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Impact of Modulation Instability on Supercontinuum Generation

Leibniz Gemeinschaft

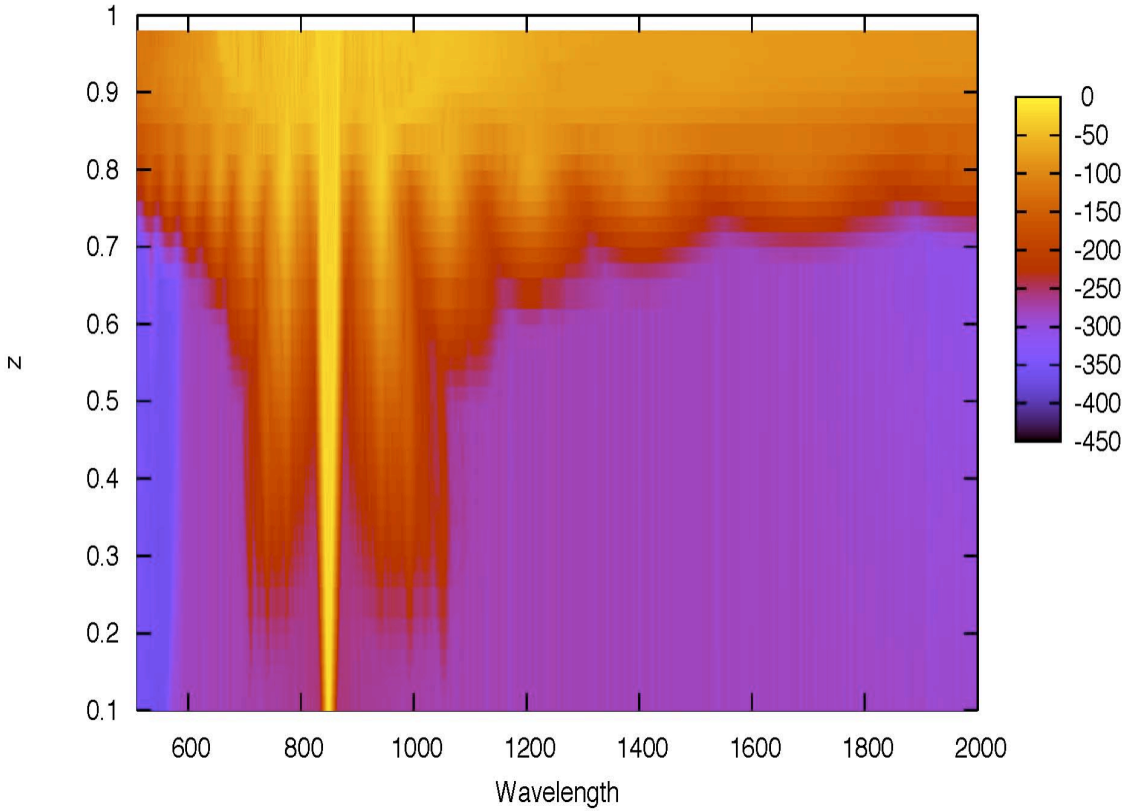
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Impact of Modulation Instability on Supercontinuum Generation

1. Introduction
2. Model
3. Soliton Fission and MI
4. SC Generation by MI
5. MI and Higher Order Effects
6. Summary

SC: bandwidth of a light bulb, with the brightness of a laser

Generated by propagation of short laser pulses through optical media under certain conditions.

