

# **A comparative study of higher order Bragg waveguide gratings using coupled mode theory and mode expansion modeling**

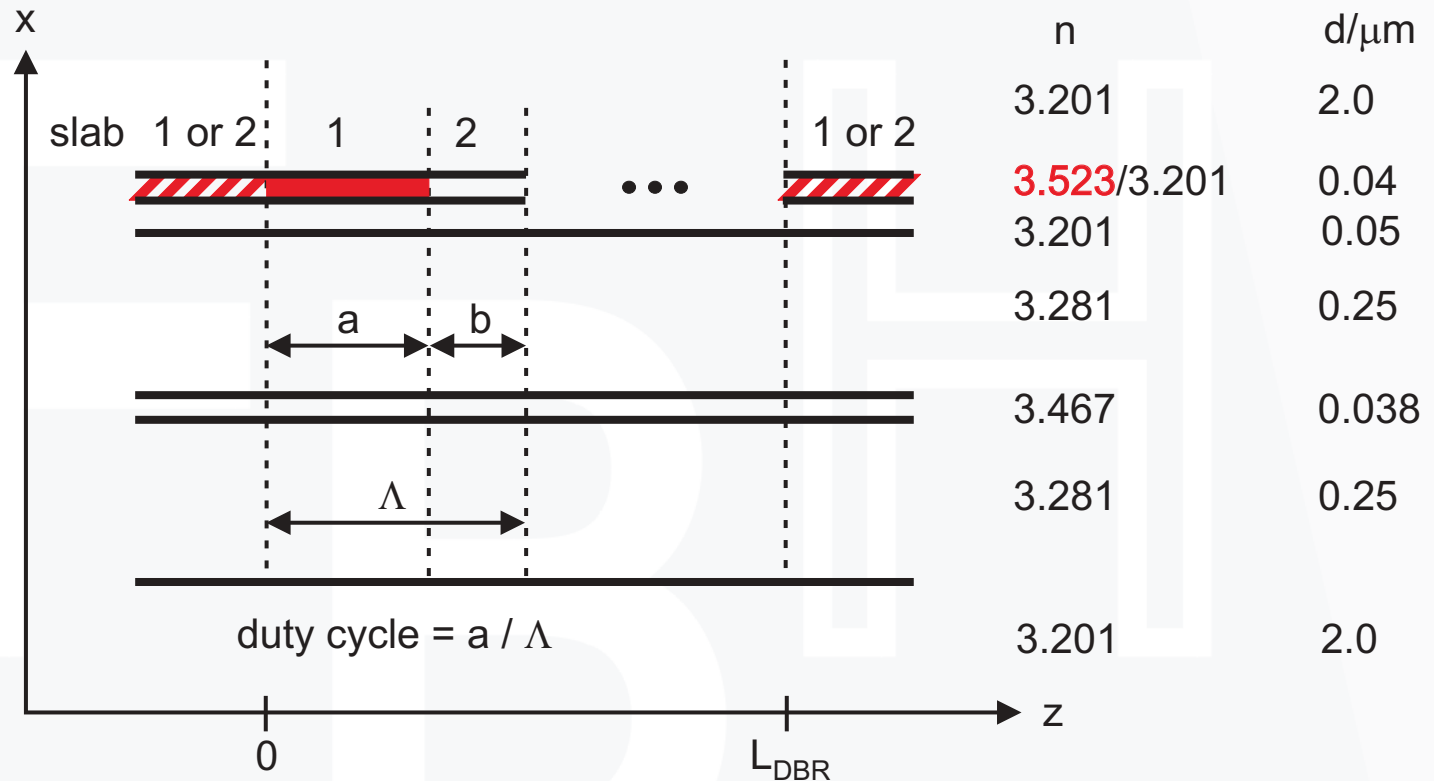
**H. Wenzel and R. Güther**

## Motivation

- Widespread application of Bragg waveguide gratings in DFB and DBR lasers
- Higher-order ( $N = 2, 3$ ) gratings needed for
  - wavelengths  $< 1 \mu\text{m}$  due to larger periods
  - surface emitting devices
- Simulation:
  - Kogelnik's and Streifer's coupled-mode theory (CMT)  $\longrightarrow$  fast, but not exact
  - direct solution of Maxwell's equations (FDTD, FDFD, MoL, bi-directional mode expansion)  $\longrightarrow$  exact, but not fast
- Question: How reliable is CMT for higher order gratings?

