



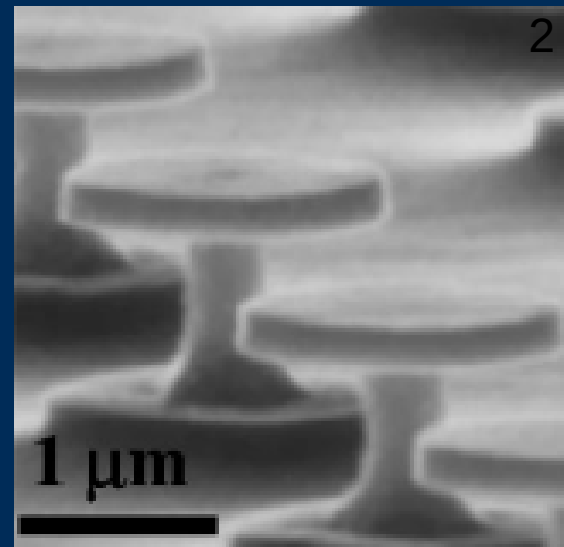
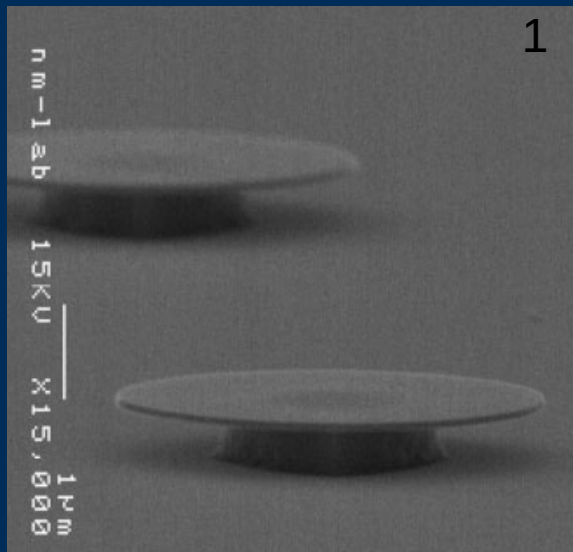
Cold-Cavity Lasing Spectra and Thresholds of a Twin Disk Photonic Molecule with Optically Coupled Whispering Gallery Modes

*Elena I. Smotrova , Alexander I. Nosich
Trevor M. Benson , and Phillip Sewell
Speaker: Mikhail Klimenko*

Outline

- Modelling of microcavity lasers
- Lasing eigenvalue problem
- Model of photonic molecule
- Numerical results
- Conclusions

