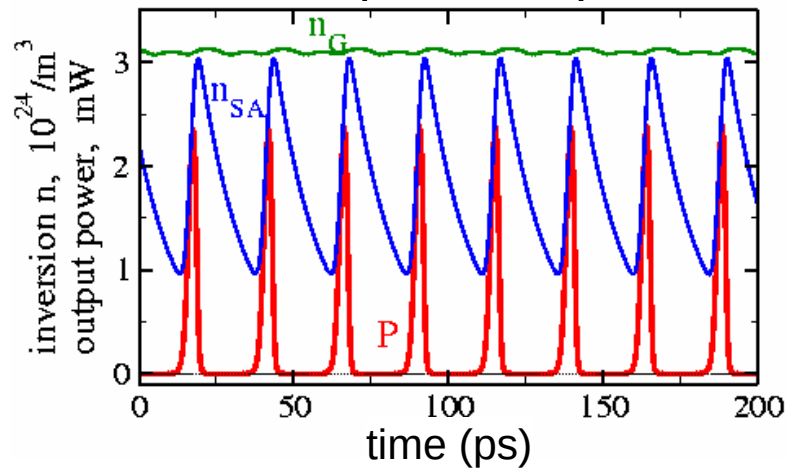
the pulsation does not lock but shows repeated phase slips

monolithic mode-locked laser

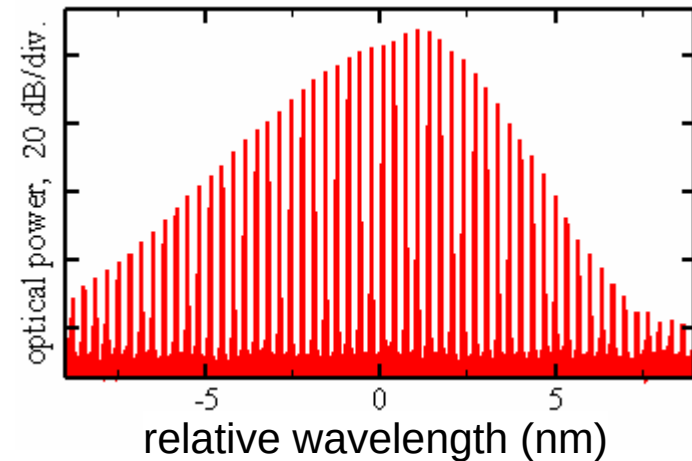
benchmark device of COST 288



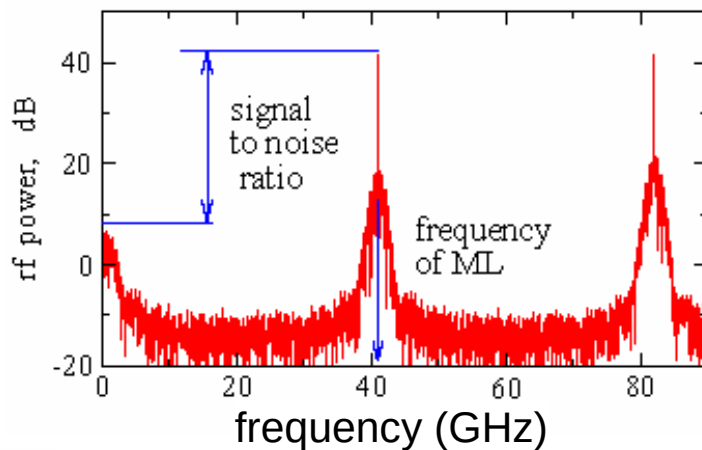
transient pulse sequence



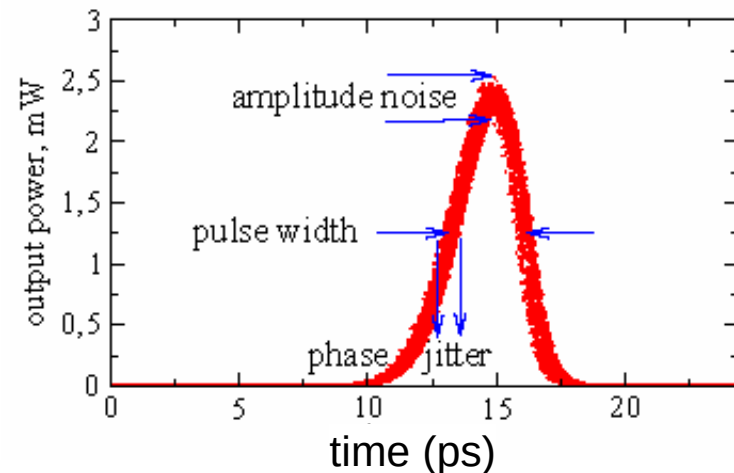
optical spectra: many locked modes



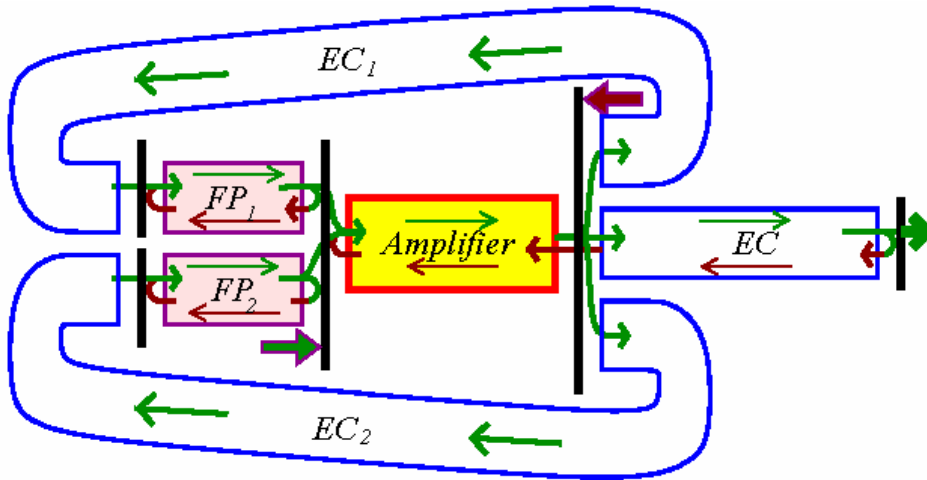
RF spectrum: SNR



sampled pulses (eye)