## Benchmark example

stack of two alternating slab waveguides (slab 1 and slab 2)



## Task

Given:

reference (Bragg) wavelength  $\lambda_0 = 980 \text{ nm}$ , length of grating  $L_{\text{DBR}} = 200 \text{ } \mu\text{m}$   
Bragg order  $N$ , duty cycle  $D$ ,

$$\text{reference waveguide : } n_{\text{ref}}^2(x) = Dn_1^2(x) + (1 - D)n_2^2(x) \implies n_{\text{eff}}$$

$$\text{grating period : } \Lambda = \frac{N\lambda_0}{2n_{\text{eff}}}$$

$$\text{lengths of slabs : } a = \Lambda D \quad \text{and} \quad b = \Lambda(1 - D)$$

Wanted:

reflectivity  $R$  and loss  $L = 1 - R - T$  of fundamental guided TE mode

## Coupled-Mode Theory for higher order gratings

Ansatz:  $E(x, z) = F(x) \left[ \psi^+(z) e^{-iN\pi/\Lambda z} + \psi^-(z) e^{+iN\pi/\Lambda z} \right] + \sum_{n=1}^{N-1} e_n(x, z) e^{-ik_{zn}z}$

with phase-matching, slowly varying amplitude + other approximations  $\Rightarrow$

coupled wave equations  $\pm \frac{\partial \psi^\pm}{\partial z} = -i\Delta\beta \psi^\pm - i\kappa \psi^\mp$

where  $\Delta\beta = \frac{2\pi n_{\text{eff}}}{\lambda} + i\frac{g - \alpha}{2} - \frac{N\pi}{\Lambda} + i\kappa_s^r$  and  $\kappa = \kappa_N + i\kappa_m^r$

+ inhomogeneous ODEs for the partial waves  $e_n$

$$\frac{\partial^2 e_n}{\partial x^2} + [n_{\text{ref}}^2 k_0^2 - k_{zn}^2] e_n(x) = -k_0^2 F(x) [\xi_n(x) \psi^+(z) + \xi_{N-n}(x) \psi^-(z)]$$

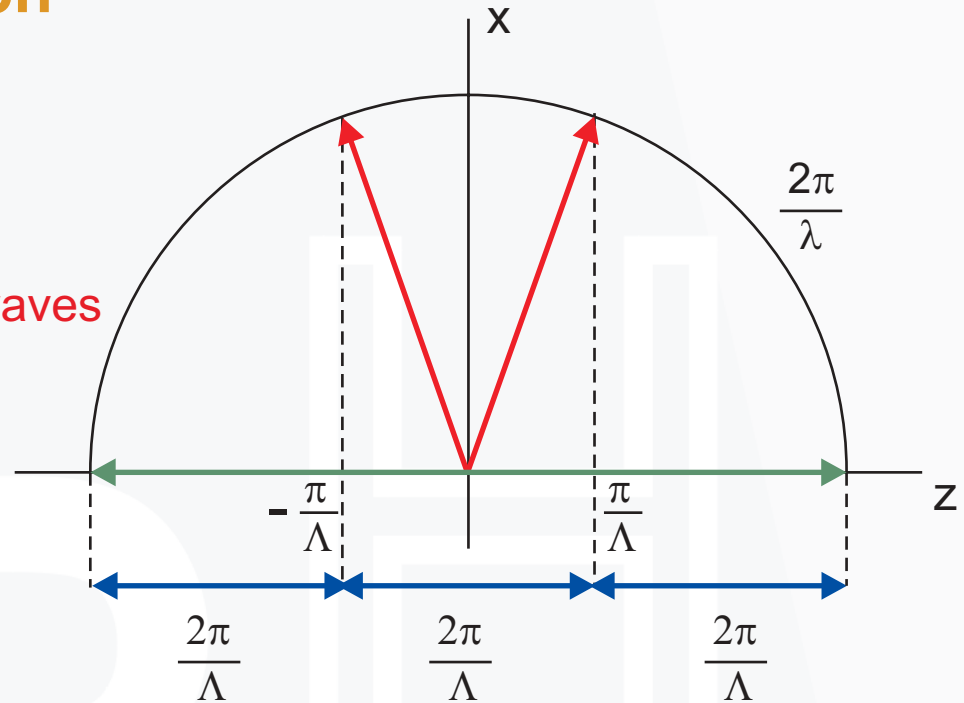
$\Rightarrow \kappa_s^r$  and  $\kappa_m^r$