Optically coupled microdisks as simple photonic molecules



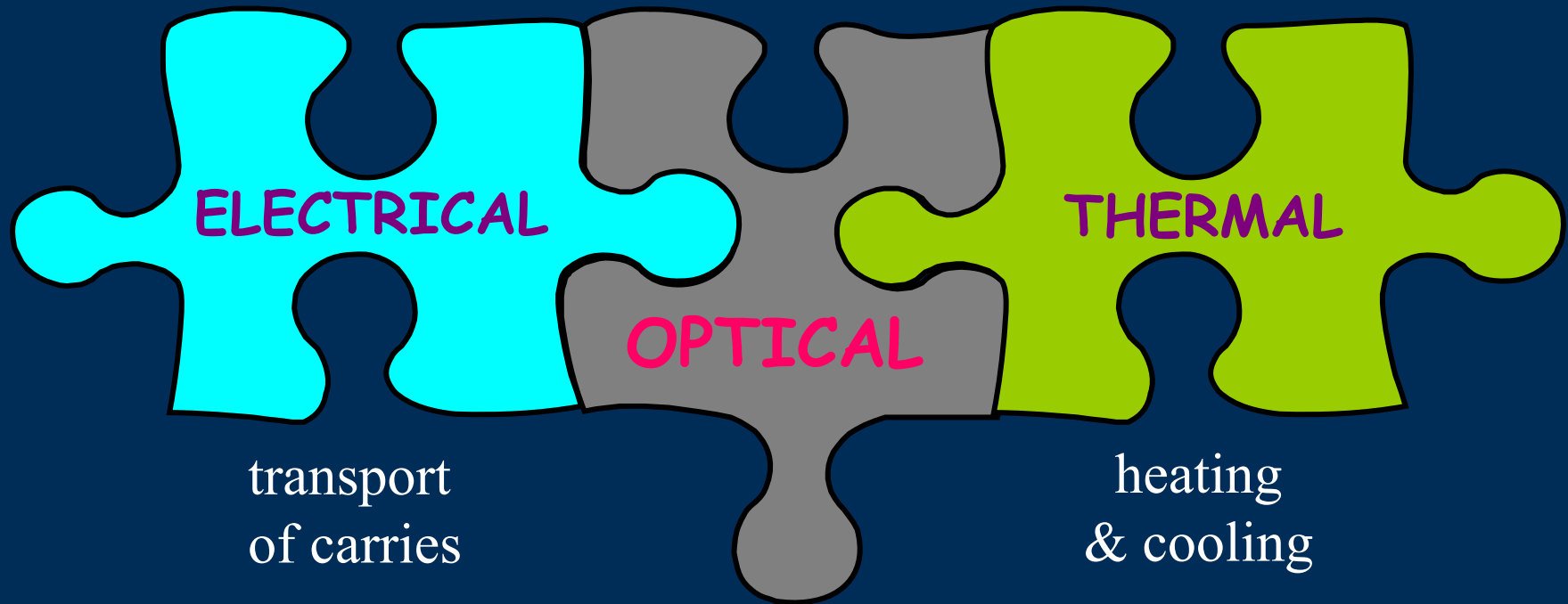
¹V. Zwiller et al., “Quantum optics with single quantum dot devices”, *New J. of Physics*, Vol. 6, No. 96, 2004

²K. Petter et al., “Fabrication of large periodic arrays of AlGaAs microdisks by laser-interference lithography and selective etching”, *Appl. Phys. Letts.*, Vol. 81, No. 4, 2002

Laser: electro-thermo-optical device

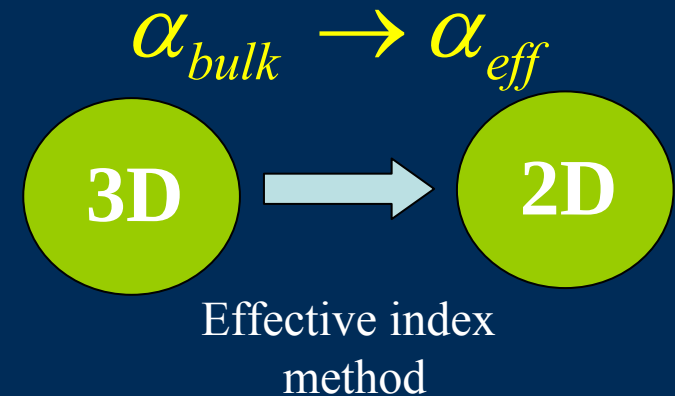
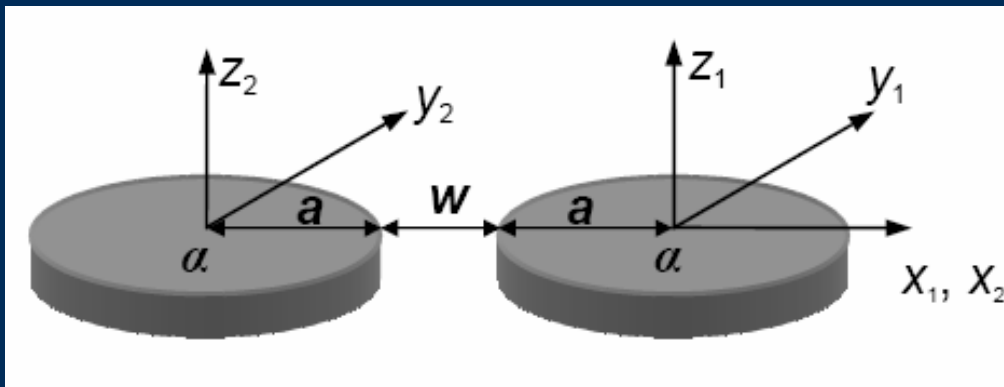
“Cold-cavity model” without / with gain