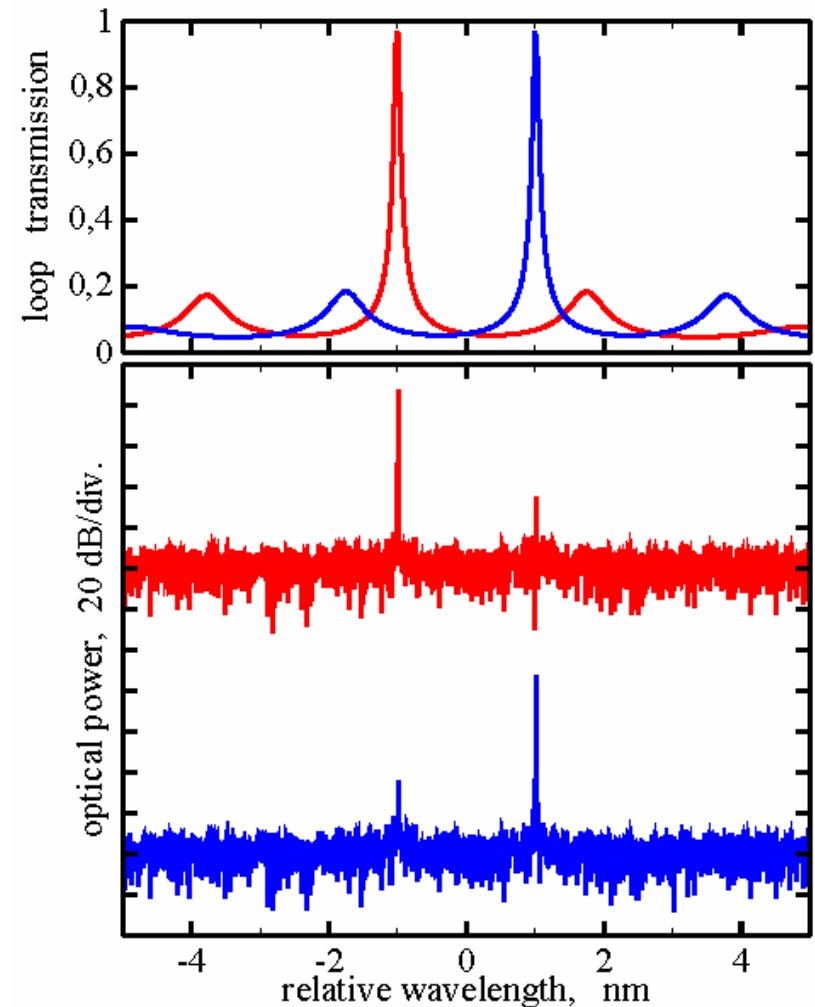


ring laser with extended cavity



optical switching between
two stable cw states

S. Zhang ..., IEEE JSTQE 10, 2004



optical fields can be represented by modes
in time and frequency domains

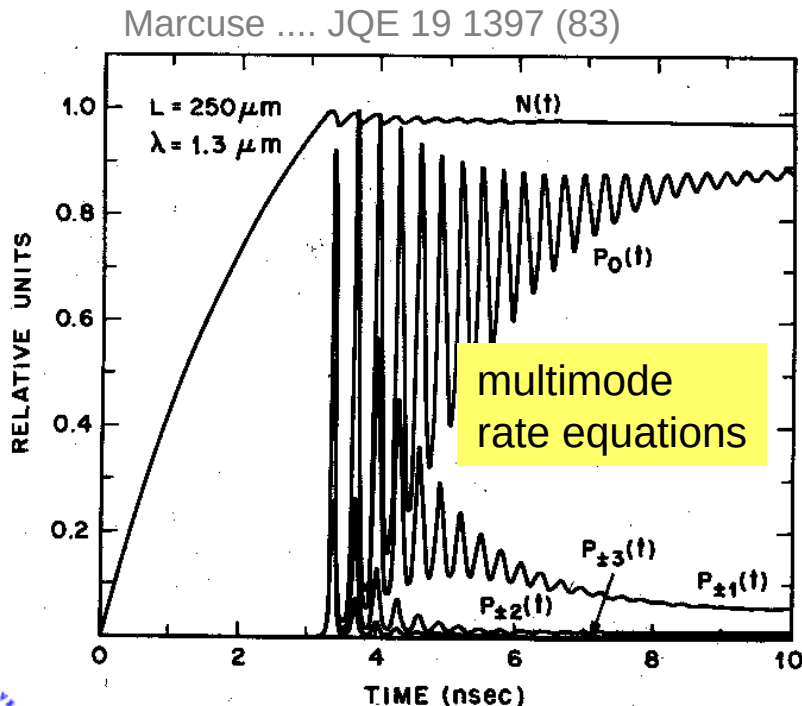
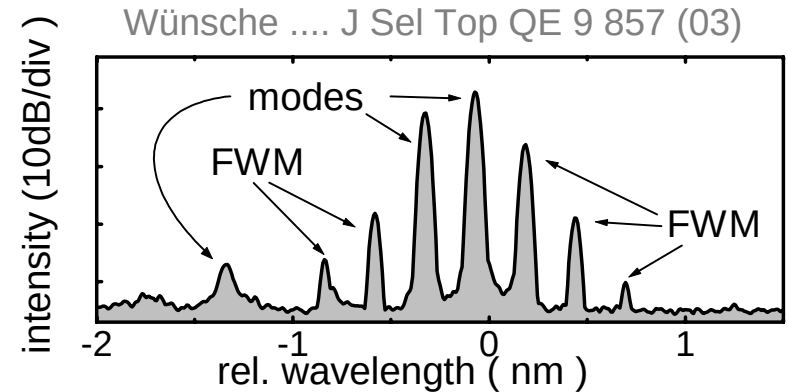
- mode definition
- how to compute modes
- meaning of complex mode frequencies
- how modes depend on inversion
- mode expansion of the field
- examples

motivation

optical modes =

natural oscillations of the laser cavity

- they govern the optical spectra
- but not all spectral lines are modes



- modes play an important role in laser dynamics
- modal intensities are subject of multimode rate equations
- how to treat modes in the traveling-wave approach?

mode definition

full TW equation: $-i\partial_t \Psi(z, t) = H(\beta) \Psi(z, t)$ +b.c.

mode equation: $H(\beta) \Theta(\beta, z) = \Omega(\beta) \Theta(\beta, z)$ +b.c.