$z$ -component of wavevector  $k_{zn}$  of radiating waves  $\longrightarrow$

# Ewald's construction

third order grating

radiating waves



radius of circle  $\equiv$  Bragg condition :  $\frac{2\pi}{\lambda} = \frac{N}{2} \frac{2\pi}{\Lambda}$

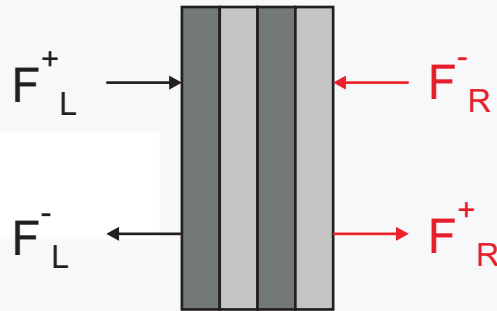
$$\Rightarrow k_{zn} = \frac{2\pi}{\lambda} - n \frac{2\pi}{\Lambda} = \frac{\pi}{\Lambda} (N - 2n), \quad n = 1, \dots, N-1$$

## Bi-directional mode expansion

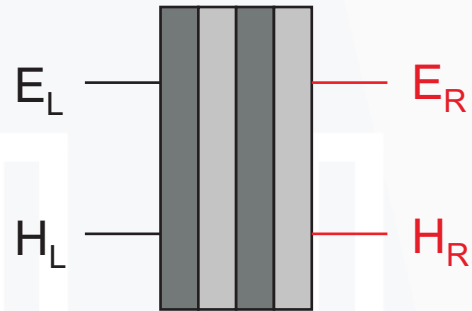
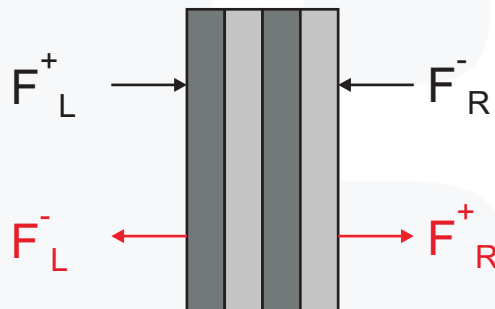
- Expansion of field in each slab onto the eigenmodes in that slab
- Simulation of radiation effects by perfectly matched layers
- Transitions conditions at the interfaces between the slabs  $\implies$  reflection and transmission matrices
- Propagation from one interface of the slab to the other interface  $\implies$  diagonal propagation matrices
- Calculation of reflection and transmission matrices of entire stack by recursive T-, R- or S-matrix algorithms