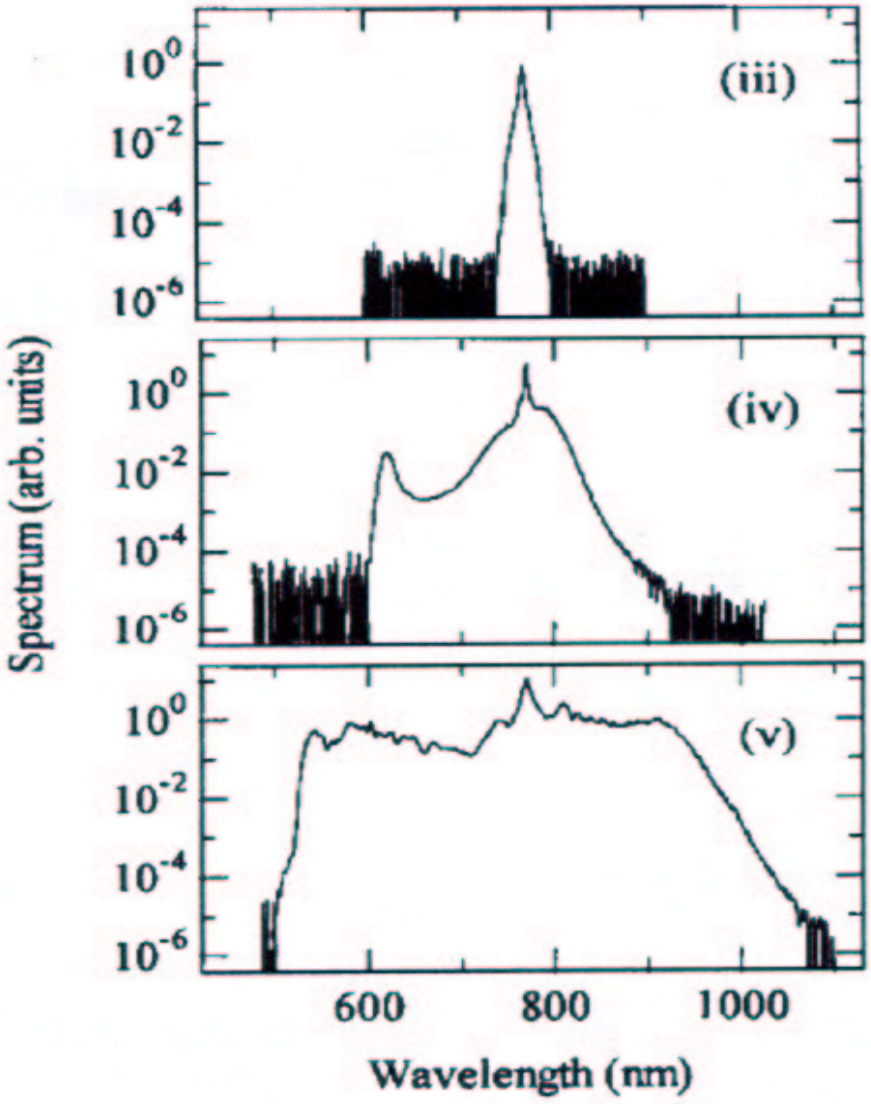


Characteristics of SC:

- ultra-wide and flat spectrum
- high degree of spectral coherence.
- SC occur in different parameter regimes (pico- or nanosecond pulses)

SC generation: an interplay of many nonlinear effects

Scenario is rich and not completely understood.



SC Generation at 770 nm
(*Ranka et al., Opt. Lett, 2000*)

- $\tau_0 = 100fs,\ L = 10cm,$
  $P_0 = 20, 220, 1600W$
- anomalous dispersion near ZDW
- soliton propagation,
  pulse compression
- ultrabroadband generation
- blue peak, pulse breakup
- negligible Raman scattering

Generalized Nonlinear Schrödinger Equation

$$\frac{\partial A}{\partial z} = \underbrace{-\frac{i}{2}\beta_2\frac{\partial^2 A}{\partial \tau^2} + \frac{1}{6}\beta_3\frac{\partial^3 A}{\partial \tau^3} + \frac{i}{24}\beta_4\frac{\partial^4 A}{\partial \tau^4}}_{\text{dispersion}} \underbrace{-\frac{1}{2}\alpha A}_{\text{loss}}$$

$$\underbrace{+i\gamma|A|^2 A}_{\text{SPM}} \underbrace{- a_1\frac{\partial}{\partial \tau}\left(|A|^2 A\right)}_{\text{self-steepening}} \underbrace{- ia_2 A\frac{\partial}{\partial \tau}\left(|A|^2\right)}_{\text{intrapulse Raman scattering}}$$

Propagation of (sub-)picosecond pulses along z
within retarded time frame $\tau = t - z/v_g$.

Pseudospectral Method

- periodic boundary conditions: $\omega_n = n\frac{2\pi}{T}\,, \quad n = 0, 1, 2, \ldots$

- FFT: $A(z,\tau) = \sum_{n=-N/2}^{N/2} \tilde{A}_n(z) \mathrm{e}^{i\omega_n \tau}$

- 8th-order Runge-Kutta scheme with adaptive stepsize control

$$\frac{\partial \tilde{A}_n(z)}{\partial z} = \frac{i}{2}\beta_2\omega_n^2\tilde{A}_n(z) - \frac{\alpha}{2}\tilde{A}_n(z) - \frac{i}{6}\beta_3\omega_n^3\tilde{A}_n(z) + \frac{i}{24}\beta_4\omega_n^4\tilde{A}_n(z) + i\gamma\tilde{\Psi}_n(z) + a_1\tilde{\Phi}_n(z) - ia_2\tilde{\tilde{\Xi}}_n(z)$$

the nonlinearities

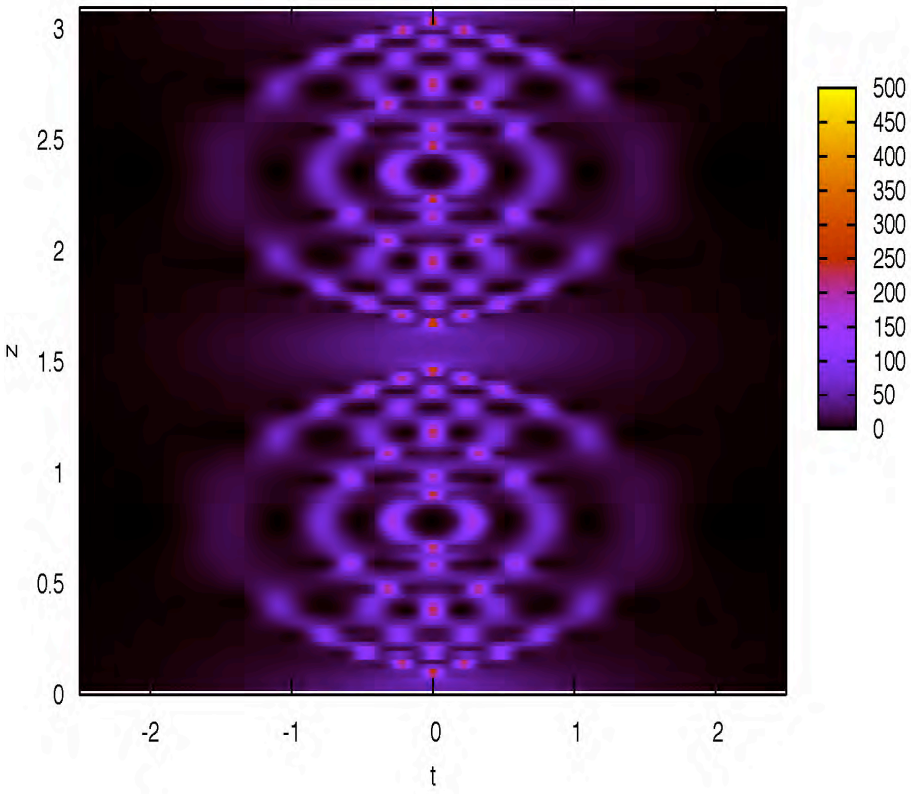
$$\Psi = |A|^2 A\,,\ \Phi = \tfrac{\partial}{\partial\tau}\left(|A|^2 A\right)\,,\ \Xi = A\tfrac{\partial}{\partial\tau}\left(|A|^2\right)$$

