

Modeling of Bidirectional Regime in Semiconductor Ring Lasers

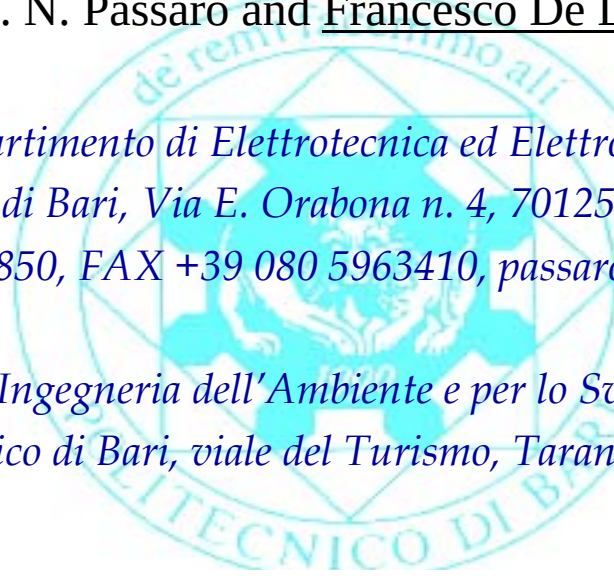
Vittorio M. N. Passaro and Francesco De Leonardis *

Dipartimento di Elettrotecnica ed Elettronica

Politecnico di Bari, Via E. Orabona n. 4, 70125 Bari - Italy

Tel. +39 080 5963850, FAX +39 080 5963410, passaro@deemail.poliba.it

** Dipartimento di Ingegneria dell'Ambiente e per lo Sviluppo Sostenibile,
Politecnico di Bari, viale del Turismo, Taranto - Italy*



PHOTONICS RESEARCH
GROUP

NUSOD 2005

19-22 September 2005, Berlin

OUTLINE

- INTEREST IN QUANTUM WELL RING LASERS
- COMPLETE MODEL OF RING LASER TIME EVOLUTION
- EACH EFFECT CONSIDERED (self-saturation, cross-saturation, self-interference, cross-interference, backscattering, pulling, pushing)
- NUMERICAL RESULTS
- CONCLUSIONS



DIPARTIMENTO DI
ELETTROTECNICA
ED ELETTRONICA

PHOTONICS RESEARCH
GROUP



THEORY: DENSITY MATRIX FORMALISM

- MODEL BASED ON A SEMI-CLASSICAL APPROACH
- LINEAR AND NONLINEAR TERMS OF POLARIZATION OBTAINED BY QUANTUM CALCULATIONS

$$P = \bar{n} \text{Tr}(\rho M) = \bar{n} \sum_{b,a} (\rho_{ba} M_{ab} + \rho_{ba} M_{ba})$$

$$\begin{aligned} \frac{\partial \rho}{\partial t} &= \left. \frac{\partial \rho}{\partial t} \right|_{stim} + \left. \frac{\partial \rho}{\partial t} \right|_{int} + \left. \frac{\partial \rho}{\partial t} \right|_{spon} + \left. \frac{\partial \rho}{\partial t} \right|_{pump} = \\ &= \frac{1}{j\hbar} [(H_0 - ME), \rho] - \frac{1}{2} [(\rho - \tilde{\rho}) \Gamma_{in} + \Gamma_{in} (\rho - \tilde{\rho})] + \\ &- \frac{1}{2} [(\rho - \tilde{\rho}) \Gamma_s + \Gamma_s (\rho - \tilde{\rho})] + \Lambda \end{aligned}$$

THEORY: DENSITY MATRIX FORMALISM

$$\rho = \begin{bmatrix} \rho_{aa} & \rho_{ab} \\ \rho_{ba} & \rho_{bb} \end{bmatrix} \text{ DENSITY MATRIX OPERATOR}$$

