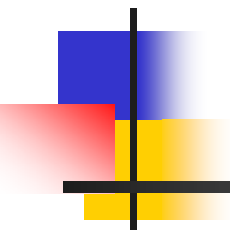


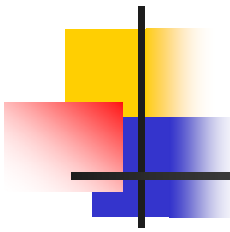
A Modified Rational-Function Method for Transmission and Reflection Calculation at Discontinuities



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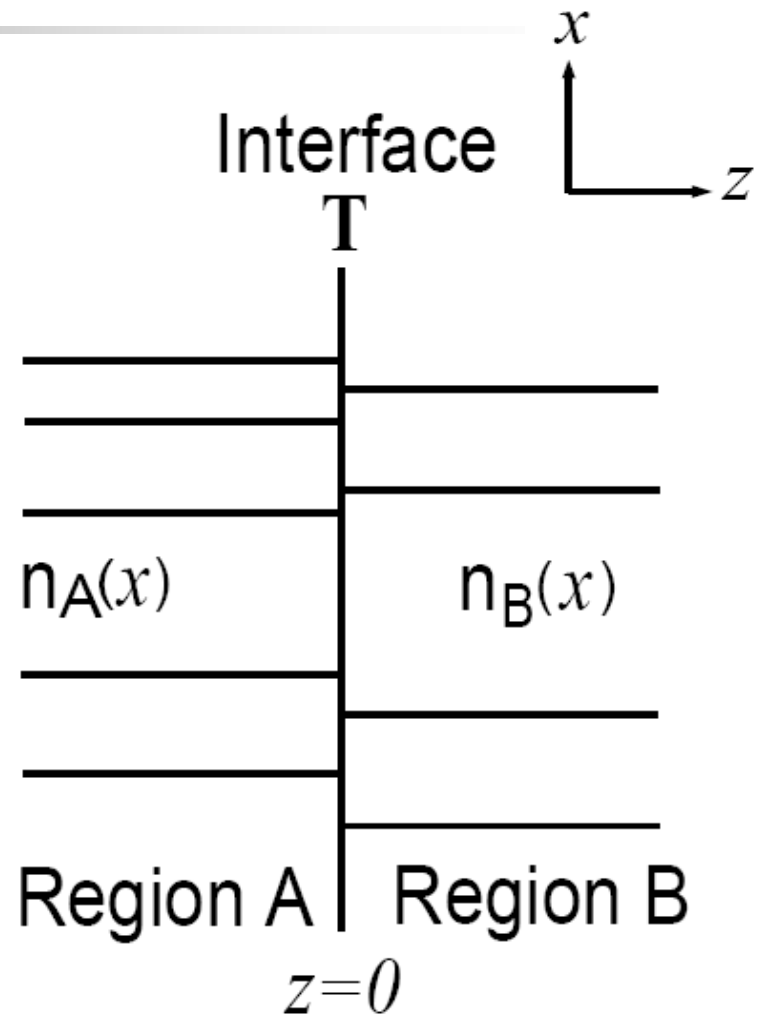
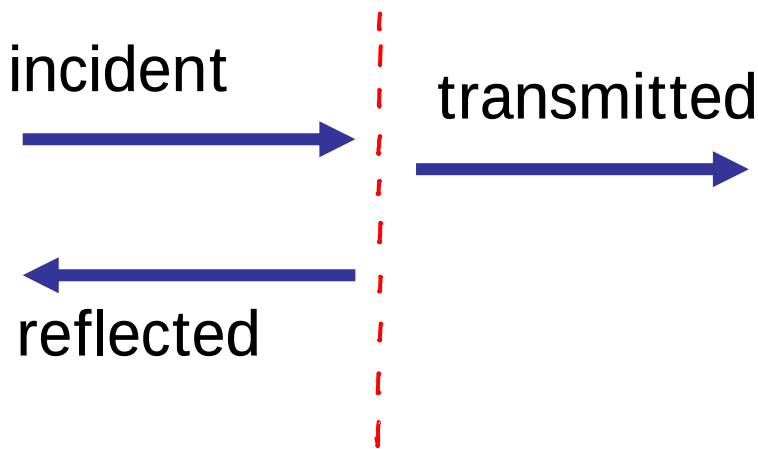


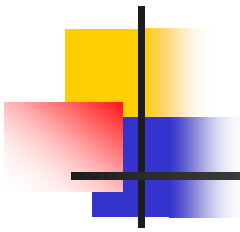
Outline

- Introduction
- Formulation
- Padé-approximant-reflective method using real reference index (PARM-R) and complex reference index (PARM-C)
- Modified Padé-approximant-reflective method using real reference index (PARM-Q)
- Summary

Introduction

- Laser facets, waveguides, gratings, connection between waveguides etc.