are evaluated in the t-domain:

$$\tilde{A}_n,\ i\omega\tilde{A}_n \xrightarrow{\text{IFFT}} A,\ \partial_\tau A \xrightarrow{\times} |A|^2 A = \Psi,\ A\partial_\tau|A|^2 = \Phi \xrightarrow{\text{FFT}} \tilde{\Psi}_n,\ \tilde{\Phi}_n,\ \tilde{\tilde{\Xi}}_n\,,$$

$$i\frac{\partial U}{\partial \xi} + sgn(\beta_2)\frac{1}{2}\frac{\partial^2 U}{\partial \theta^2} + N^2|U|^2U = 0$$

Nonlinear Schrödinger Equation



$\tau_0 = 1$ps, $N = 7$

with $\theta = \frac{\tau}{\tau_0}$, $\xi = \frac{z|\beta_2|}{\tau_0^2}$

and normalized amplitude

$U(z,\theta) = A(z,\theta)/\sqrt{P_0}$.

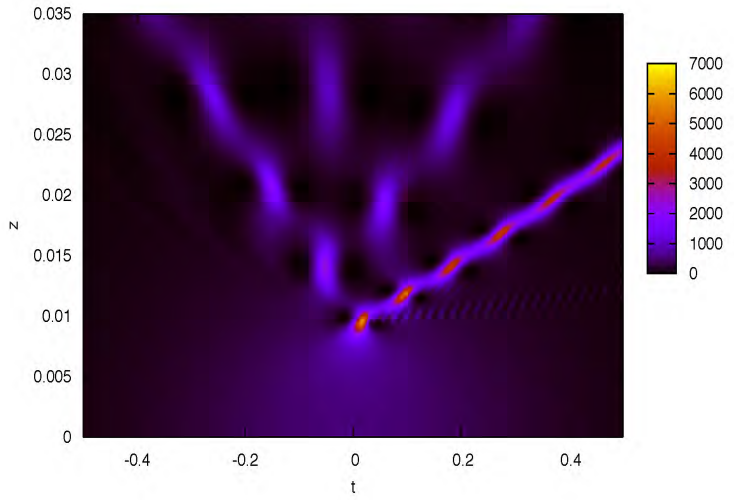
Soliton order: $N^2 = \frac{\gamma P_0 \tau_0^2}{|\beta_2|}$.

Fundamental soliton solution

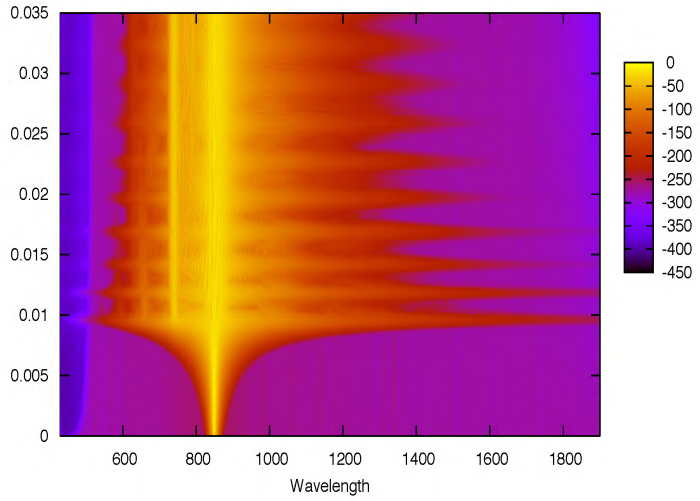
$$U(\xi,\theta) = \mathrm{sech}(\theta)\exp(i\xi/2).$$

near ZDW, visible regime, small intensity, N=7, $\gamma = 150 km^{-1}W^{-1}$,$\tau_0 = 300 fs$, $\beta_3 = 0.01 ps^3/km$, $\beta_4 = 0$.

time domain:



spectrum:

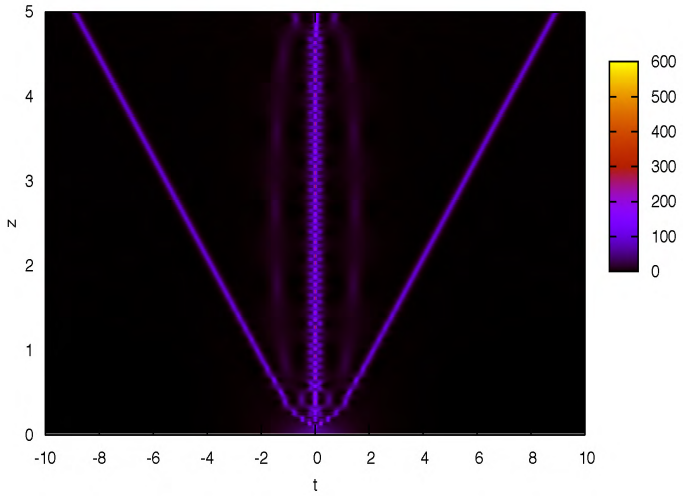


- fission
- nonsolitonic radiation

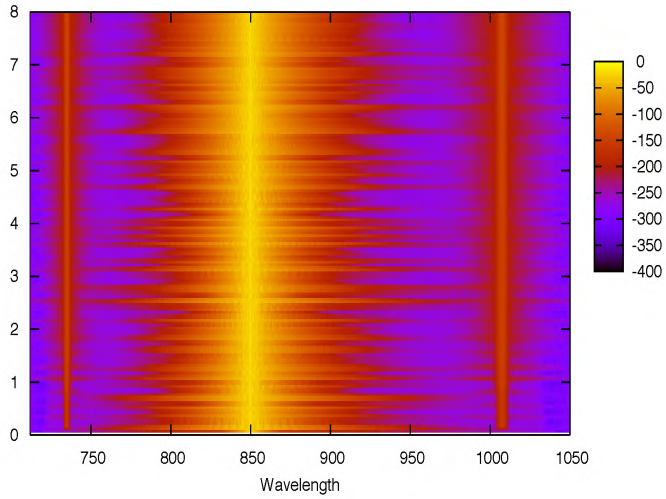
- blue peak
- not flat, but saturates

near ZDW, visible regime, small intensity, $N = 7$, $\gamma = 150 km^{-1}W^{-1}$,$\tau_0 = 1ps$, $\beta_3 = 0$, $\beta_4 = 5 \cdot 10^{-5} ps^4/km$.

time domain:



spectrum:



- fission
- 3 parts

- blue and red peak
- not flat, but saturates

SC generation by fission of higher-order solitons in PCF
(*Husakou and Hermann, Phy. Rev. Lett., 2001*)

Corresponds to our findings:

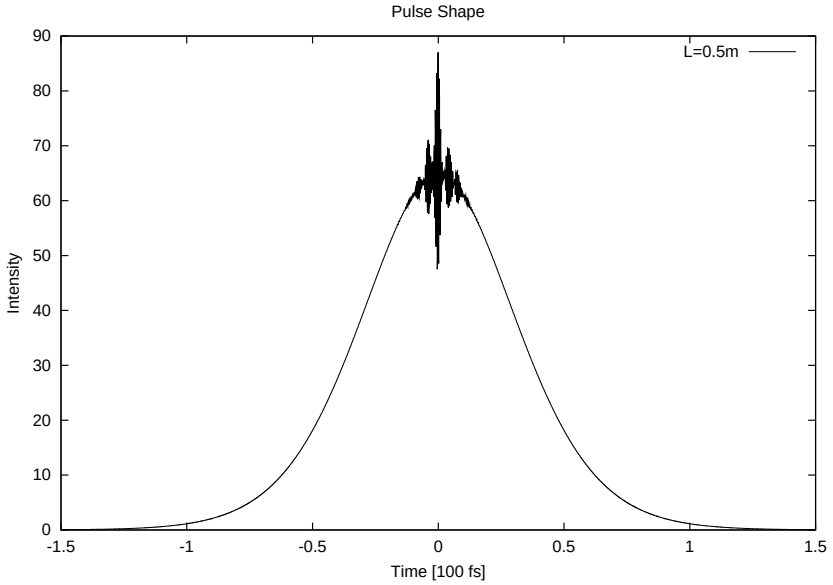
- fission of higher-order solitons
- blueshifted nonsolitonic radiation
- strong influence of β_3 near ZDW

Furthermore, negligible SPM indicated:

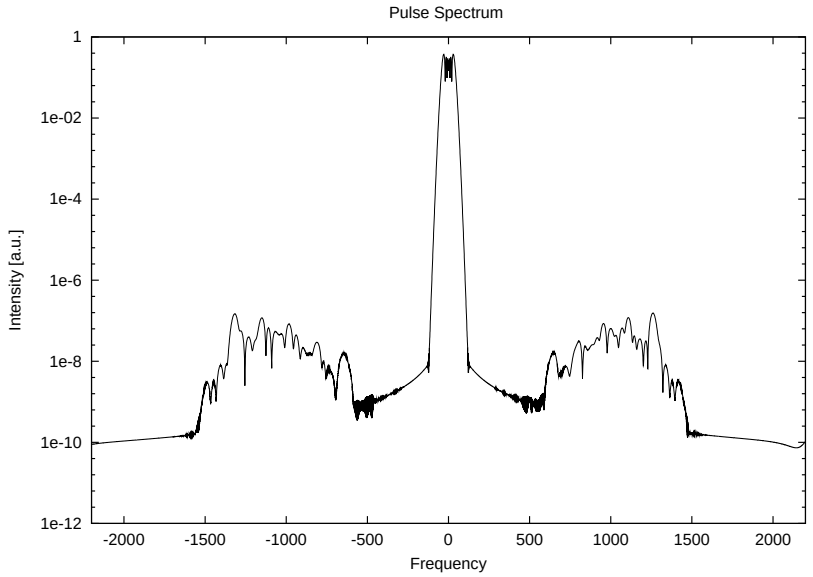
- shorter pulse with same intensity yields a narrower spectrum

MI: amplitude modulation. Interplay nonlinearity/anomalous dispersion.