H_0 UNPERTURBED HAMILTONIAN OF THE MQW STRUCTURE

M DIPOLE MOMENT OPERATOR

Γ_{in} OPERATOR FOR INTRA-BAND RELAXATION EFFECT

Γ_s OPERATOR FOR SPONTANEOUS EMISSION

Λ OPERATOR FOR THE ELECTRON (HOLE) INJECTION

$\tilde{\rho}$ FERMI-DIRAC DISTRIBUTION IN QUASI-EQUILIBRIUM

$\tilde{\rho}$ FERMI-DIRAC DISTRIBUTION IN THERMAL EQUILIBRIUM



THEORY: DENSITY MATRIX FORMALISM

SOLVING THE DENSITY MATRIX RATE EQUATION USING A PERTURBATION APPROACH (*):

$$P_n^{(1)}(t) = \frac{j\bar{n} [\rho_{bb}^{(0)} - \rho_{aa}^{(0)}]}{N_m} |M|^2 E_n(t) \mathcal{C} \left(W_n - W_{ba}, \frac{1}{\tau_{in}} \right) \int_{V_I} |F(r)|^2 dr$$

$$P_n^{(3)}(t) = \frac{\varepsilon}{\omega_{ba}} \sum_k \sum_m \sum_q E_m E_k^* E_q e^{-j\Psi_{nqkm}} \Phi_{nqkm}$$

$$\Psi_{nqkm} = (\omega_n - \omega_q + \omega_k - \omega_m)t + (\phi_n - \phi_q + \phi_k - \phi_m)$$

$$\Phi_{nqkm} = \frac{\omega_{ba} |M|^4 \bar{n}}{j\varepsilon} [\rho_{bb}^{(0)} - \rho_{aa}^{(0)}] \tilde{N}_{nqkm} \mathcal{C}_{qkm}$$

$$\tilde{N}_{nqkm} = \frac{\int_{V_I} F_n(r)^* F_q F_k^* F_m dr}{N_n}$$

(*) F. De Leonardis and V.M.N. Passaro, "Accurate physical modelling of multi quantum well ring lasers," *Laser Physics Letters*, vol. 2, n. 2, pp. 59-70, 2005.

THEORY: RATE EQUATIONS

SUBSTITUTING THE POLARIZATION COMPONENTS IN THE MAXWELL EQUATIONS:

$$\frac{dI_1}{dt} = 2I_1(\alpha_1 - \tilde{\beta}_1 I_1 - \tilde{\theta}_{12} I_2) - 2\sqrt{I_1 I_2}(\xi_1 I_1 + \eta_{12} I_2) + b\sqrt{I_1 I_2} \cos(\psi_{tot} + \delta)$$

$$\frac{dI_2}{dt} = 2I_2(\alpha_2 - \tilde{\beta}_2 I_1 - \tilde{\theta}_{21} I_1) - 2\sqrt{I_1 I_2}(\xi_2 I_2 + \eta_{21} I_1) + b\sqrt{I_1 I_2} \cos(\psi_{tot} - \delta)$$

$$\begin{aligned} \frac{d\psi_{tot}}{dt} = & (\Omega_2 - \Omega_1) - (\omega_2 - \omega_1) - (\gamma_2 - \gamma_1) + (\nu_2 I_2 - \nu_1 I_1) \\ & + (\nu_{21} I_1 - \nu_{12} I_2) + \sqrt{I_1 I_2}(\chi_{21} - \chi_{12}) + \left(\sigma_{21} I_1 \sqrt{\frac{I_1}{I_2}} - \sigma_{12} I_2 \sqrt{\frac{I_2}{I_1}} \right) \\ & - \frac{b}{2} \left(\sqrt{\frac{I_2}{I_1}} \sin(\psi_{tot} + \delta) + \sqrt{\frac{I_1}{I_2}} \sin(\psi_{tot} - \delta) \right) \end{aligned}$$

I_1, I_2

INTENSITIES OF COUNTER-PROPAGATING MODES

ψ_{tot}

PHASE DIFFERENCE BETWEEN TWO MODES IN THE RING



DIPARTIMENTO DI
ELETTROTECNICA
ED ELETTRONICA

PHOTONICS RESEARCH
GROUP



THEORY: RATE EQUATION PARAMETERS

ALL PARAMETERS IN THE RATE EQUATIONS EVALUATED NUMERICALLY:

$$\alpha_n = \frac{1}{2} \frac{\omega_n}{\varepsilon} \sum_{b,a} \bar{n} \left[\rho_{bb}^{(0)} - \rho_{aa}^{(0)} \right] \frac{\left(\frac{\hbar}{\tau_{in}} \right)^2}{\left(W_n - W_{ba} \right)^2 + \left(\frac{\hbar}{\tau_{in}} \right)^2} \frac{\int_{V_f} |F(r)|^2 dr}{N_m} - \frac{\omega_n}{2Q_n} = g_n - \frac{\omega_n}{2Q_n} \quad \text{NET GAIN}$$