Introduction

- Modes from analytical solution
 - Simple structures
- Fourier series
 - Discontinuous field across the abrupt interfaces
- Finite-difference
 - Time and memory consuming

Formulation

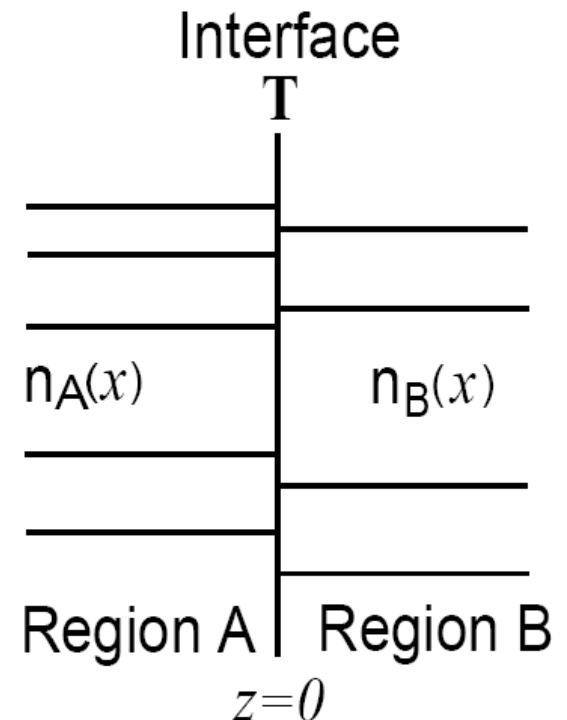
- 2D, TE, lossless

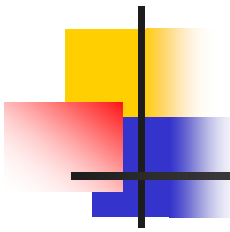
$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial z^2} + k_0^2 n^2(x) E_y = 0$$

- Finite-difference discretization

$$\frac{\partial^2 E}{\partial z^2} + AE = 0, \quad A\phi_i = \beta_i^2 \phi_i$$

$$E = [E_{y1} \ E_{y2} \ \cdots \ E_{yN}]^T, \quad A: \text{characteristic matrix}$$





Formulation

$$\frac{\partial^2 E}{\partial z^2} + AE = 0, \quad A\phi_i = \beta_i^2 \phi_i$$

$$E^+ = \sum a_i^+ \phi_i(x), \quad E^- = \sum a_i^- \phi_i(x)$$

Total field

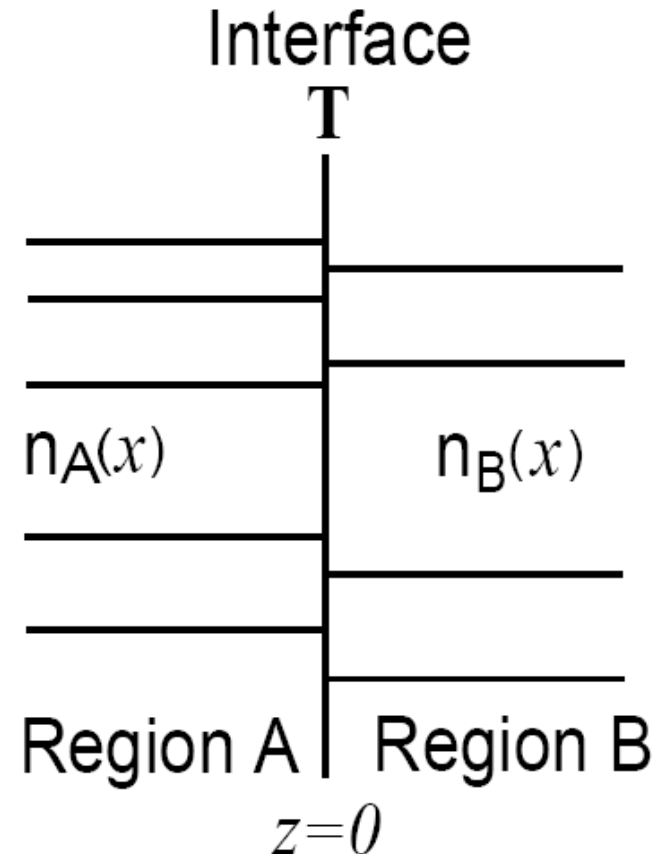
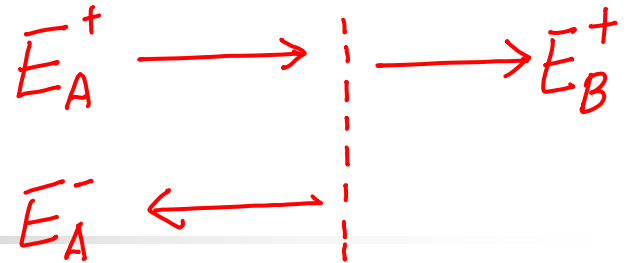
$$\begin{aligned} E(x, z) &= \sum (a_i^+ e^{-j\beta_i z} + a_i^- e^{+j\beta_i z}) \phi_i(x) \\ &= e^{-j\sqrt{A}z} E^+ + e^{+j\sqrt{A}z} E^- \end{aligned}$$

Formulation

- After matching the boundary conditions

$$E_A^- = \frac{\sqrt{A_A} - \sqrt{A_B}}{\sqrt{A_A} + \sqrt{A_B}} E_A^+ \equiv [R_{TE}] E_A^+$$

$$E_B^+ = \frac{2\sqrt{A_A}}{\sqrt{A_A} + \sqrt{A_B}} E_A^+ \equiv [T_{TE}] E_A^+$$