Time domain:



Spectrum:

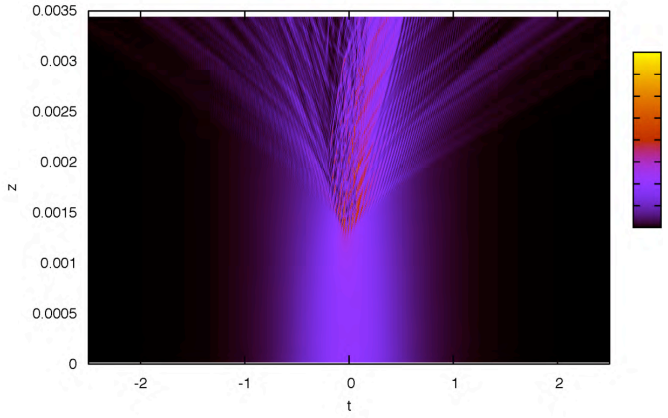


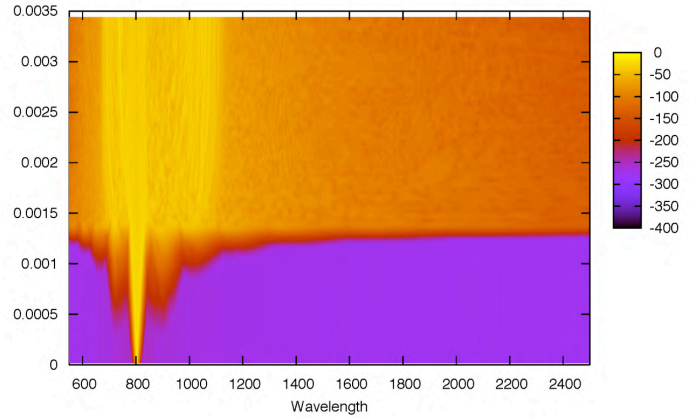
Ripples at center of the pump pulse.

Emergence of symmetric sidebands.

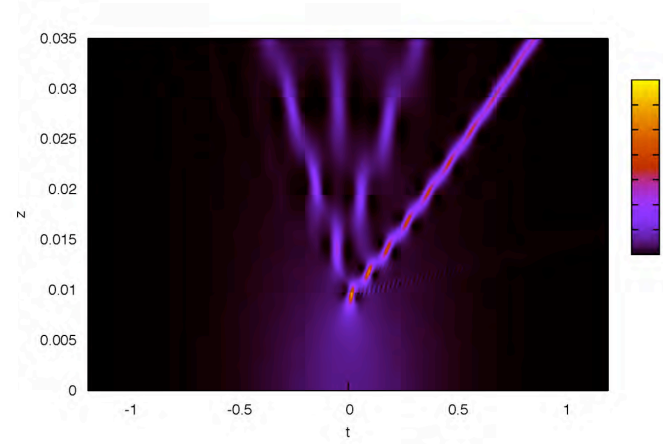
Nakazawa et al, PRA 1989

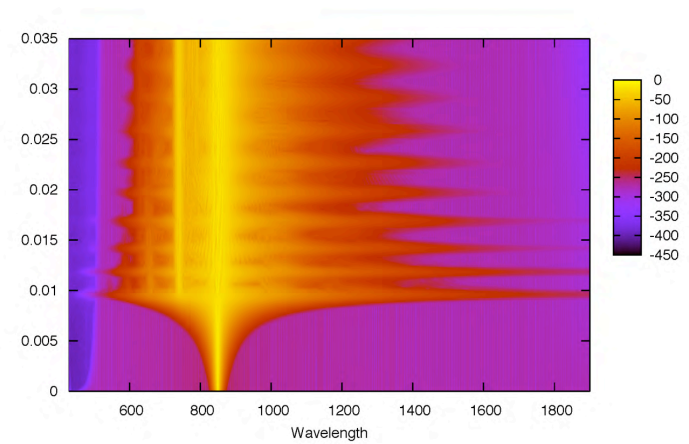
Modulation Instability:

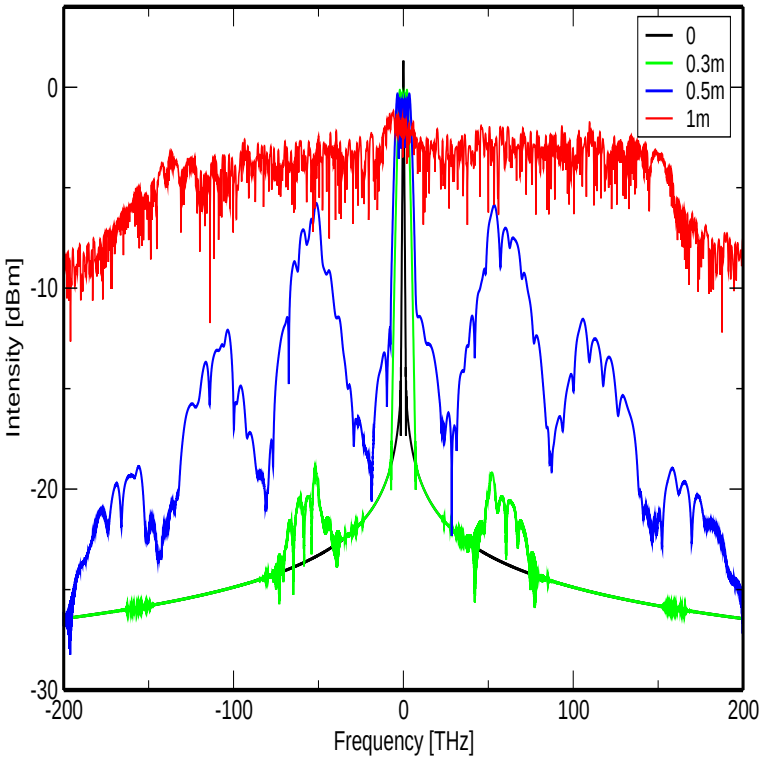




Soliton Fission:







Spectrum of an initially 1ps sech-pulse.

$P_0 = 400W$,$\gamma = 150km^{-1}W^{-1}$, $\beta_2 = -1ps^2/km$,

$\beta_3 = 0.01ps^3/km$, $\beta_4 = 5 \times 10^{-5}ps^4/km$ and $T_R = 3$ fs.