$$\tilde{\beta}_i = \frac{\omega_i}{2\varepsilon} |M|^4 \tilde{N}_{iii} \mathbf{I}_{iii} \quad \text{SELF-SATURATION COEFFICIENT}$$

$$\tilde{\theta}_{ij} = \frac{\omega_i}{2\varepsilon} |M|^4 \left[\tilde{N}_{ijj} \mathbf{I}_{ijj} + \tilde{N}_{jji} \mathbf{I}_{jji} + \tilde{N}_{ijj} \mathbf{I}_{ijj} \right] \quad \text{CROSS-SATURATION COEFFICIENT}$$

$$\xi_i = \frac{\omega_i}{2\varepsilon} |M|^4 \left[\tilde{N}_{iij} \mathbf{I}_{iij} + \tilde{N}_{iji} \mathbf{I}_{iji} + \tilde{N}_{ijj} \mathbf{I}_{ijj} \right] \quad \text{SELF-INTERFERENCE COEFFICIENT}$$

$$\eta_{ij} = \frac{\omega_i}{2\varepsilon} |M|^4 \tilde{N}_{ijj} \mathbf{I}_{ijj} \quad \text{CROSS-INTERFERENCE COEFFICIENT}$$

THEORY: RATE EQUATION PARAMETERS

$$I_{nqkm} = \int_{W_g}^{\infty} g_{cv}(W_{ba}) [f_c(W_{ba}) - f_v(W_{ba})] \text{Im} \left\{ j e^{-j\Psi_{nqkm}} C_{qkm} \right\} dW_{ba}$$

$$\gamma_i = \frac{\omega_i}{2\varepsilon} \frac{\int_{V_l} |F(r)|^2 dr}{N_m} \int_{W_g}^{\infty} |M|^2 g_{cv}(W_{ba}) [f_c(W_{ba}) - f_v(W_{ba})] \frac{(W_i - W_{ba})}{(W_i - W_{ba})^2 + \left(\frac{\hbar}{\tau_{in}}\right)^2} dW_{ba} \quad \text{PULLING EFFECT}$$

$$V_i = \frac{\omega_i}{2\varepsilon} \tilde{N}_{iii} R_{iii} \quad \text{SELF-PUSHING EFFECT}$$

$$V_{ij} = \frac{\omega_i}{2\varepsilon} \left[\tilde{N}_{ijj} R_{ijj} + \tilde{N}_{iji} R_{iji} + \tilde{N}_{ijj} R_{ijj} \right] \quad \text{CROSS-PUSHING EFFECT}$$

$$\chi_{ij} = \frac{\omega_i}{2\varepsilon} \left[\tilde{N}_{ijj} R_{ijj} + \tilde{N}_{iji} R_{iji} + \tilde{N}_{ijj} R_{ijj} \right] \quad \text{NONLINEAR EFFECT}$$

$$\sigma_{ij} = \frac{\omega_i}{2\varepsilon} \tilde{N}_{ijj} R_{ijj} \quad \text{NONLINEAR EFFECT}$$

$$R_{nqkm} = \int_{W_g}^{\infty} |M|^4 g_{cv}(W_{ba}) [f_c(W_{ba}) - f_v(W_{ba})] \text{Re} \left\{ j e^{-j\Psi_{nqkm}} C_{qkm} \right\} dW_{ba}$$

THEORY: RATE EQUATIONS

TO COMPLETE THE MODEL THE CARRIER DYNAMICS IS TAKEN INTO ACCOUNT:

$$\begin{aligned} \frac{dN}{dt} = & \frac{J}{ed} - AN - BN^2 - CN^3 - \frac{2\varepsilon_0 n_{eff}^2}{h\omega_1} \left[I_1 \left(g_1 - \tilde{\beta}_1 I_1 - \tilde{\theta}_{12} I_2 \right) - \sqrt{I_1 I_2} \left(\xi_1 I_1 + \eta_{12} I_2 \right) \right] \\ & - \frac{2\varepsilon_0 n_{eff}^2}{h\omega_2} \left[I_2 \left(g_2 - \tilde{\beta}_2 I_2 - \tilde{\theta}_{21} I_1 \right) - \sqrt{I_1 I_2} \left(\xi_2 I_2 + \eta_{21} I_1 \right) \right] \end{aligned}$$