# Algorithms

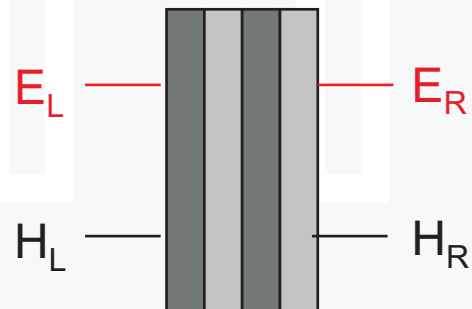
T-matrix algorithms: unstable



S-matrix algorithm:  
unconditionally stable



R-matrix algorithm:  
conditionally stable



## Tools

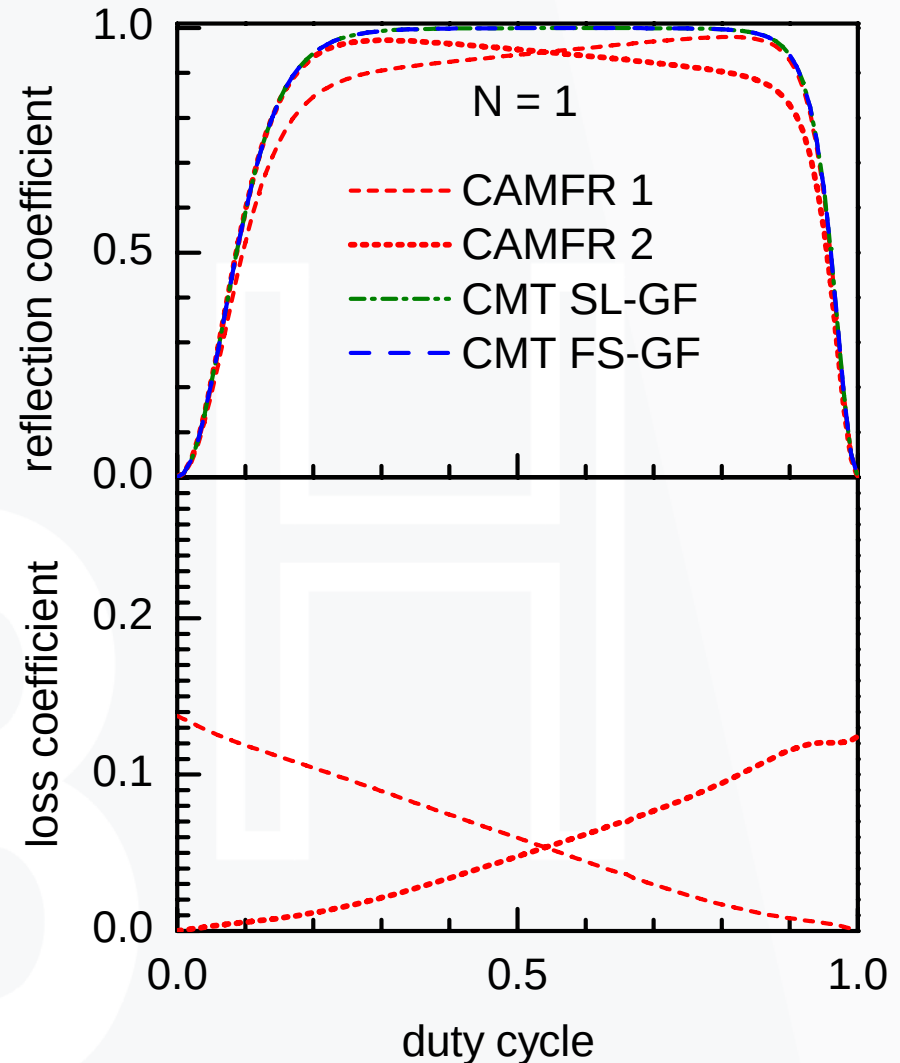
- CMT: two different methods for solving the inhomogeneous ODEs for partial waves
  - using exact slab Green's function\*
  - using approximate free space Greens's function
- Mode expansion: CAMFR\*\* with stack enclosed by
  - slab 1
  - slab 2

\* Ali Shams, Photonami Inc.

\*\* <http://camfr.sourceforge.net> (P. Bienstman, Ghent University)

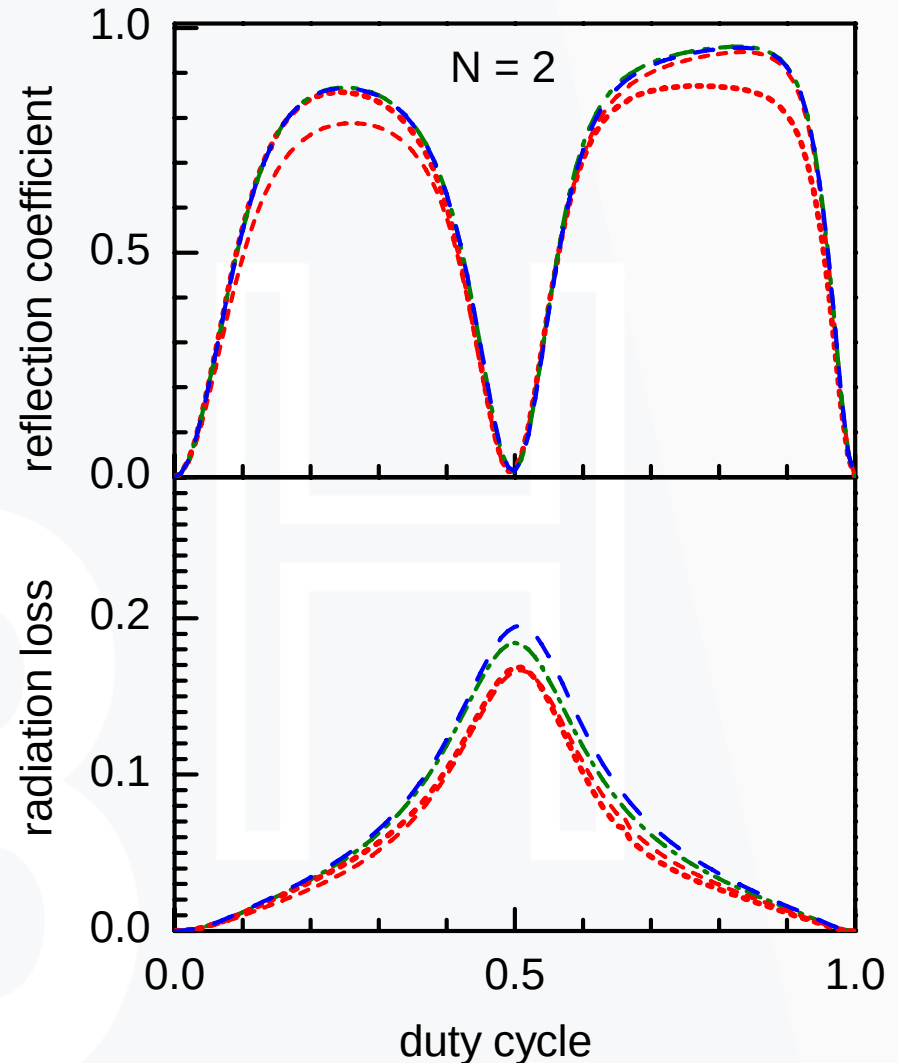
## Maximum reflectivity and loss for $N=1$