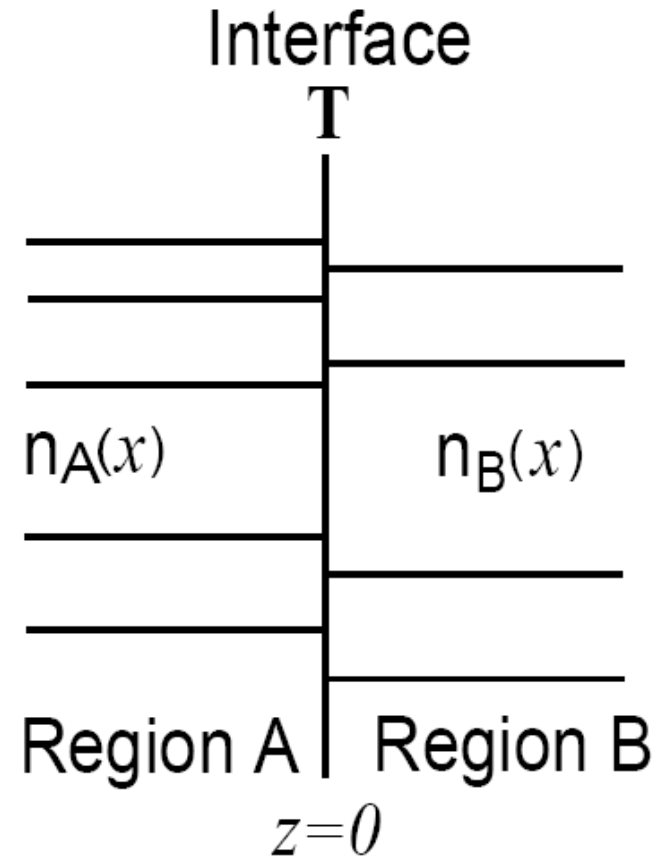
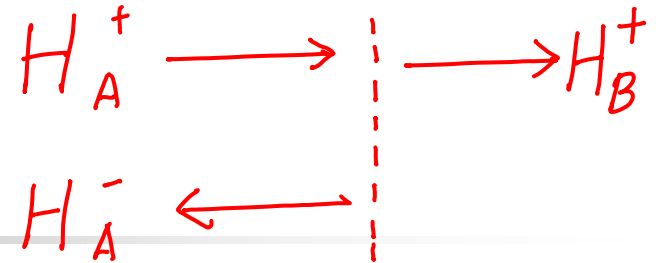


Formulation

■ TM case

$$H_A^- = \frac{\frac{1}{n_A^2} \sqrt{A_A} - \frac{1}{n_B^2} \sqrt{A_B}}{\frac{1}{n_A^2} \sqrt{A_A} - \frac{1}{n_B^2} \sqrt{A_B}} H_A^+ \equiv [R_{TM}] H_A^+$$

$$H_B^+ = \frac{\frac{2}{n_A^2} \sqrt{A_A}}{\frac{1}{n_A^2} \sqrt{A_A} - \frac{1}{n_B^2} \sqrt{A_B}} H_A^+ \equiv [T_{TM}] H_A^+$$



Formulation

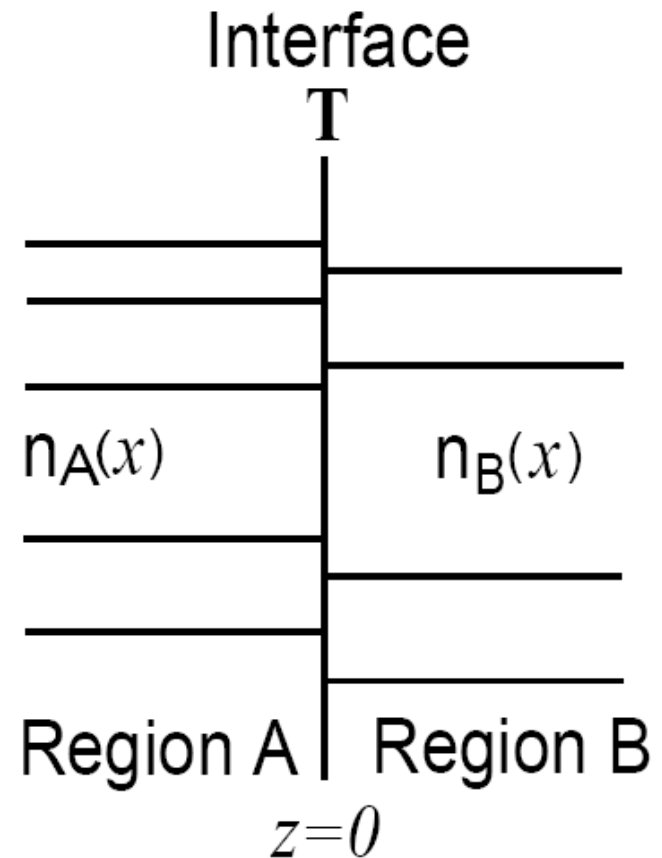
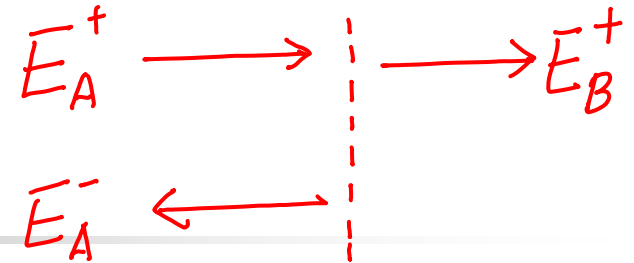
$$E_A^- = \frac{\sqrt{A_A} - \sqrt{A_B}}{\sqrt{A_A} + \sqrt{A_B}} E_A^+ \equiv [R_{TE}] E_A^+$$