(Demircan and Bandelow, Opt. Comm. , 2005)

SC generation initiated by MI:

- MI sidebands, further FWM $\Longrightarrow$
- approximately symmetric and flat spectrum, spanning an octave of frequencies
- spectral densities varies less than 10 dB over a bandwidth of 480 – 950 nm
- negligible role of higher-order effects

Supercontinuum and four-wave mixing (Wadsworth et al, Optics Express (2004))

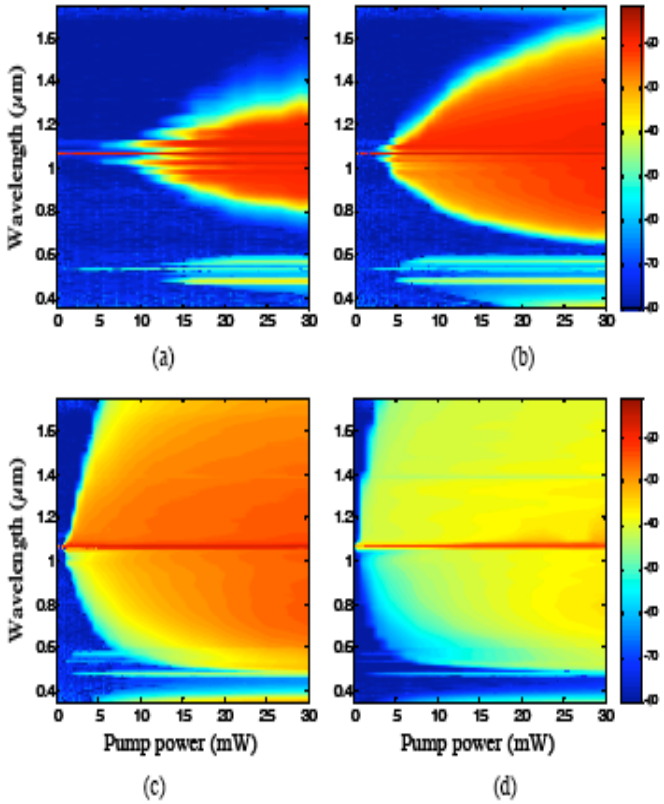
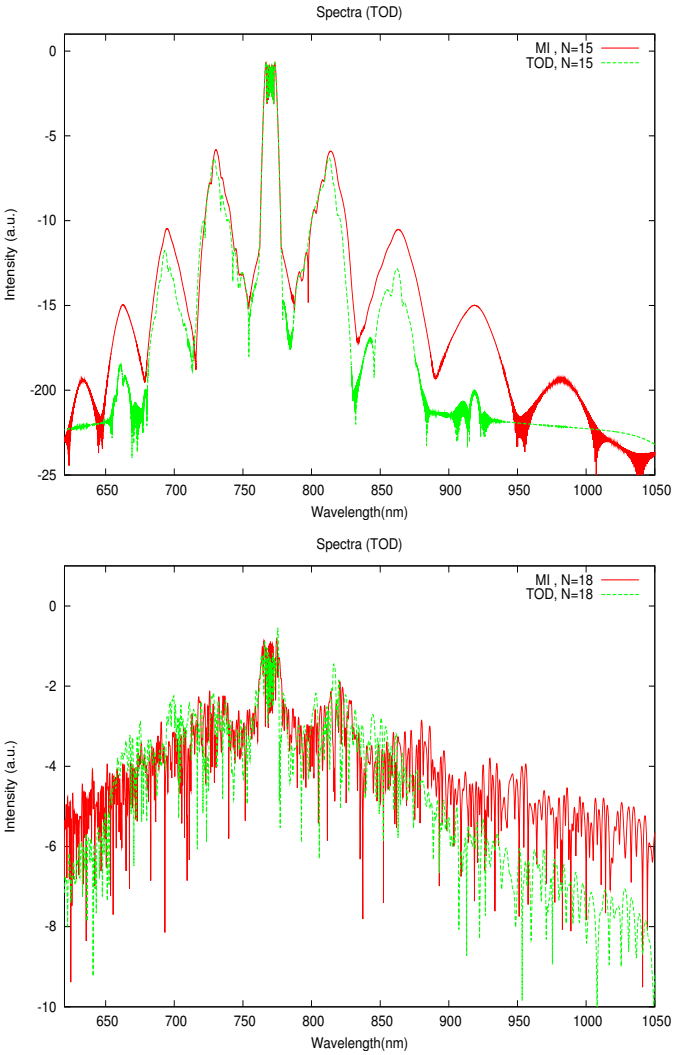


Fig. 8. Measured output continuum spectra from (a) 1 m, (b) 3m, (c) 20 m and (d) 100 m of fibre P. False colour scale in dBm/5nm bandwidth.

- sub-ns pulses (600 ps)
- PCF with ZDW=1064 nm
- MI as SC generator identified
- also: MI in the normal dispersion regime
- shift of the MI peaks



- anomalous dispersion ($\beta_2 < 0$)

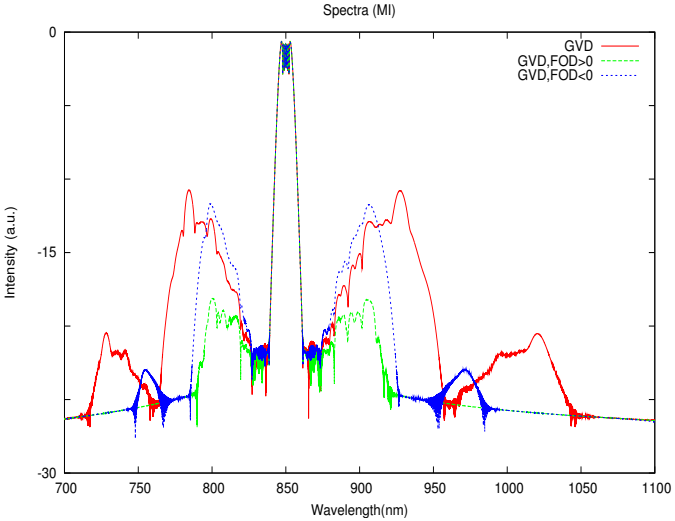
  $\lambda_p = 770nm$, $\gamma = 100km^{-1}W^{-1}$,