mode function

mode frequency (complex)

characteristic equ.:

$$(r_L, -1) M(\Omega; \beta) \begin{pmatrix} -r_0^* \\ 1 \end{pmatrix} = 0$$

facet reflectivities

overall transfer matrix

how to compute modes

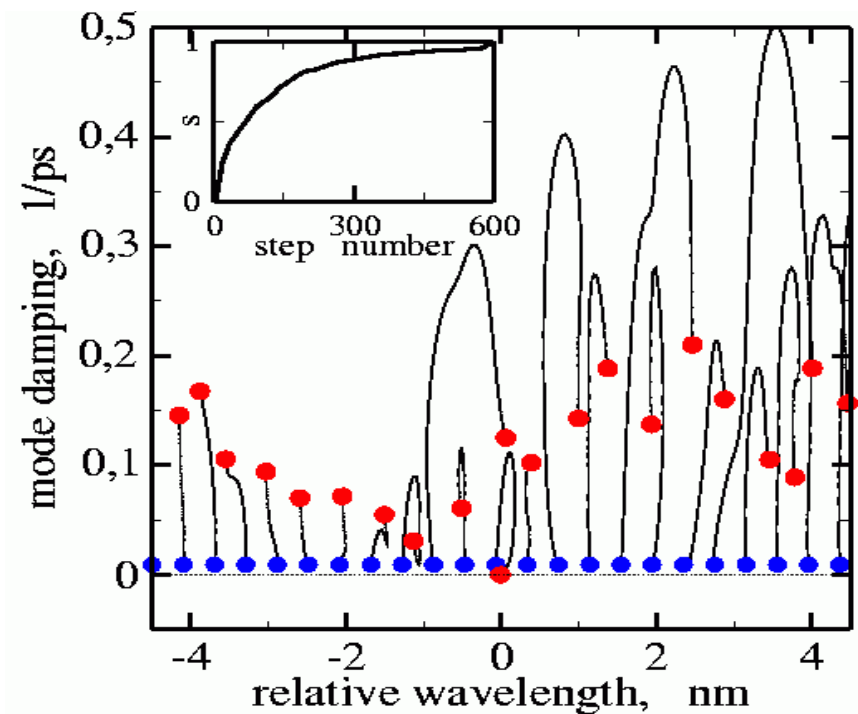
$$(r_L, -1)M(\Omega; \beta) \begin{pmatrix} -r_0^* \\ 1 \end{pmatrix} = 0$$

FP laser: analytic solution $\Im m \Omega = -[2\Im m \beta L + \ln(|r_0 r_L|)] / \tau$
 $\Re e \Omega = [\arg(-r_0^* r_L) - 2\Re e \beta L + 2\pi k] / \tau$

Other lasers:

homotopy method,
starting from FP

M. Aubry,
Homotopy Theory and Models
Boston, MA: Birkhäuser, 1995.



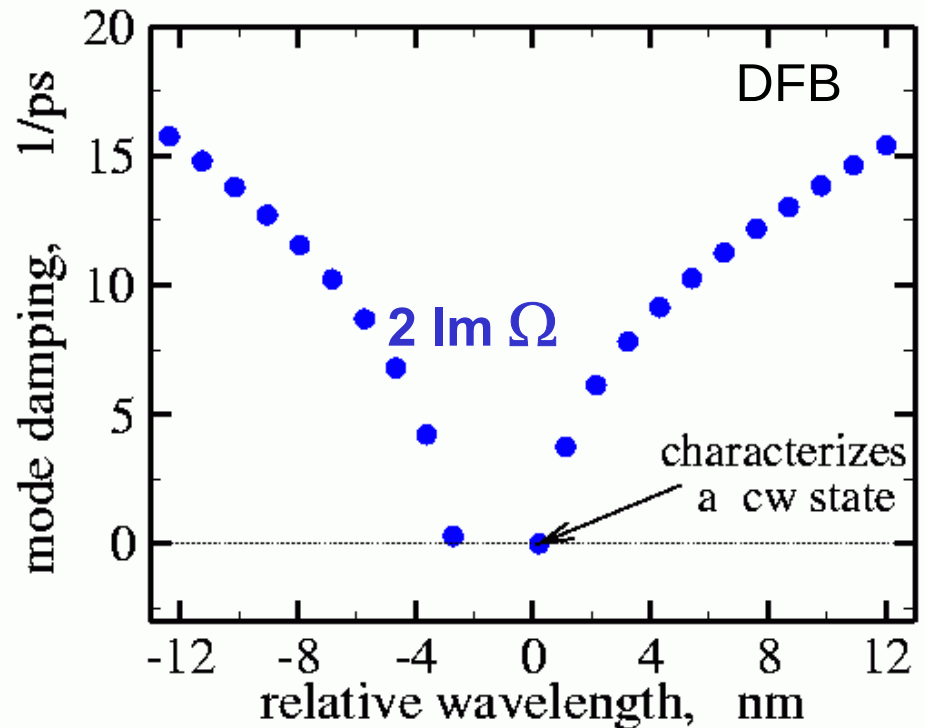
meaning of complex mode frequencies Ω_m

full mode field $\sim \Theta^m(\beta, z)e^{i\Omega_m t}$

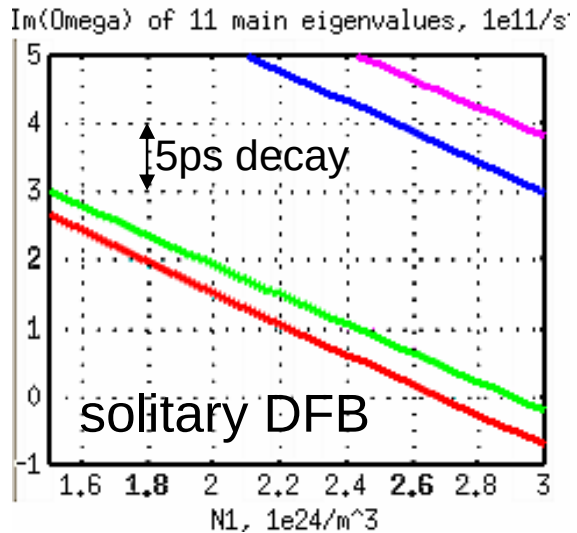
intensity $\sim e^{-2\text{Im}\Omega_m t}$

$2 \text{Im } \Omega_m = \text{modal decay rate}$

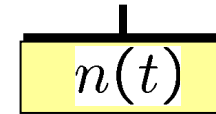
$\text{Re } \Omega_m = \text{mode frequency}$
 $\Leftrightarrow \text{wavelength}$