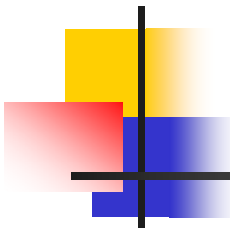
$$E_B^+ = \frac{2\sqrt{A_A}}{\sqrt{A_A} + \sqrt{A_B}} E_A^+ \equiv [T_{TE}] E_A^+$$

■ Method of Lines (MoL)

$$\sqrt{A} = X \sqrt{D} X^{-1}$$

- Eigenvalue problem
- Accurate but time and memory consuming





Padé Approximation

- Approximate the square root by a rational function around the reference wavevector $k_0 \bar{n}$
- Frequently used in wide-angle (WA) beam propagation method (BPM)

$$E_A^- = \frac{\sqrt{A_A} - \sqrt{A_B}}{\sqrt{A_A} + \sqrt{A_B}} E_A^+ \approx \frac{S_A - S_B}{S_A + S_B} E_A^+$$

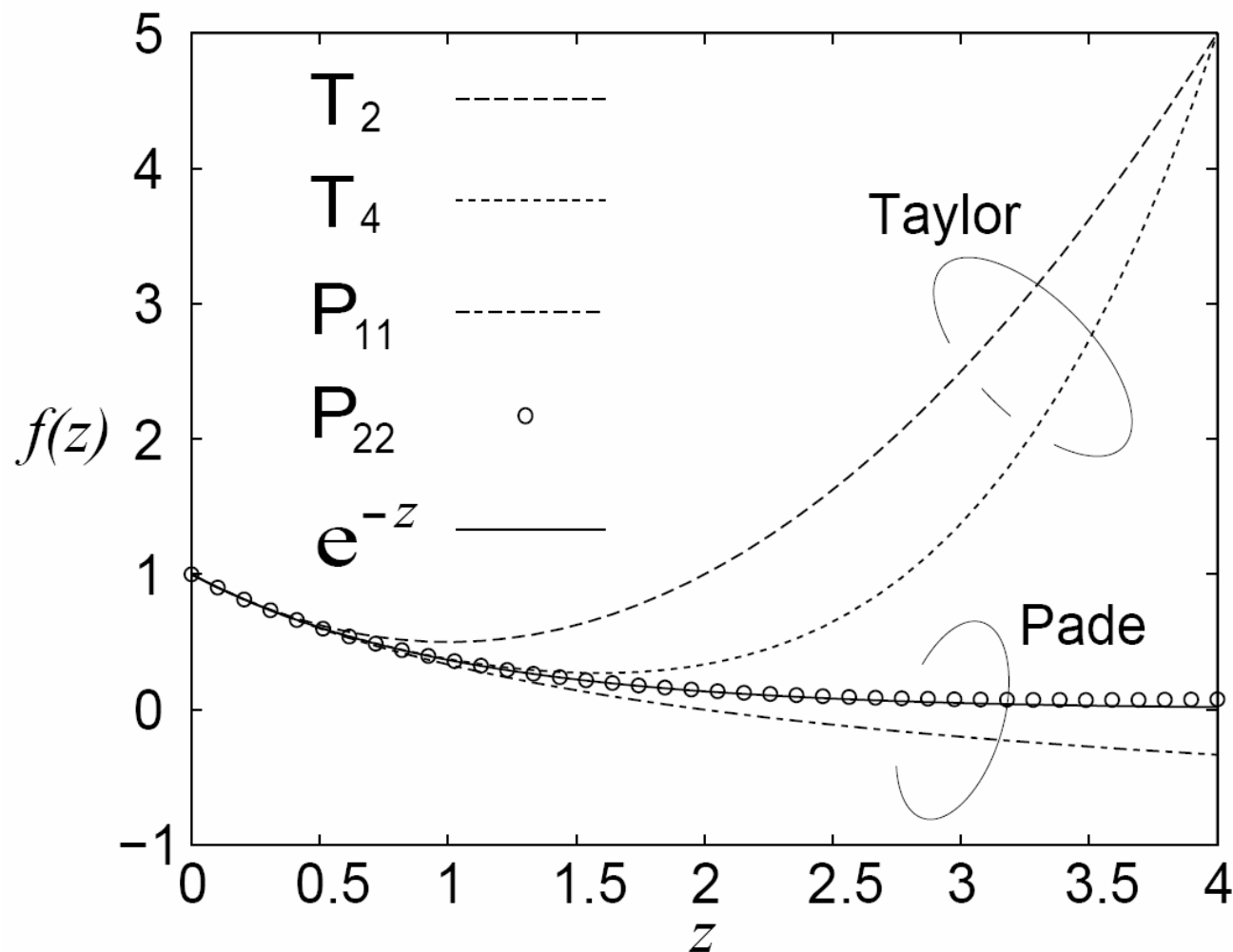
$$E_B^+ = \frac{2\sqrt{A_A}}{\sqrt{A_A} + \sqrt{A_B}} E_A^+ \approx \frac{2S_A}{S_A + S_B} E_A^+$$