- No radiation loss but insertion loss, only obtained with CAMFR
- Maximum insertion loss for  $D=0$  or  $D=1$  depending on surrounding slabs (1 or 2, respectively)



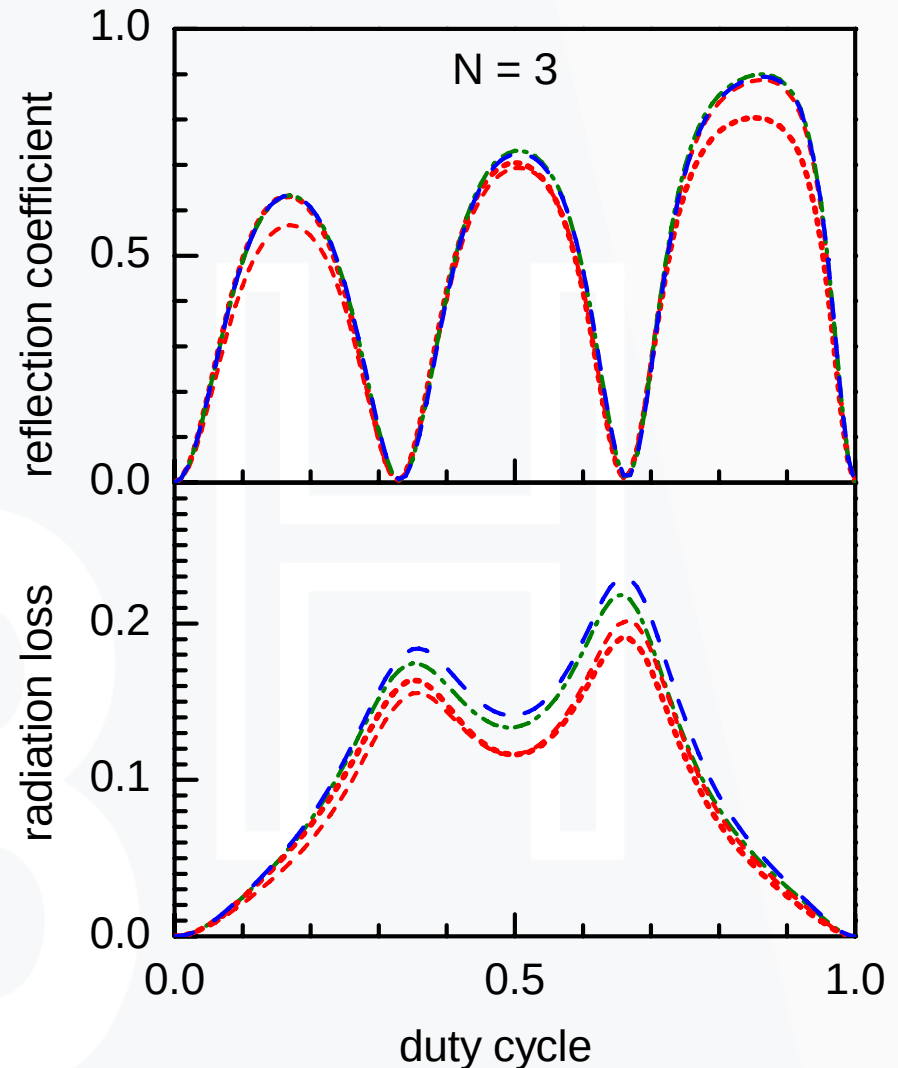
## Maximum reflectivity and loss for $N=2$