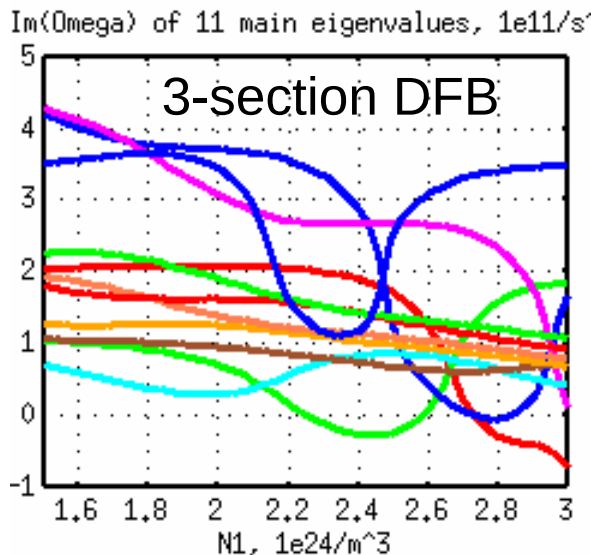
how modes depend on inversion



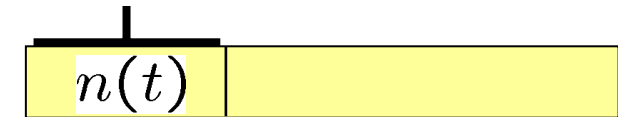
simple laser



- $\Theta^m(z)$ independent of n !
- $\Omega_m(n) = \Omega_m(n_0) - v_g(\beta(n) - \beta(n_0))$
identical variation of all modes



compound laser



- $\Theta^m(n, z)$ changes with n !
 $n = n(t) \Rightarrow$ "moving modes"
- $\Omega_m(n)$: different nonlinear functions of n

mode expansion of the optical field

mode functions form basis of the Hilbert space

→ optical field can be expanded in a mode series

$$\Psi(z, t) = \sum_k f_k(t) \Theta^k(\beta, z)$$

$$f_k(t) = [\Theta^{k\dagger}(\beta, z), \Psi(z, t)]$$

adjoint mode

Hilbert space scalar product

rigorous formulations:

J. Rehberg ... ZAMM 77, 75 (97)

J. Sieber, PhD Thesis, HU Berlin (01)

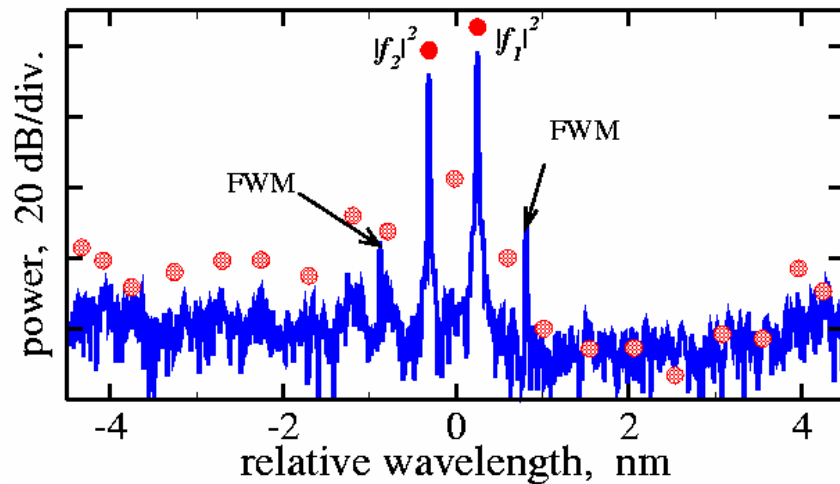
such expansion is possible for any choosen β

best choice: $\beta = \beta(t)$ **instantaneous modes**

the instantaneous modes adapt themselves adiabatically to slow changes of the optical cavity.

examples

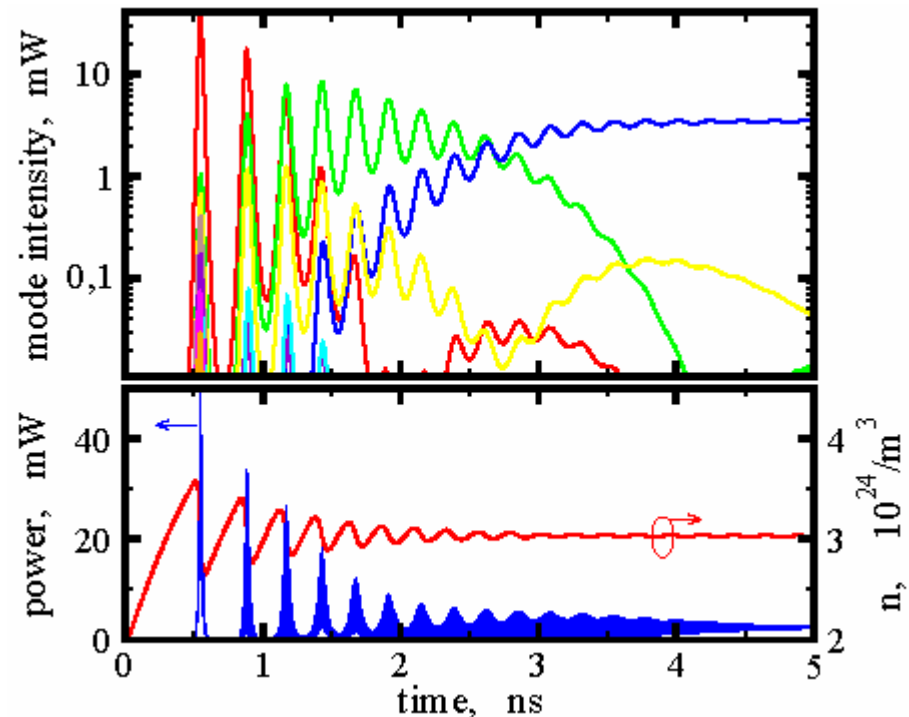
stable two-mode pulsation



mode expansion allows:

- to identify dominant modes
- to identify which peaks of the optical spectra belong to modes
- to interpret properly nontrivial transients of the field

turn-on spiking FP laser



field projection onto moving subspaces
spanned by few instantaneous modes

- amplitude equations
- the simple case of solitary lasers
- restriction to essential modes
- precision of mode approximations
- math foundation: center manifold theorem

amplitude equations

$$\Psi(z, t) = \sum_k f_k(t) \Theta^k(\beta, z)$$

moving modes: $\beta = \beta(n(t))$

how evolve the mode amplitudes?