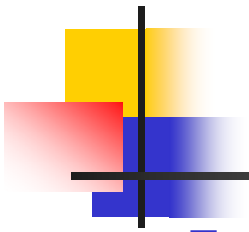
$$T_2 = 1 - z + \frac{z^2}{2}$$

$$P_{11} = \left(1 - \frac{z}{2}\right)^{-1} \left(1 + \frac{z}{2}\right)$$

$$T_4 = 1 - z + \frac{z^2}{2} - \frac{z^3}{6} + \frac{z^4}{24}$$

$$P_{22} = \left(1 - \frac{z}{2} + \frac{z^2}{12}\right)^{-1} \left(1 + \frac{z}{2} + \frac{z^2}{12}\right)$$





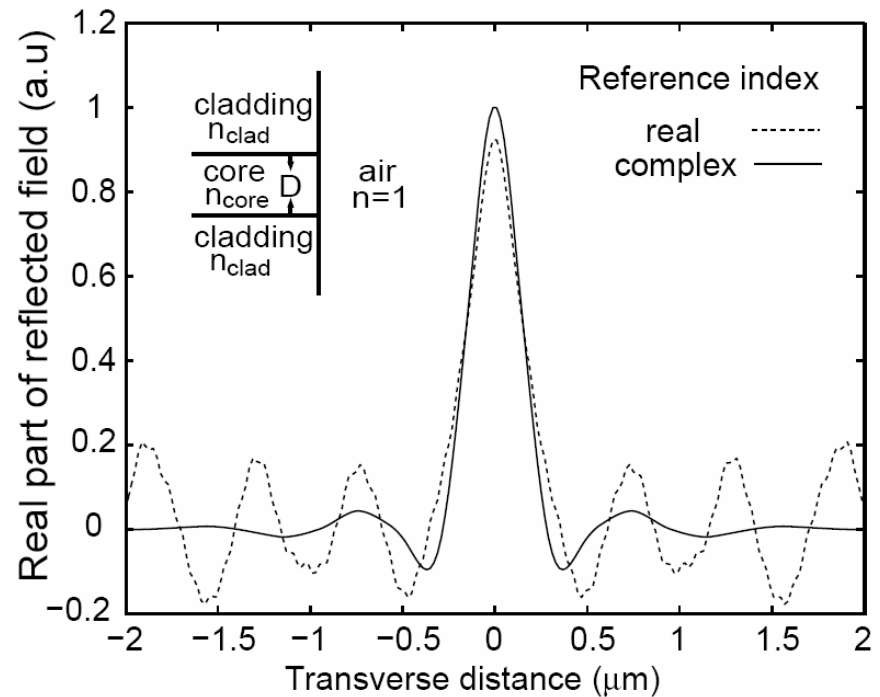
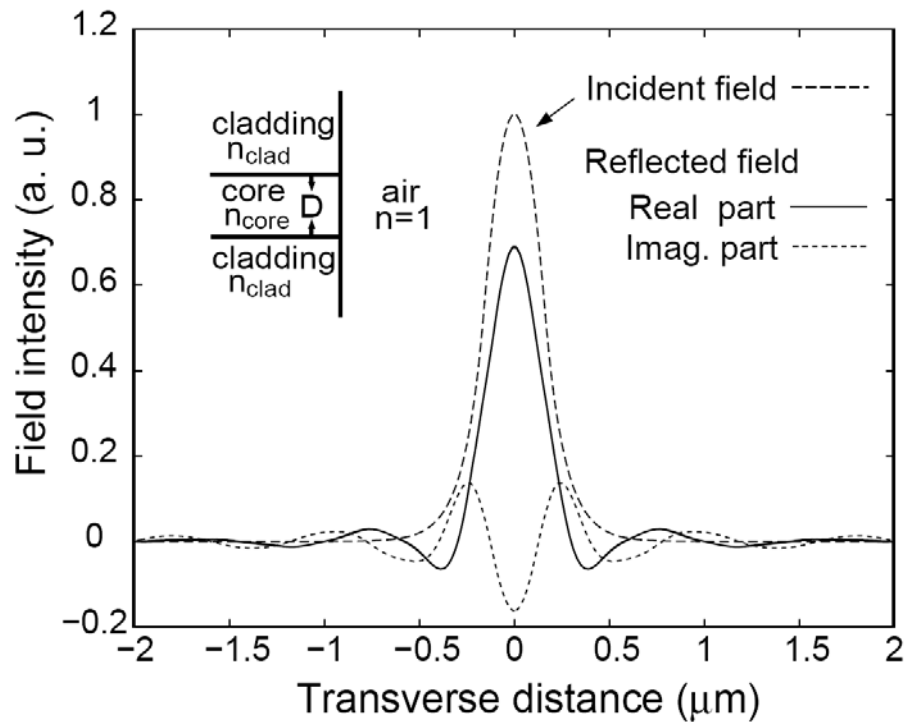
PARM-R and PARM-C