EM field confinement



Necessary Parts of Comprehensive Simulation ⁴

Reduction to 2D problem in disk plain



For example, if disks are made of GaAs and disk thickness is 0.1 of radius,

$$\alpha = 3.374, \lambda = 1.55 \mu m$$

E_z polarization $\alpha_{eff}^E = 1.31$

H_z polarization $\alpha_{eff}^H = 2.63$



in thin disks, only H_z pol. modes can be studied



2-D lasing eigenvalue problem ~ cold cavity with gain ~

$$\sim e^{-i\omega t}$$



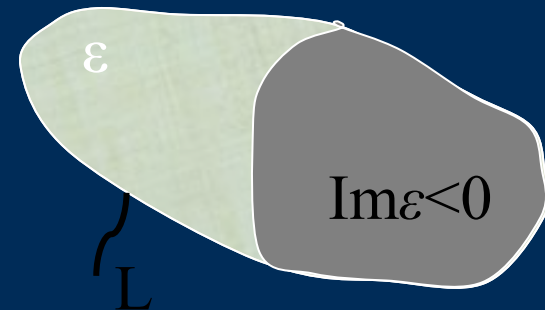
Helmholtz equation off the boundary

$$(\Delta + k^2 \nu^2) U(r, \varphi) = 0$$

$$\sqrt{\varepsilon} = \nu = \alpha_{\text{eff}} - i\gamma$$

α_{eff} is effective refractive index

γ is threshold material gain



Transparent boundary conditions



Sommerfeld radiation condition at infinity

The real-valued (κ_j, γ_j) are the eigenparameters

Single microdisk: fundamental features

All modes in a single microdisk are double degenerate

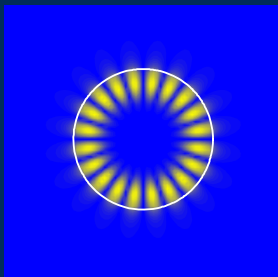
periodicity of lasing frequencies:

WGM &
non-WGM

$$\kappa_{mn}^{E,H} \sim \frac{\pi}{2\alpha} \left(m + 2n \pm \frac{1}{2} \right)$$

super-low thresholds:

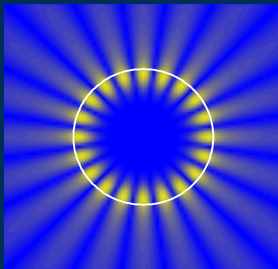
WGM



$$\gamma_{mn}^{E,H} \sim C_{E,H} e^{-2m \ln(2m / \kappa_{mn}^{E,H})}$$

high thresholds:

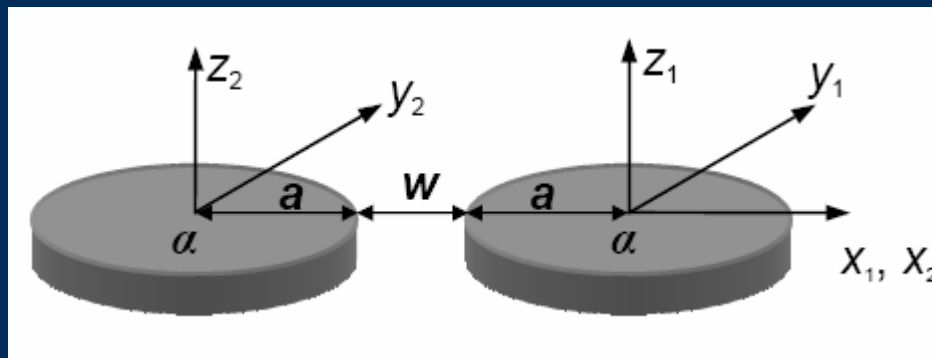
Non-WGM



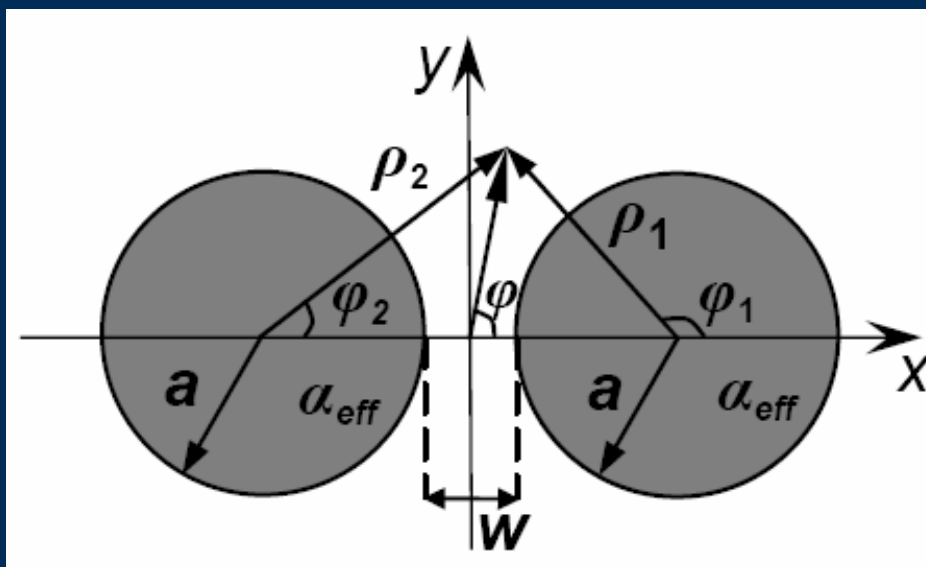
$$\gamma_{mn}^{E,H} \sim \pi \ln[(\alpha + 1) / (\alpha - 1)] / 2\kappa_{mn}^{E,H}$$

Geometry of two-disk photonic molecule

3D

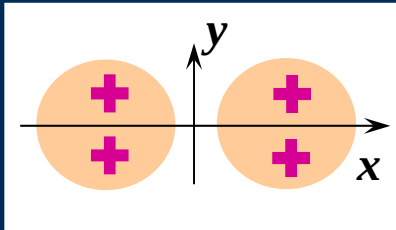


2D



Four symmetry classes

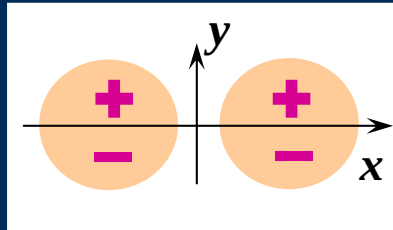
even-even (EE)



$$A_p = B_{-p}; A_p = (-1)^p A_{-p}$$

$$C_p = D_{-p}; C_p = (-1)^p C_{-p}$$

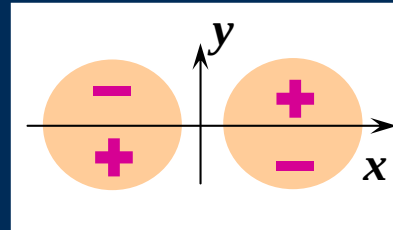
odd-even (OE)



$$A_p = B_{-p}; A_p = (-1)^{p+1} A_{-p}$$

$$C_p = D_{-p}; C_p = (-1)^{p+1} C_{-p}$$

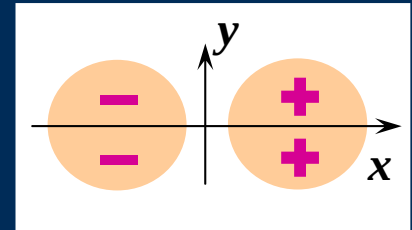
odd-odd (OO)