- Overall good agreement between the models
- Insertion loss is subtracted from loss coefficients computed by CAMFR



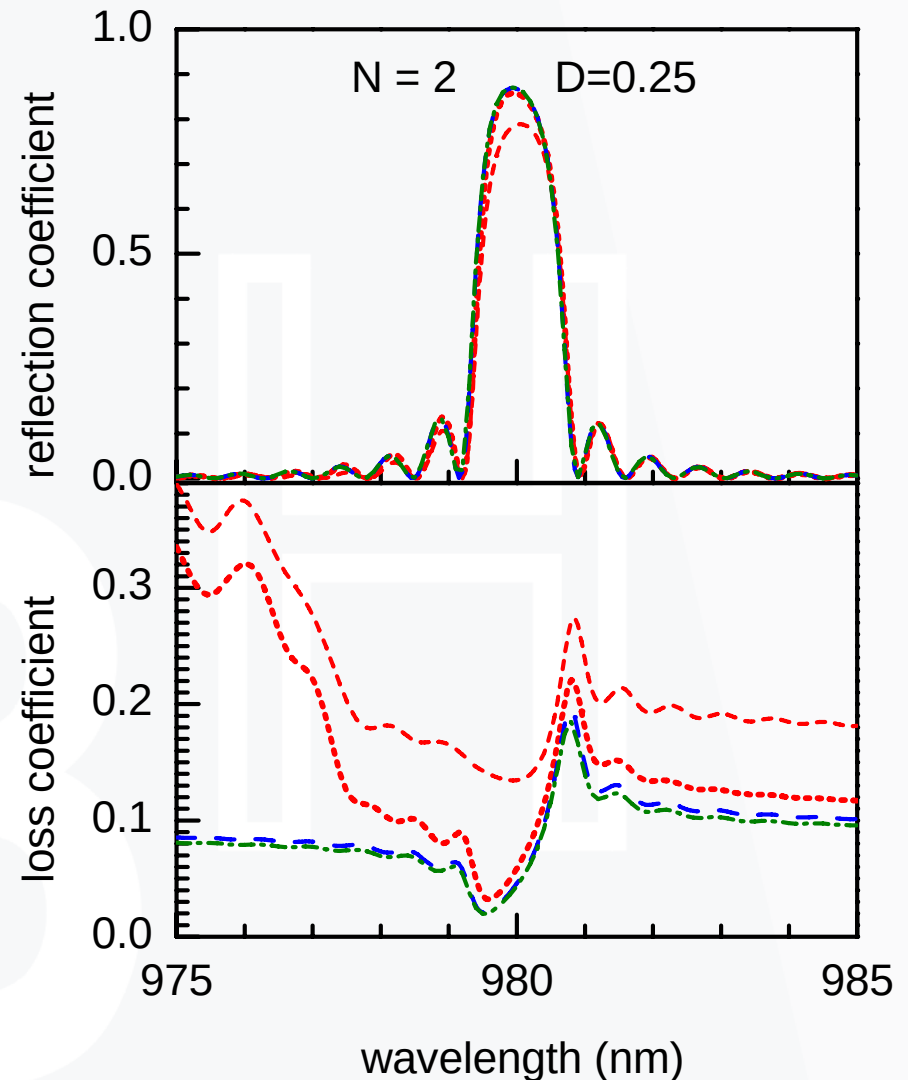
## Maximum reflectivity and loss for $N=3$

- Position and values of maxima and minima of reflection and loss coefficients reproduced by all models
- Larger deviations between CAMFR and CMT for  $D < 0.5$  and surrounding slab 1 as well as for  $D > 0.5$  and surrounding slab 2