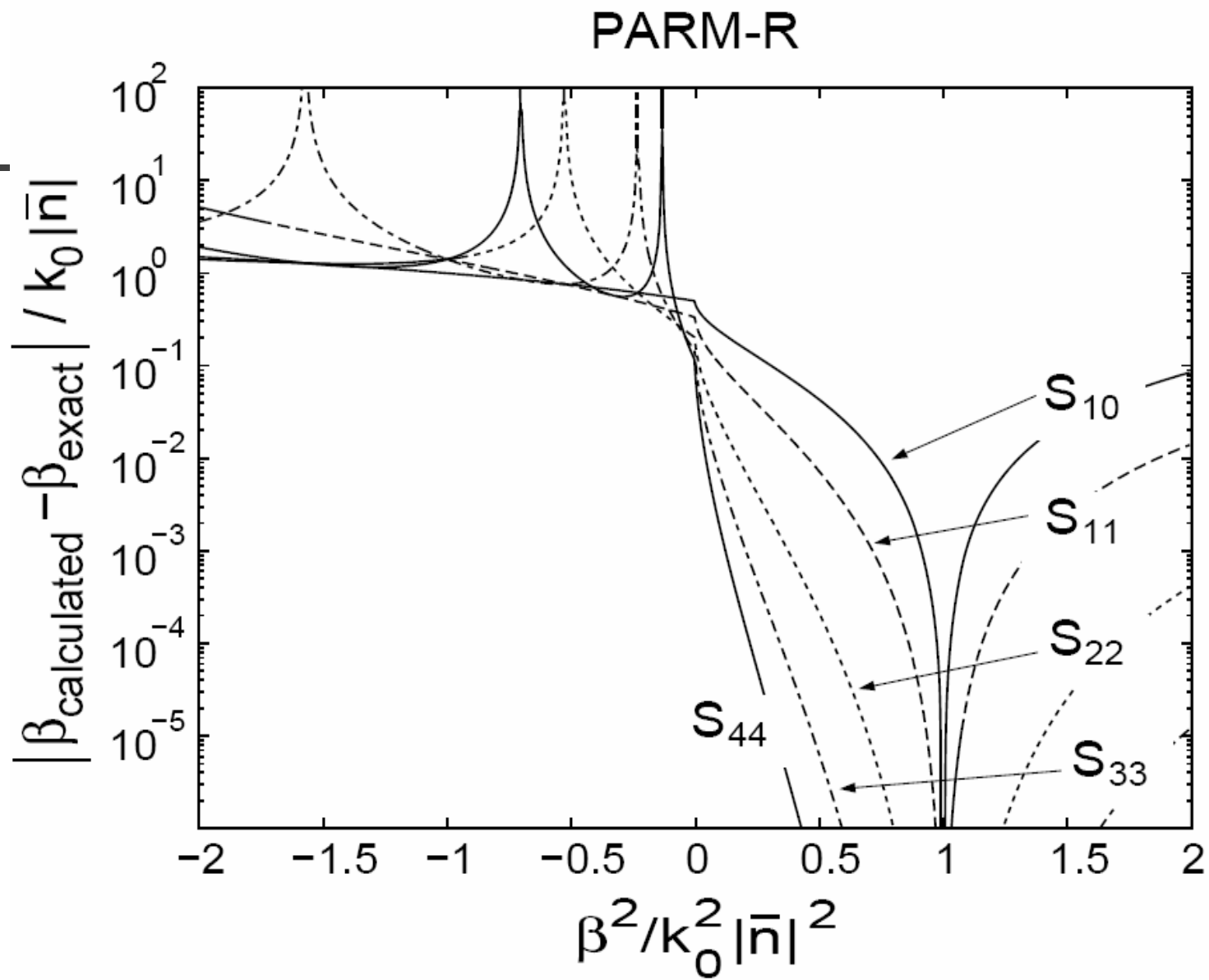
- Issue of Padé approximants -- Singularities
 - e.g.

$$\sqrt{A} \approx S_{11} = \left(k_0 \bar{n}\right) \frac{3A + k_0^2 \bar{n}^{-2}}{A + 3k_0^2 \bar{n}^{-2}}$$