$$A_p = -B_{-p}; A_p = (-1)^p A_{-p}$$

$$C_p = -D_{-p}; C_p = (-1)^p C_{-p}$$

even-odd (EO)



$$A_p = -B_{-p}; A_p = (-1)^{p+1} A_{-p}$$

$$C_p = -D_{-p}; C_p = (-1)^{p+1} C_{-p}$$

Series expansions of the field function in local coordinates

$$U(\rho, \varphi) = \begin{cases} \sum_{p=-\infty}^{\infty} \{A_p, B_p\} J_p(k\sqrt{\varepsilon}\rho_{1,2}) e^{ip\varphi_{1,2}}, \rho_{1,2} < a \\ \sum_{p=-\infty}^{\infty} C_p H_p^{(1)}(k\rho_1) e^{ip\varphi_1} + \sum_{p=-\infty}^{\infty} D_p H_p^{(1)}(k\rho_2) e^{ip\varphi_2}, (\rho_1 > a) \wedge (\rho_2 > a) \end{cases}$$

Coupling leads to quartets of similar non-degenerate modes

Infinite matrix equations

boundary conditions



Sommerfeld radiation condition at infinity



Fredholm second kind equation

$$(I + A^{\pm\pm}(\kappa, \gamma))X = 0$$

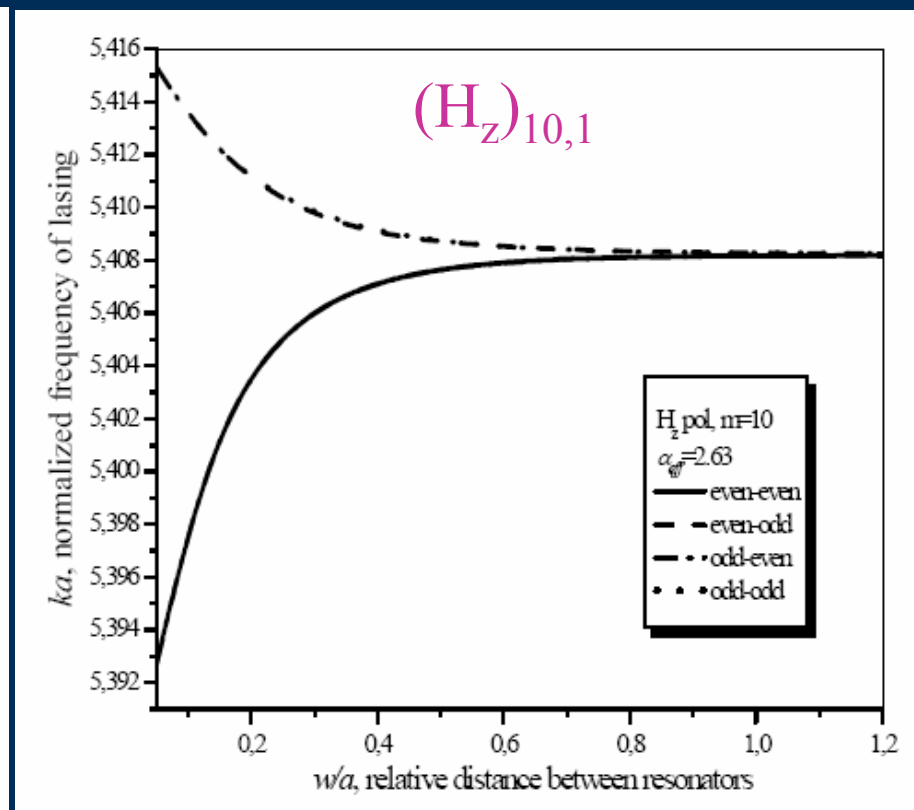
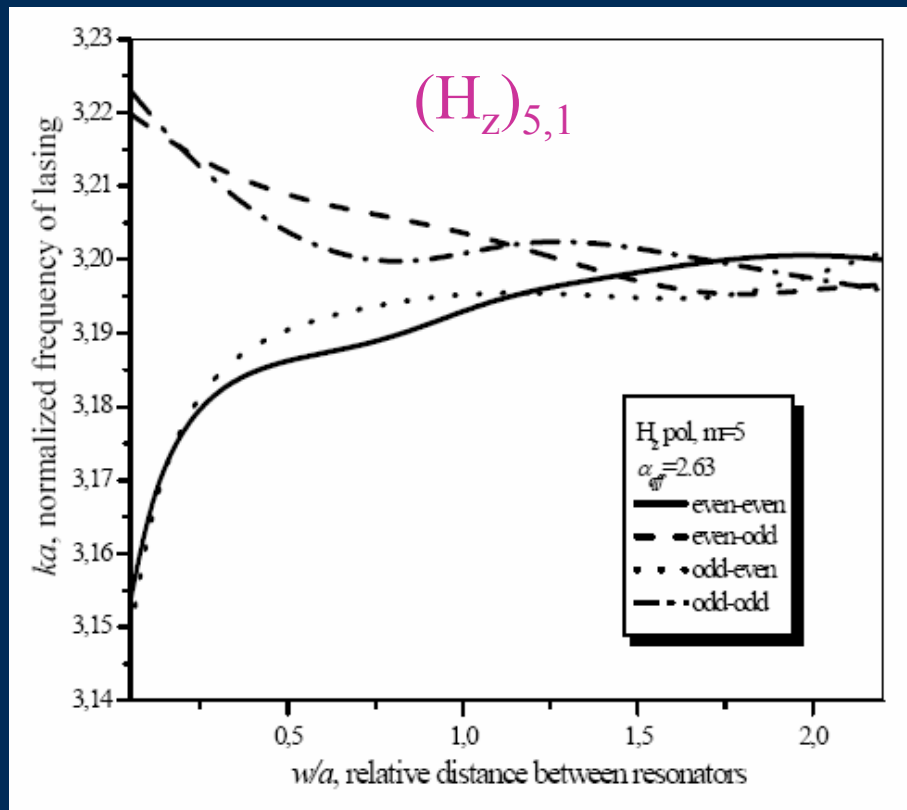
$$A_{mp}^{(\pm, \pm)} = \pm \mu_p \frac{J_m(\kappa) V_p(\kappa, \nu)}{F_p(\kappa, \nu) J_p(\kappa)} \{H_{m+p}^{(1)}(\kappa(w/a+2)) \pm (-1)^p H_{m-p}^{(1)}(\kappa(w/a+2))\}$$

$$x_n = A_n / (F_n(\kappa \nu) J_n(\kappa)), \quad \kappa = ka; \quad \mu_p = 1/2, \text{ if } p = 0 \text{ and } \mu_p = 1, \text{ if } p \neq 0$$

$$F_m^{H,E}(\kappa) = J_m(\kappa \nu) H_m'^{(1)}(\kappa) - \beta^{H,E} J_m'(\kappa \nu) H_m^{(1)}(\kappa), \quad V_m^{H,E}(\kappa, \nu) = J_m(\kappa \nu) J_m'(\kappa) - \beta^{H,E} J_m'(\kappa \nu) J_m(\kappa)$$