- A LEAKAGE RECOMBINATION COEFFICIENT
- B BIMOLECULAR RECOMBINATION COEFFICIENT
- C AUGER COEFFICIENT

THEORY: BIDIRECTIONAL REGIME

➤ STABILITY CRITERION FOR BIDIRECTIONAL REGIME OF SUCH A GAIN-COUPLED TWO-MODE SYSTEM EQUIVALENT TO HAVE BOTH NEGATIVE **EIGENVALUES OF RELEVANT JACOBIAN**:

$$\frac{d}{dt} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} = \mathbf{T} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix}$$

ε_1 *TWO ARBITRARY PERTURBATIONS OF THE*
 ε_2 *STATIONARY SOLUTION*

$\mathbf{T}$ *MATRIX DEPENDENT ON LASER COEFFICIENTS*

THEORY: BIDIRECTIONAL REGIME

SIGN OF BOTH EINGEVALUES DEPENDING ON A **MODIFIED COUPLING CONSTANT C** (INCLUDING INTERFERENCE TERMS AND BACKSCATTERING RATE COEFFICIENT):

$$C = \frac{\left[\tilde{\theta}_{12} + \sqrt{I_2^s/I_1^s} \eta_{12} - (b/2) \cos(\psi_{tot} + \delta) \left(1/\sqrt{I_1^s I_2^s} \right) \right]}{\left[\tilde{\beta}_1 + \sqrt{I_2^s/I_1^s} \xi_1 \right]} \times \frac{\left[\tilde{\theta}_{21} + \sqrt{I_1^s/I_2^s} \eta_{21} - (b/2) \cos(\psi_{tot} - \delta) \left(1/\sqrt{I_1^s I_2^s} \right) \right]}{\left[\tilde{\beta}_2 + \sqrt{I_1^s/I_2^s} \xi_2 \right]}$$

STABILITY CONDITION FOR BIDIRECTIONALITY: $C < 1$

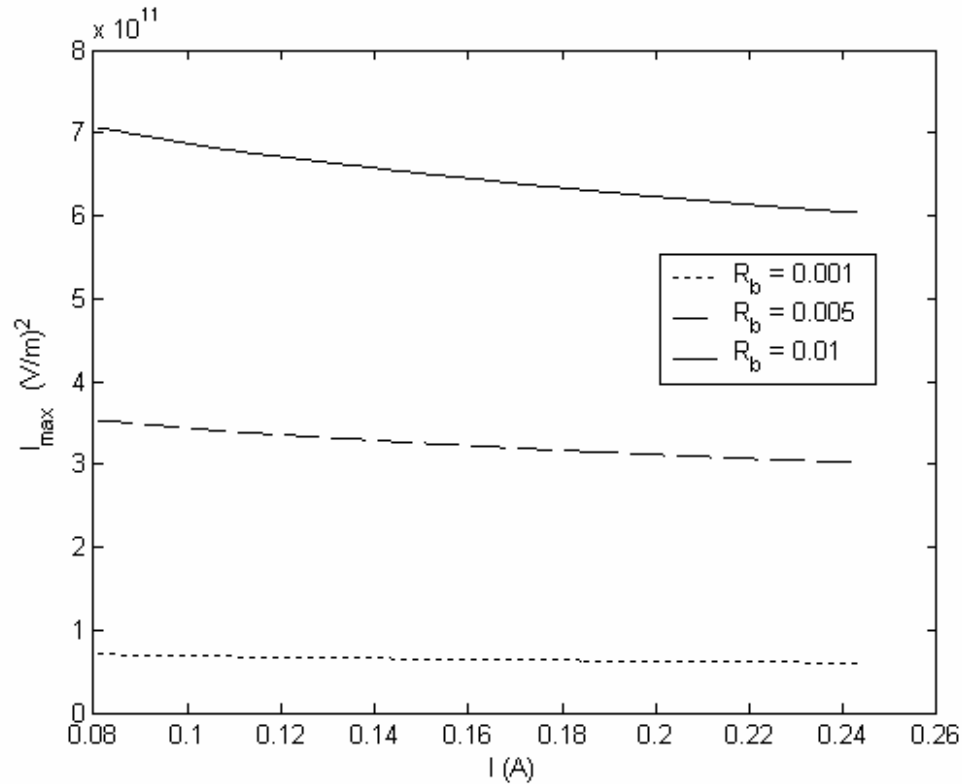
THEORY: BIDIRECTIONAL REGIME

CONDITION $C < 1$ IMPLIES CONSTRAINTS ON THE WAVE INTENSITIES:

$$I_{\min} = \frac{b \cos(\delta)}{2 \left[\left(\tilde{\theta}_{ij} + \eta_{ij} \right) + \left(\tilde{\beta}_i + \xi_i \right) \right]} \leq I^s \leq \frac{b \cos(\delta)}{2 \left[\left(\tilde{\theta}_{ij} + \eta_{ij} \right) - \left(\tilde{\beta}_i + \xi_i \right) \right]} = I_{\max}$$

INTENSITY RANGE OF TWO BEAMS WHERE **THE BIDIRECTIONAL BEHAVIOUR IS STABLE**

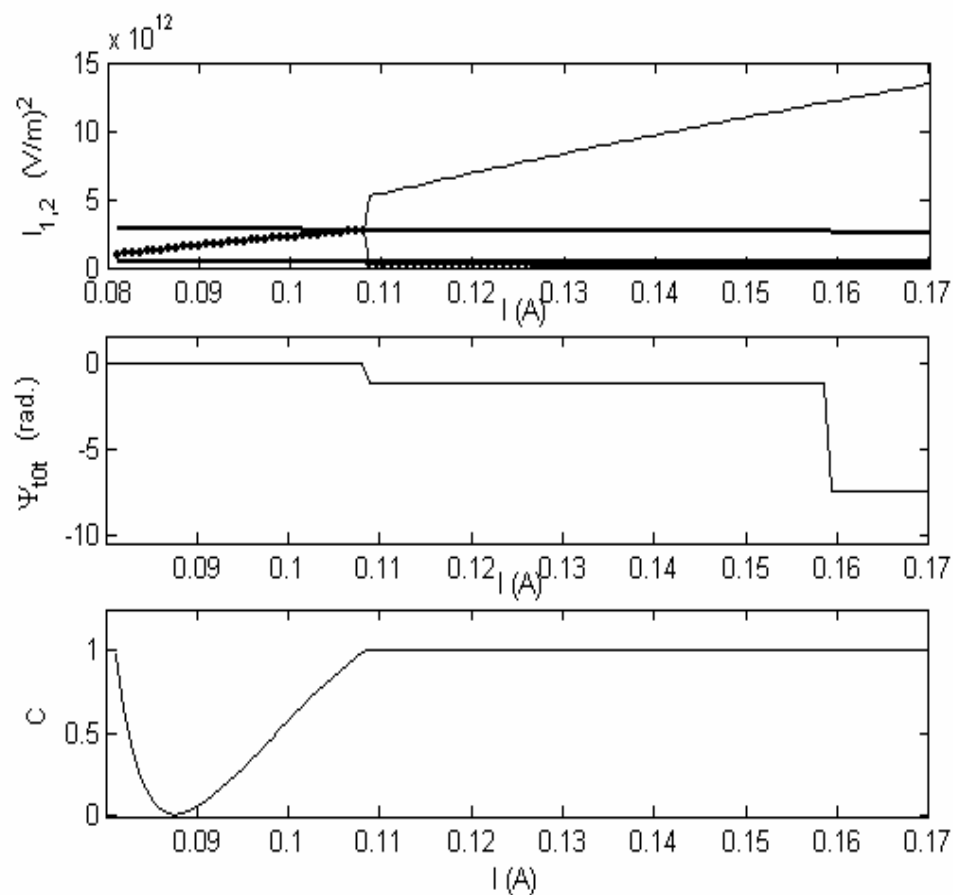
NUMERICAL RESULTS



R_b

*BACKSCATTERING
AMPLITUDE
REFLECTIVITY*

NUMERICAL RESULTS



CONCLUSIONS

- COMPLETE PHYSICAL MODEL OF MQW RING LASER
- BOTH AMPLITUDE AND PHASE
- QUANTUM MECHANICS EVALUATION OF NON LINEAR LASER COEFFICIENTS
- STUDY OF THE BACKSCATTERING INFLUENCE
- EVALUATION OF THE CRITERION TO ACHIEVE A STABLE BIDIRECTIONAL REGIME



DIPARTIMENTO DI
ELETTROTECNICA
ED ELETTRONICA

PHOTONICS RESEARCH
GROUP