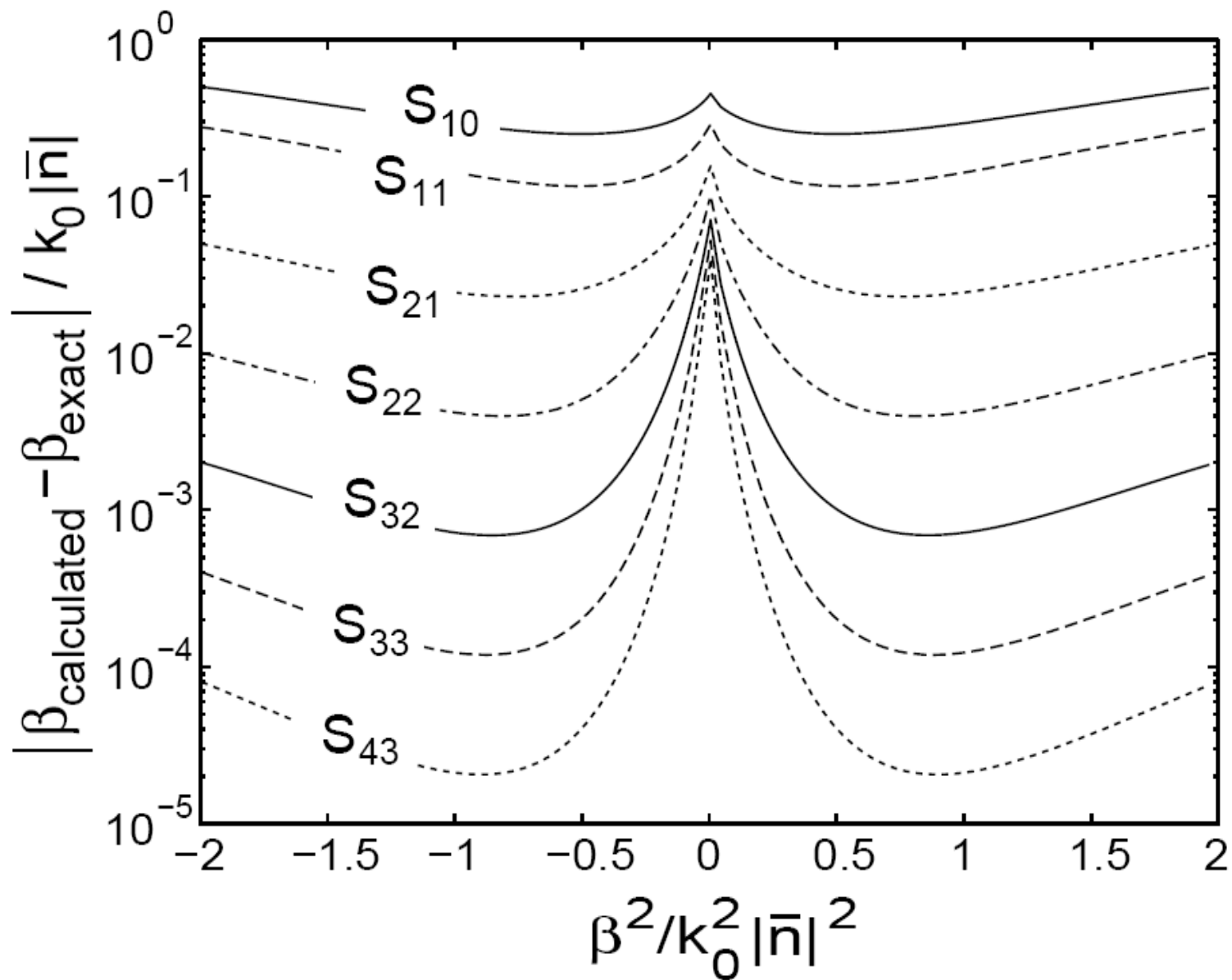
- Singularity at $A + 3k_0^2 \bar{n}^{-2} = 0$
 - Corresponding to excitation of evanescent waves
- Non-convergent results
- Complex $\bar{n}$ to avoid singularities