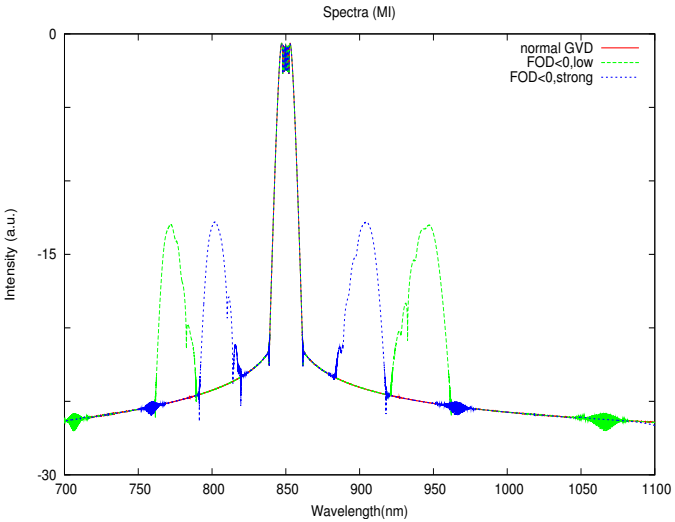
  $\beta_2 = -1ps^2/km$, $L = 0.5m$, $\beta_3 = 0.001ps^3/km$

- β_3 tends to suppress MI: decelerated SC generation

- asymmetry in the spectral shape

- normal dispersion $+$ $\beta_3 \Rightarrow$ no MI!





- anomalous dispersion ($\beta_2 < 0$)
  $\beta_4 = \pm 10^{-5} ps^4/km$

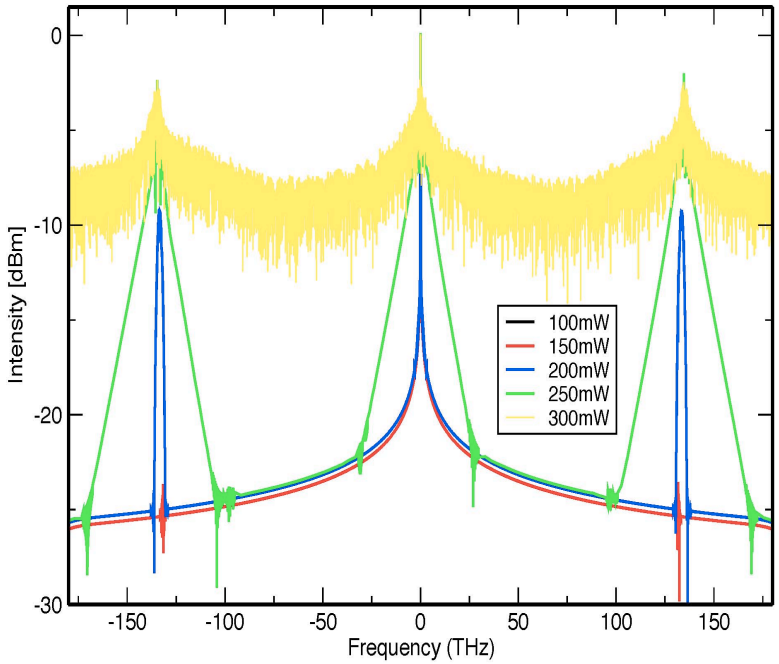
  - reduced effect

- normal dispersion ($\beta_2 > 0$)

  - $\beta_4 < 0$: MI sidebands!
    widely seperated from the pump beam

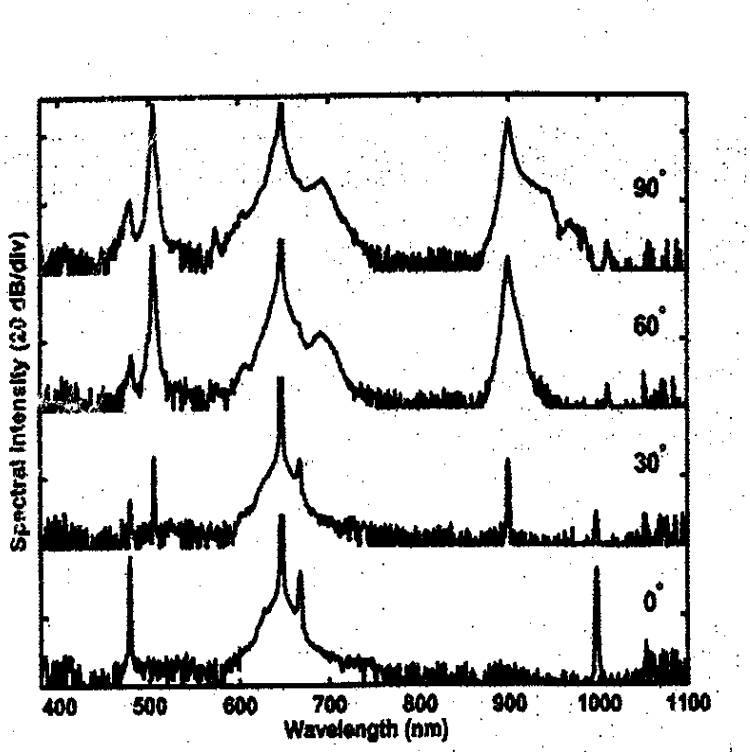
  - $\beta_4 > 0$: no MI

SC Generation by MI in the Normal Dispersion Regime: Numerical Example



- shift of the MI peaks with increasing power
- shift vanishes with high peak powers
- generation of new MI sidebands next to the former MI peaks
- SC generation only by FWM/MI