H. Wenzel ... IEEE JQE 32, 69 (96)

$$\dot{f}_k = i\Omega_k f_k + \sum_{l=1}^{\infty} K_{kl} f_l + F_k$$

$$K_{kl} = -[\Theta^{k\dagger}, \partial_t \Theta^l][\Theta^{k\dagger}, \Theta^k]^{-1}$$

mode coupling factors

$$F_k = i[\Theta^{k\dagger}, F_{sp}][\Theta^{k\dagger}, \Theta^k]^{-1}$$

spontaneous emission
into mode k

the simple case of solitary lasers

$$\Theta^k(z) \text{ independent of } t \rightarrow K_{kl} = -[\Theta^{k\dagger}, \overset{=0}{\partial_t \Theta^l}] [\Theta^{k\dagger}, \Theta^k]^{-1} = 0$$

↓ (drop noise for simplicity)

$$\dot{f}_k = i\Omega_k f_k$$

↓

noncoupled rate equations
for photon number $S_k \sim |f_k|^2$

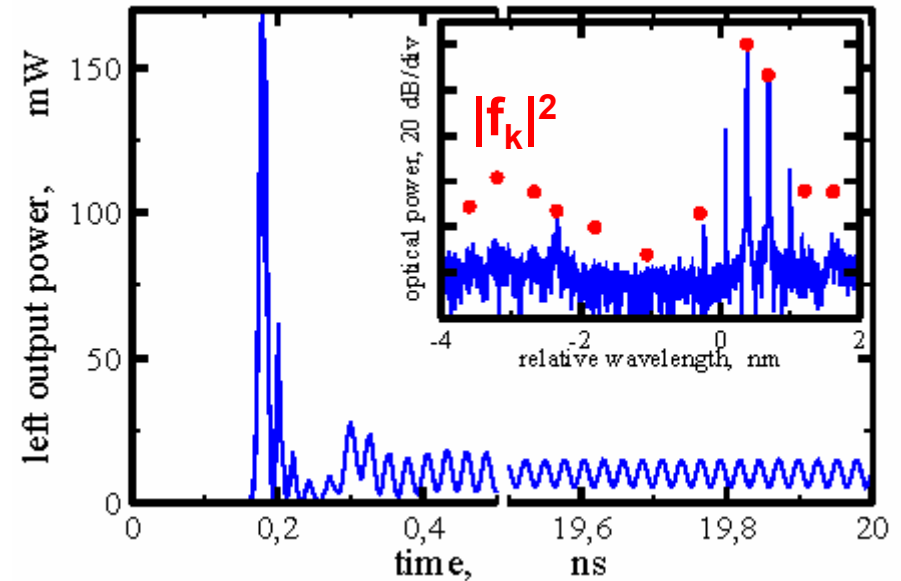
$$\dot{S}_k = -2\text{Im}\Omega_k S_k$$

Marcuse JQE 19 1397 (83)

mode coupling by K_{kl} is a specific effect of compound lasers

restriction to essential modes

$t \rightarrow \infty$: only few modes with noticeable amplitude



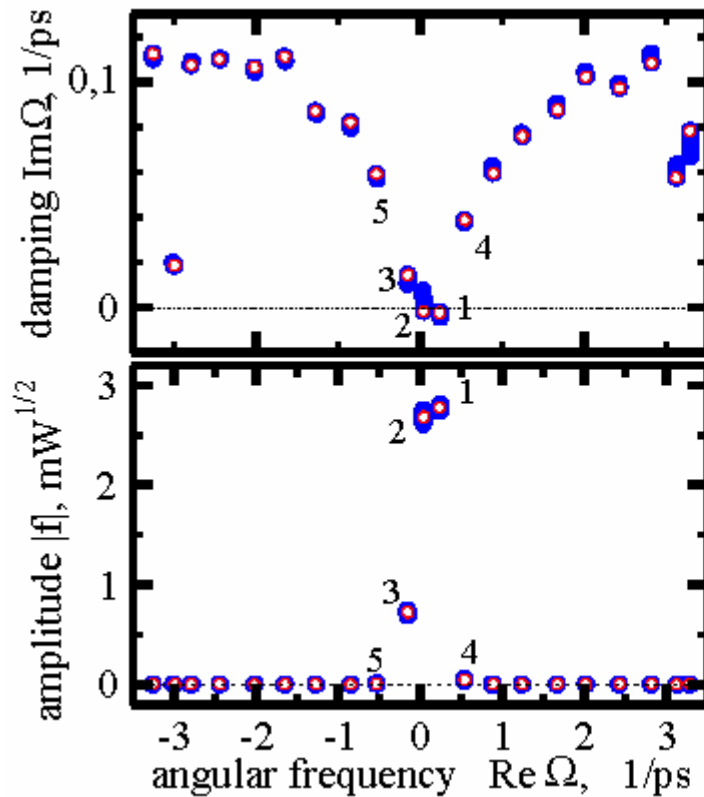
restriction to these essential modes yields closed set of ODE:

$$\begin{aligned} \dot{f}_k &= i\Omega_k(n)f_k + \sum_{l=1}^q K_{kl}(n, \dot{n})f_l \\ \dot{n}_r &= \varepsilon [I_r - R_r(n_r) - \Re e(f, L^r(n)f)] \end{aligned}$$