$$\text{Det}(I + A^{\pm\pm}(\kappa, \gamma)) = 0$$

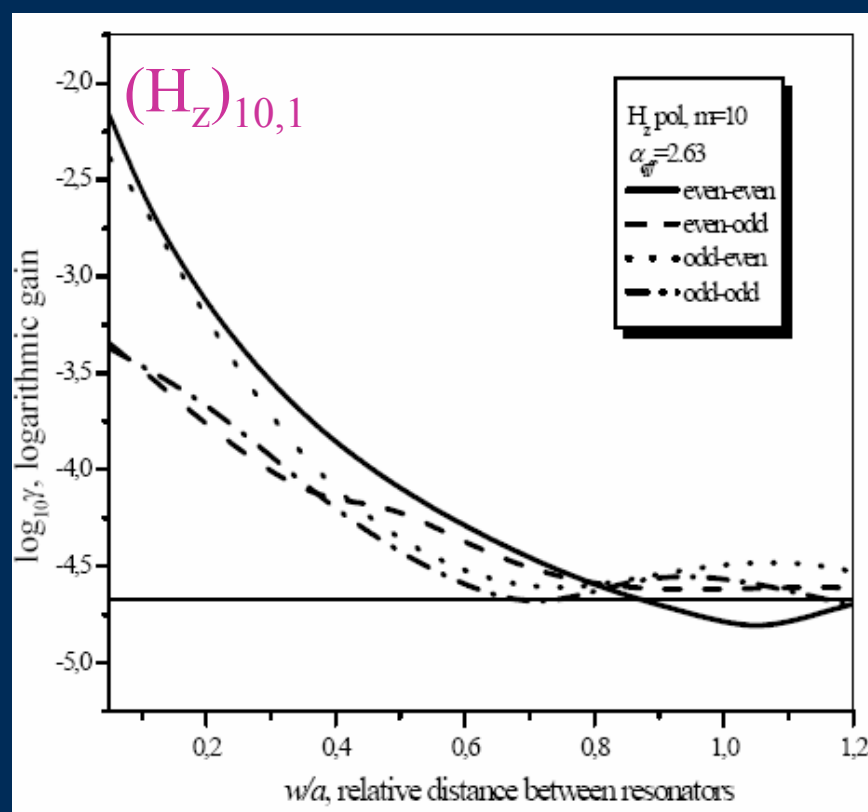
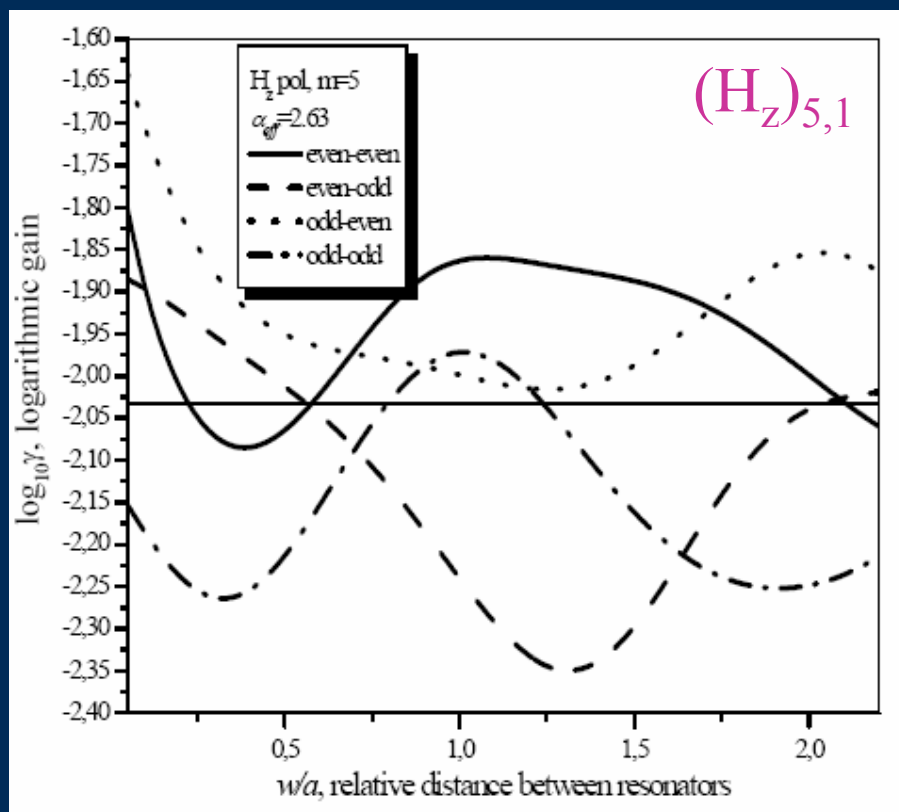
Dependences of frequencies on the distance between resonators, H_z polarization