(*Harvey et al, OL, Nov. 2003*)



$T_0 = 70ps$, $\lambda_p = 647nm$, $z = 1m$

Positions of MI sidebands:

$\beta_2 = 2.5ps^2/km$ $\quad$ $\beta_4 = -4.49 \times 10^{-5} ps^4/km$

	λ_a [nm]	λ_s [nm]
Predicted	505	900
Experimental	504	904
Numerical	503.84	903.79

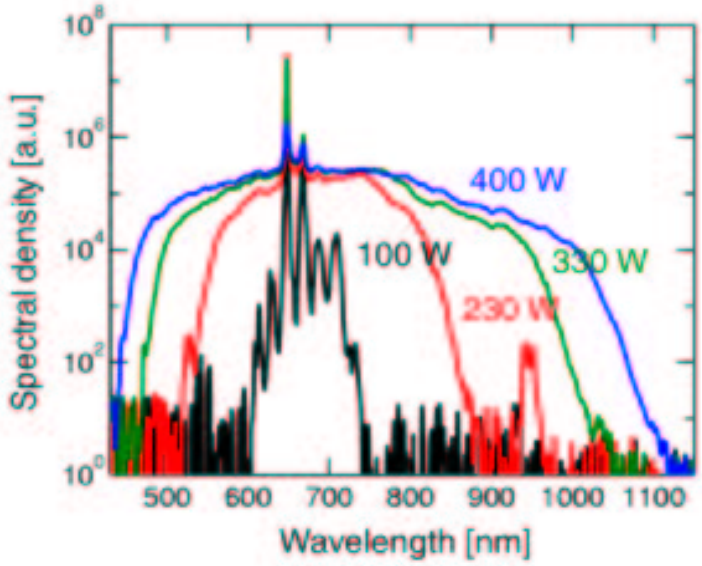
$\beta_2 = 4.48ps^2/km$ $\quad$ $\beta_4 = -5.90 \times 10^{-5} ps^4/km$

Predicted	487	963
Experimental	478	1003
Numerical ($\beta_4 = -5.09 \times 10^{-5} ps^4/km$)	477.89	1002.96

Numerics:

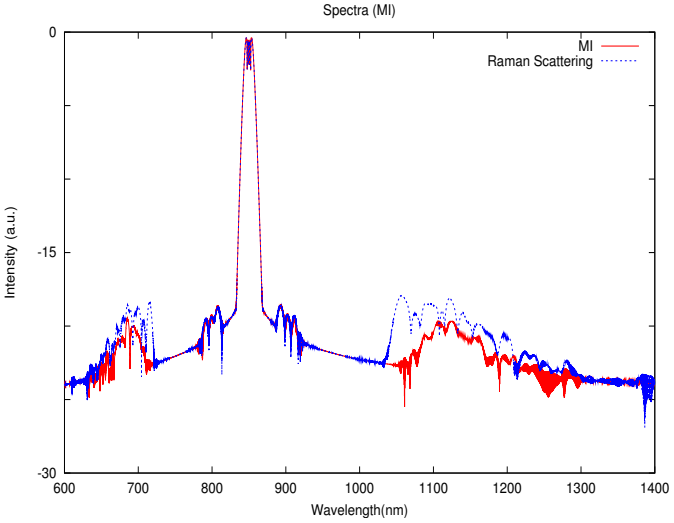
- Time window $T = 700ps \Longrightarrow$ Resolution 2^{19}

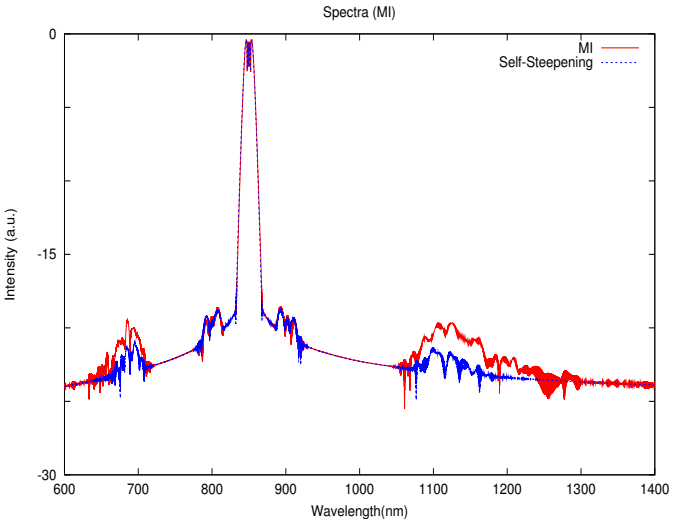
Supercontinuum generation with picosecond pulses (*Coen et al, Opt. Lett., 2001*)



- normal dispersion near ZDW
- negligible SPM
- strong Raman-Stokes lines
- parametric FWM
- non-phase-matched process

$\tau_0 = 60ps,\ L = 3m,\ \lambda = 647nm$





- MI interacts with Raman scattering:
  - MI intensified for low Raman contribution
  - Raman scattering $\Rightarrow$ asymmetric spectra

- self-steepening usually negligible, but:

  suppresses MI when a_1 is drastically increased

Summary

- SC Generation inspected.
- Generators: Soliton Fission vs. MI.
- MI inherent property of the NLSE: occurs in different parameter regimes.
- SC generation initiated almost by the MI.
- Higher-order effects: minor role for SC generation.