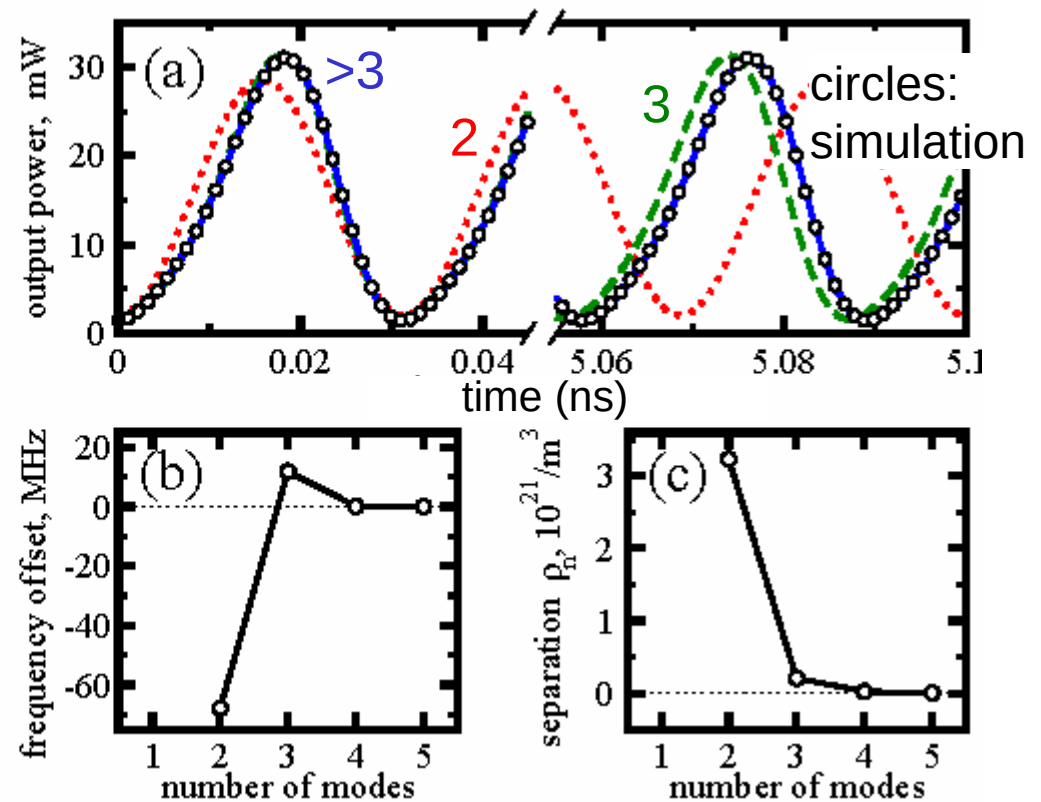
$k = 1, \dots, q$ $r = \text{subsection number}$

precision of mode approximations

most relevant modes



dependence on number q of modes



consider the TW model

$$\frac{d}{dt}\Psi = iH(n)\Psi; \quad \frac{d}{dt}n = \varepsilon [J(n, t) - \Re e\langle \Psi; G(n)\Psi \rangle].$$

It holds:

- only finitely many eigenvalues $i\Omega(n)$ of $iH(n)$ are critical (close to imaginary axis or with positive real part).
- All they have finite algebraic multiplicity
- it exists an exponentially attracting smooth invariant manifold

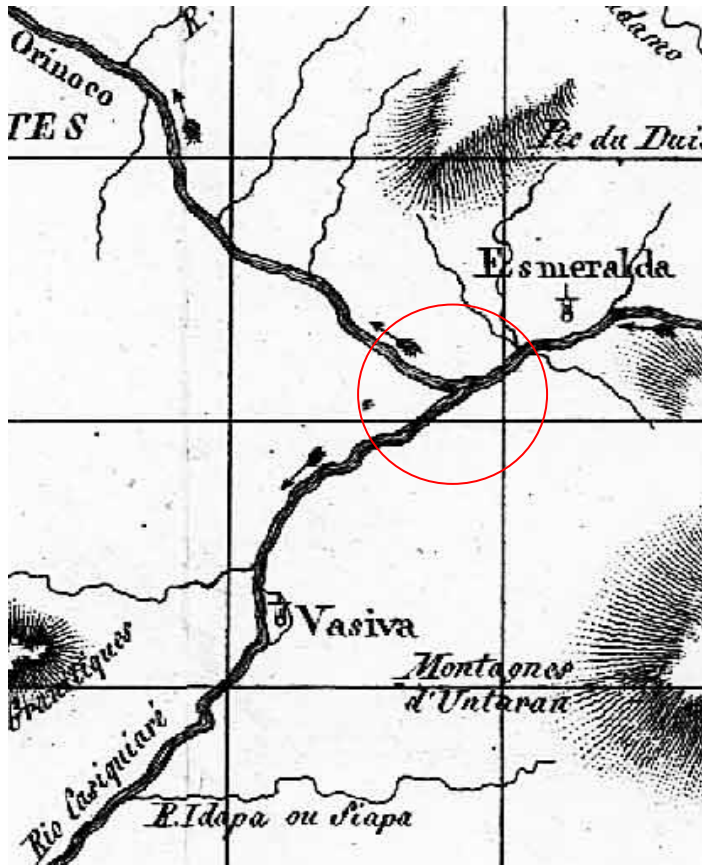
J. Sieber, PhD Thesis, HU Berlin, 2001

→ asymptotic motion on a finite dimensional center manifold

an elegant way to understand the inherent nonlinearities

- what is a bifurcation?
- why bifurcation analysis?
- how doing bifurcation analysis of TW models?
- example
- live demonstration

what is a bifurcation?



a famous old example:

river Orinoco splits in two arms

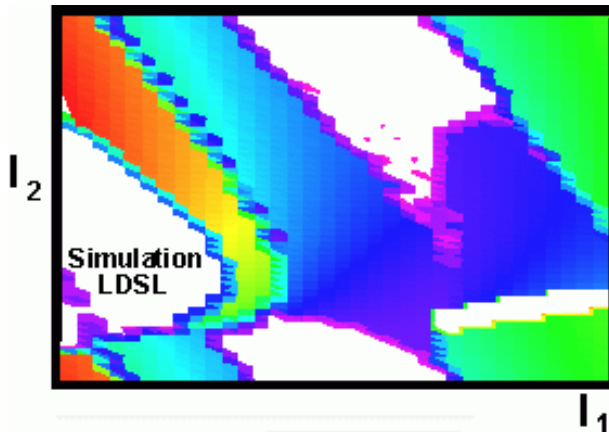
in our context:

- qualitative change of a solution under smooth variation of a control parameter
- border in parameter space between regions of different behaviour

drawing by Humboldt, from

<http://www.uni-potsdam.de/u/romanistik/humboldt/hin/leitner-HINIII.htm>

why bifurcation analysis?



How do operation regimes depend on parameters?

You can scan parameters by simulation, but:

- this procedure is mostly very laborious
- possible multistabilities (hysteresis) require to check different initial conditions.