Mode quartets in photonic molecule form two closely spaced pairs



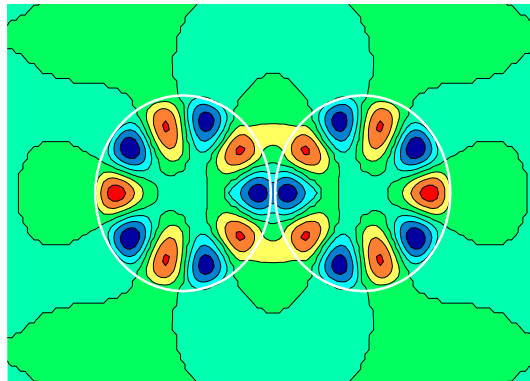
Dependences of thresholds on the distance between resonators, H_z polarization



Thresholds in PM can be slightly lower than in a single microdisk 12

Near-field patterns, $(H_z)_{5,1}$

EE

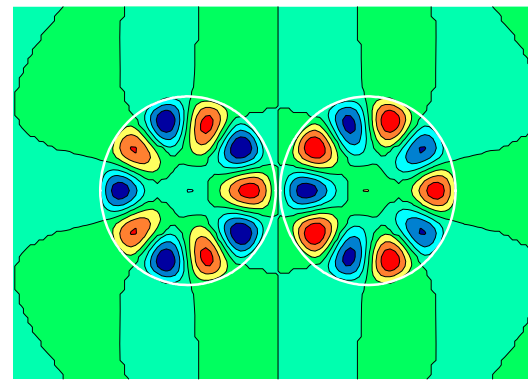


$$ka=3.15$$

$$\gamma=1.58*10^{-2}$$

$$w/a=0.05$$

EO

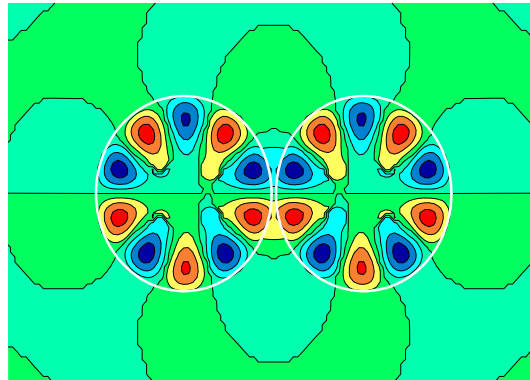


$$ka=3.22$$

$$\gamma=1.3*10^{-2}$$

$$w/a=0.05$$

OE

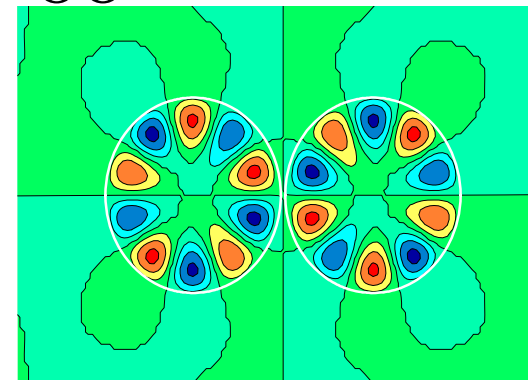


$$ka=3.15$$

$$\gamma=2.29*10^{-2}$$

$$w/a=0.05$$

OO



$$ka=3.22$$

$$\gamma=7.03*10^{-3}$$

$$w/a=0.05$$

Conclusions

- ✎ Each mode in a PM of N microdisks splits into $2N$ new coupled ones
- ✎ Modes obtain frequency shifts - half are redshifted and other half are blueshifted
- ✎ It is possible to achieve a threshold that is slightly lower than for an isolated disk

Thank you for
your attention