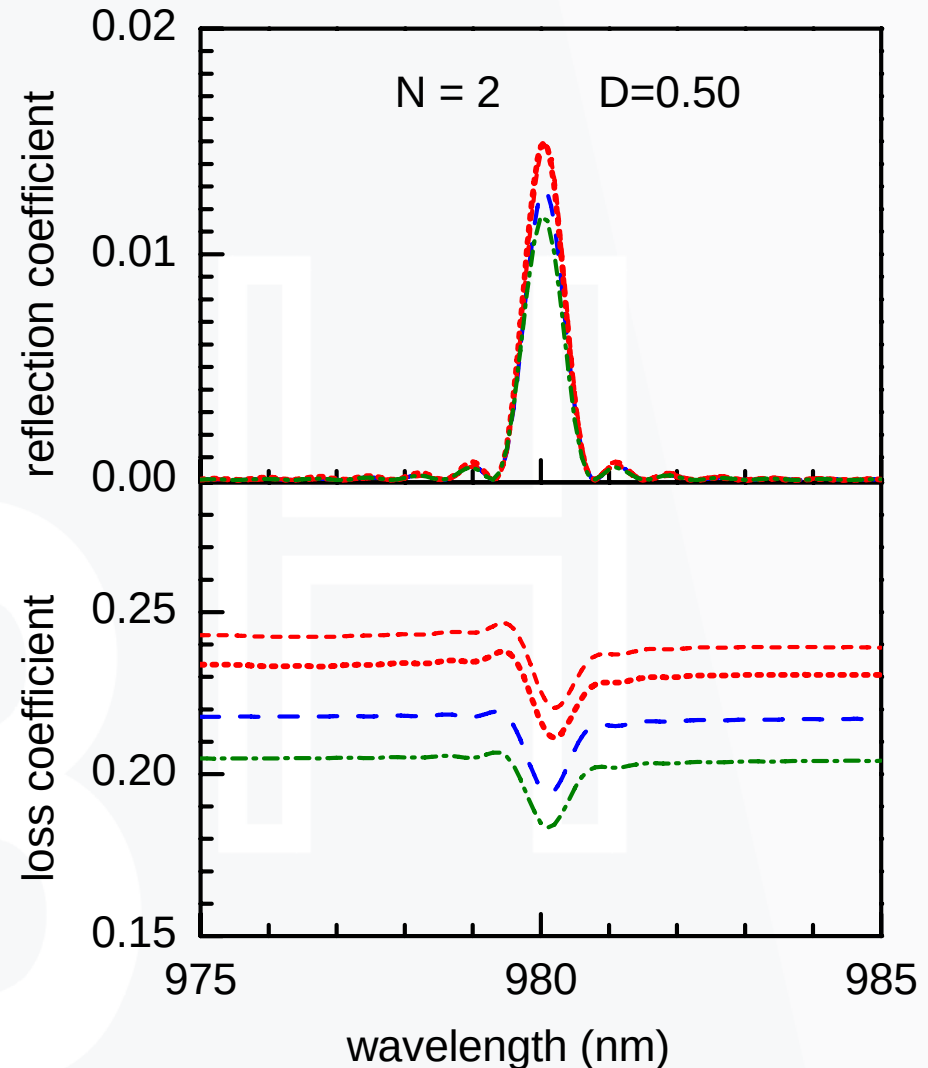
## Reflectivity and loss spectra for $N=2$ and $D=0.25$

- Good agreement between the reflectivity and loss spectra in the vicinity of the maximum of the reflectivity and at longer wavelengths for surrounding slab 2
- Large deviation between CAMFR and the CMT models at shorter wavelengths



## Reflectivity and loss spectra for $N=2$ and $D=0.5$

- Overall good agreement
- Reflectivity underestimated by CMT models
- Larger loss obtained with CAMFR due to insertion loss



## Conclusions

- Good mutual agreement  $\implies$  CMT applicable and CAMFR capable to model higher order Bragg waveguide gratings
- Additional loss contribution due to the mismatch between the field distributions in the grating and in the enclosing waveguides
- Correct simulation of radiation fields by PML boundary condition as implemented in CAMFR
- Approximation of Green's function needed to compute the coupling coefficients by that of a homogeneous unbounded medium possible with sufficient accuracy