Bifurcation analysis is more elegant:

- it calculates only the border lines between regimes
- it allows to classify the different bifurcations

B. Krauskopf in "Fundamental issues of Nonlinear Laser Dynamics", AIP 2000

how doing bifurcation analysis of TW models?

no possibility yet for a direct bif-analysis of the full set of partial-differential TW equations.

way out:

use finite dimensional mode approximation models

$$\begin{aligned}\dot{f}_k &= i\Omega_k(n)f_k + \sum_{l=1}^q K_{kl}(n, \dot{n})f_l \\ \dot{n}_r &= \varepsilon [I_r - R_r(n_r) - \Re e(f, L^r(n)f)]\end{aligned}$$

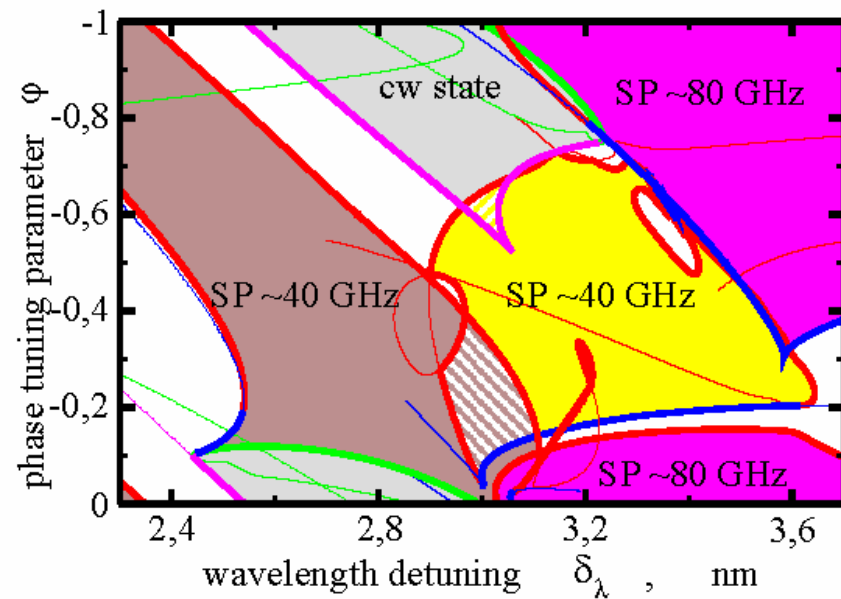
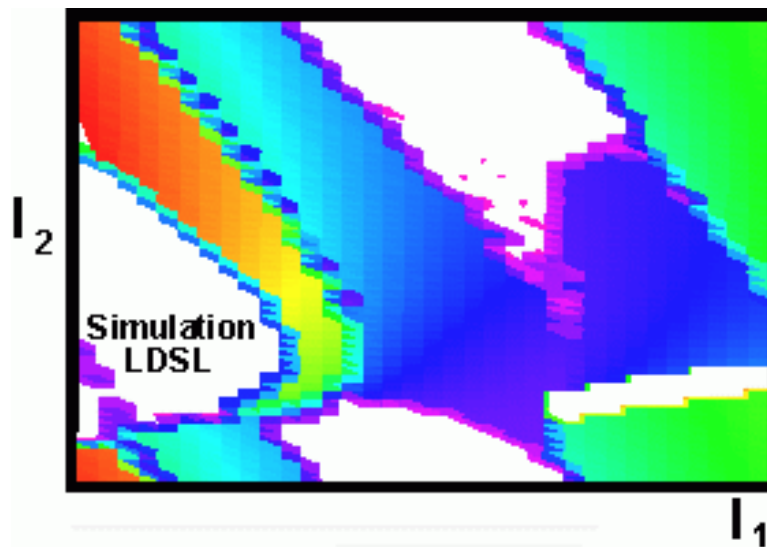
and apply some standard software for numerical continuation and bifurcation analysis of ODE's,

e.g., AUTO:

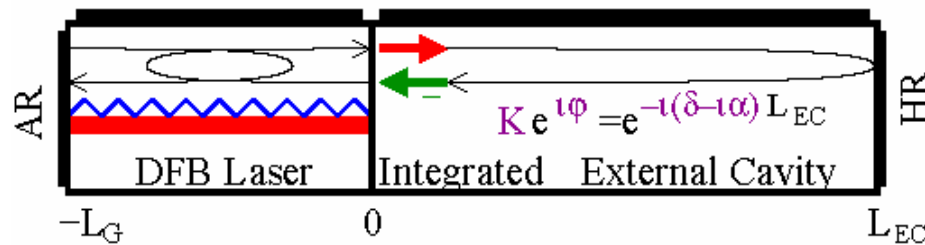
E. J. Doedel ... Int. J. Bif. and Chaos 1, 493 & 745 (91),
<http://cmvl.cs.concordia.ca/auto/>

example

Thick lines: bifurcations where stable orbits are losing stability; Thin lines: bifurcations of unstable orbits.



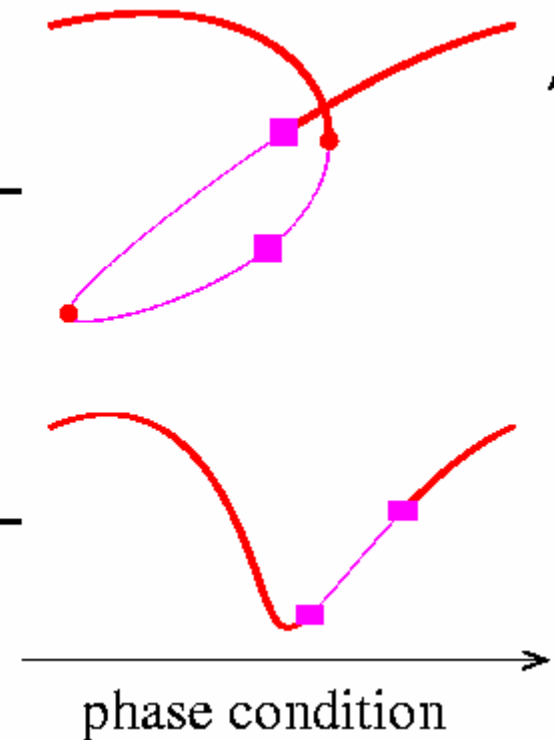
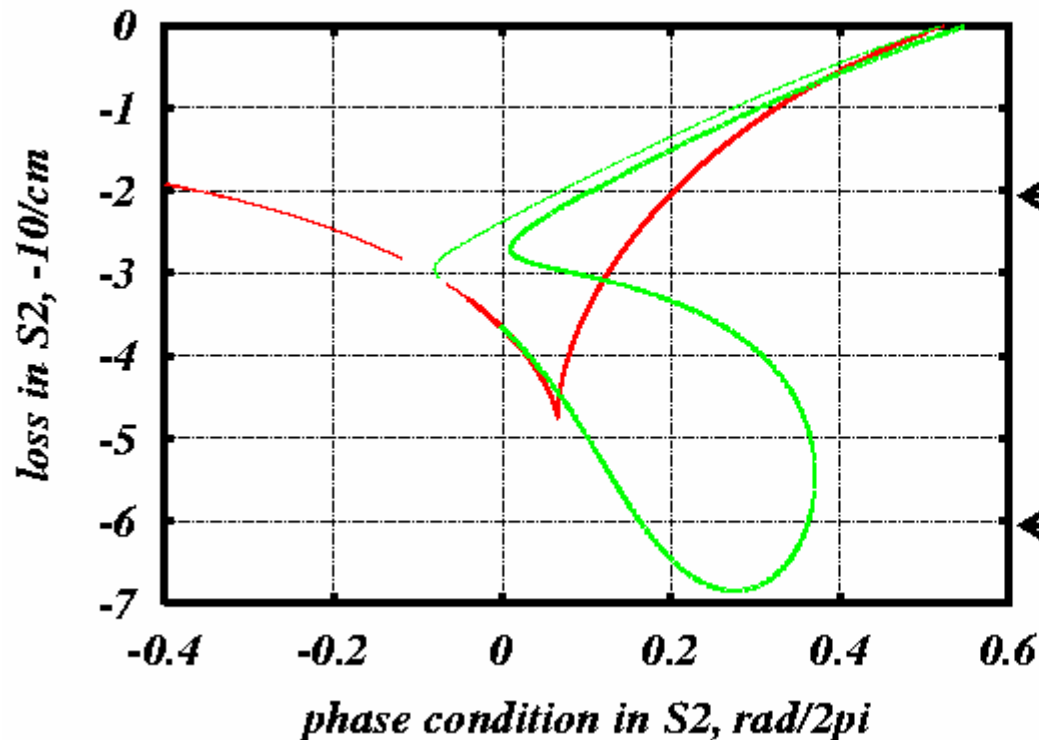
live demonstration: DFB laser with integrated external cavity



bifurcation parameters:

$$\varphi = -\delta L_{EC} \text{ (phase)}$$

$$\alpha = -\ln(K)/L_{EC} \text{ (loss)}$$



summary of the tutorial

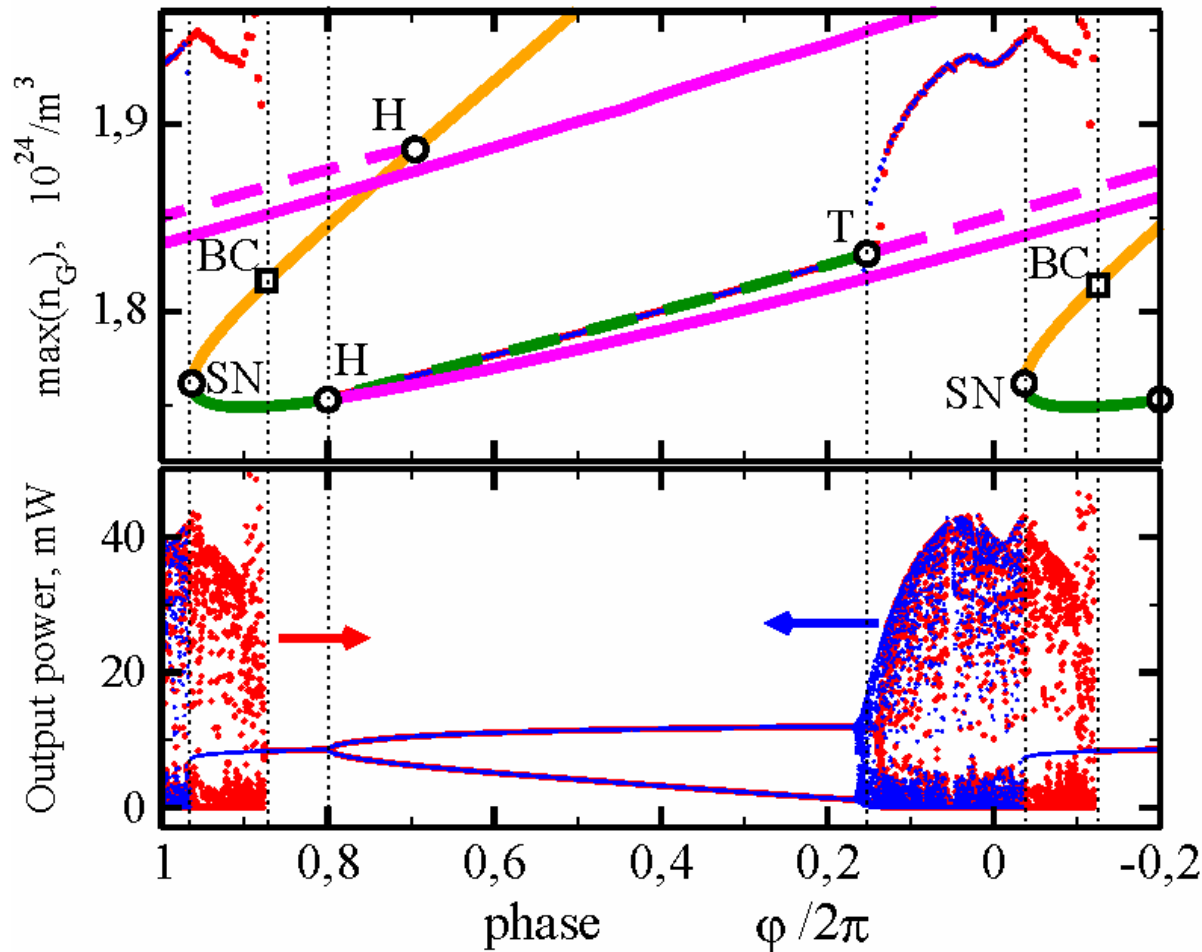
traveling-wave modeling

- bases on transverse-longitudinal separation
- resolves longitudinal coordinate z and time t
- is the method of choice for compound-cavity devices
- but is useful for simpler configurations, too.

more information on the used simulation tool LDSL:

<http://www.wias-berlin.de/software/ldsl/>

one-parameter bifurcation diagrams



1-parameter bifurcation diagram (above).
Green: stable orbits.
Violet and orange: main unstable branches.
Bullets: main related bifurcations.

Path-following performed with software tool AUTO