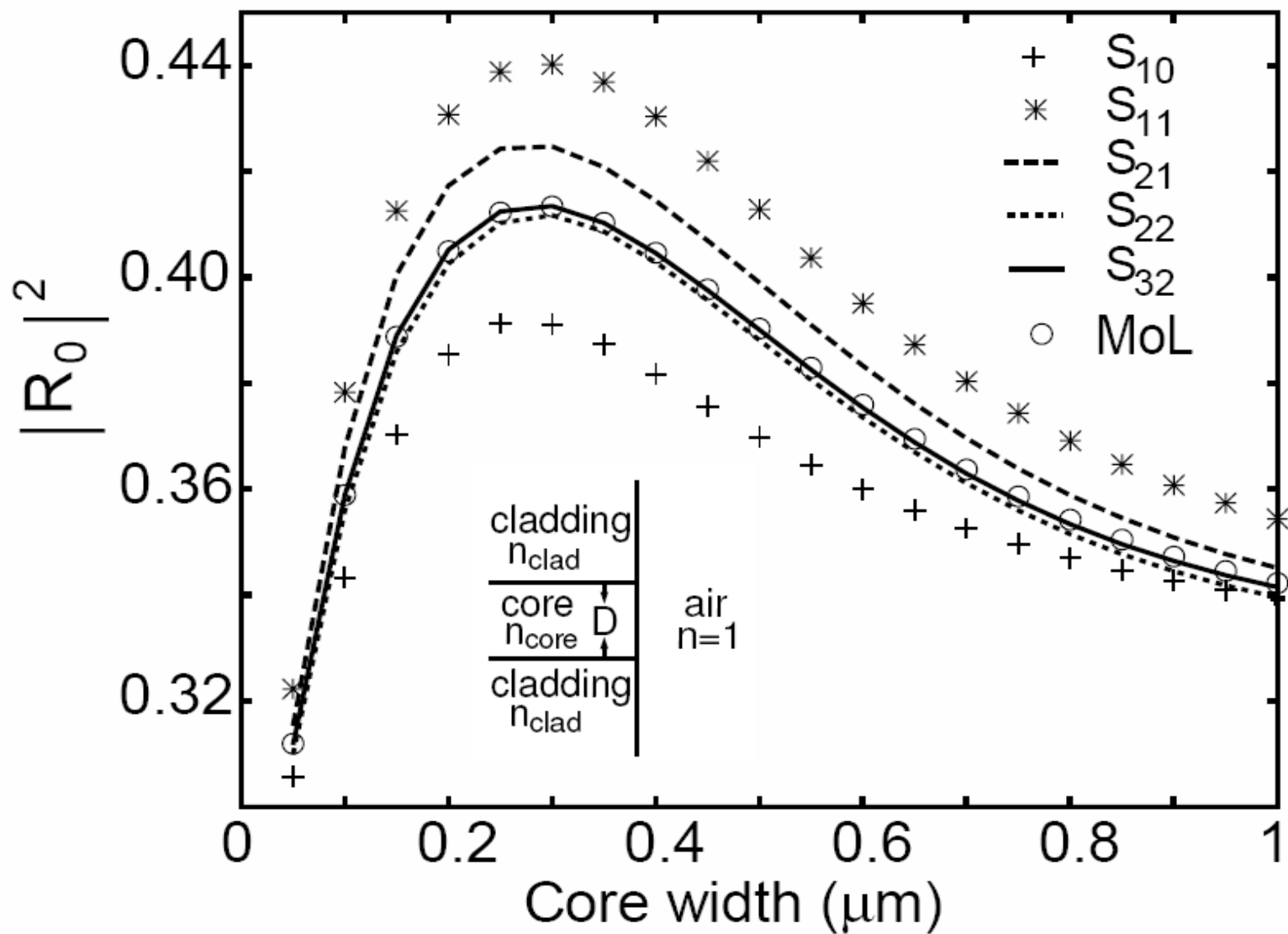
PARM-R and PARM-C

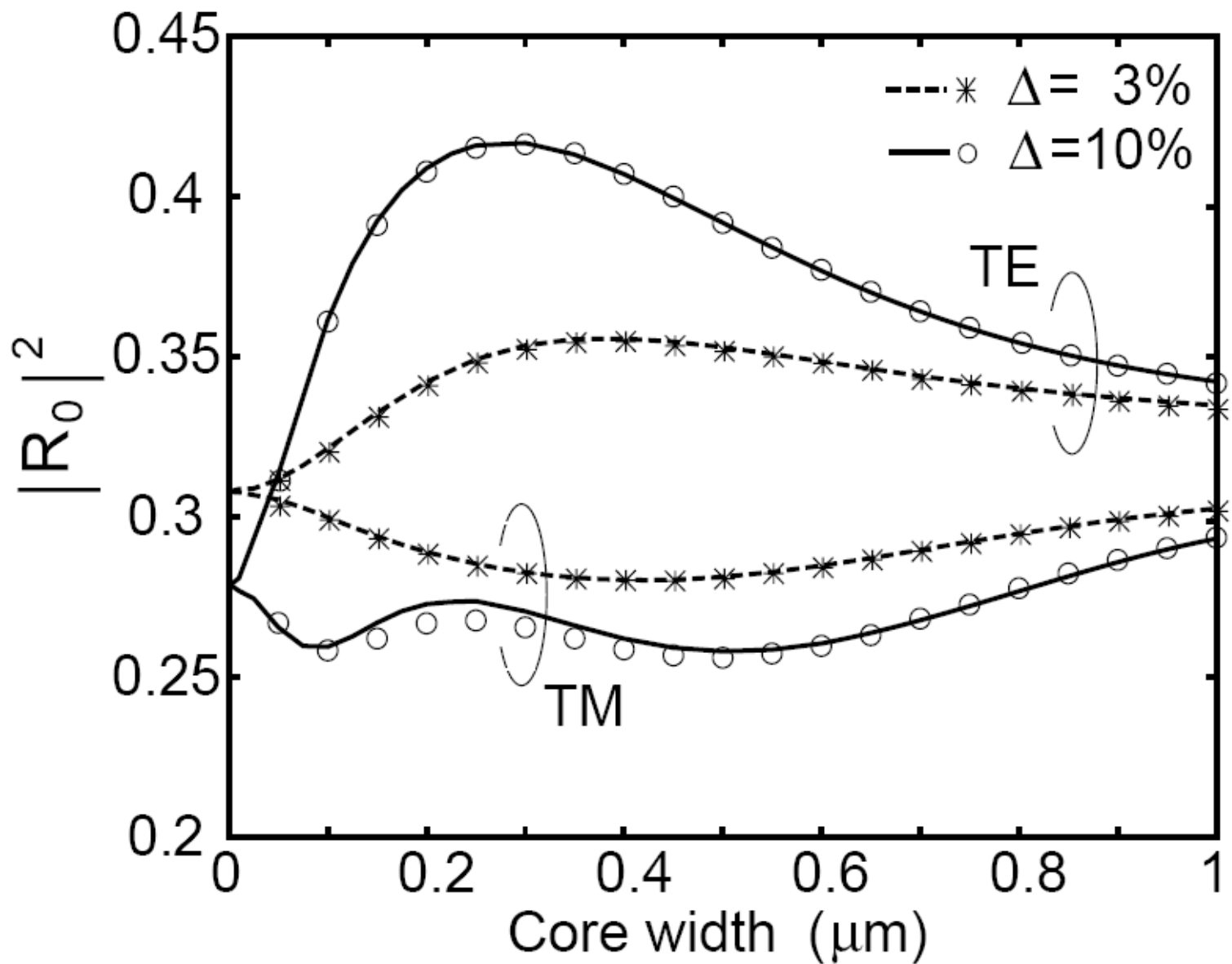




PARM-C







Another Approximation

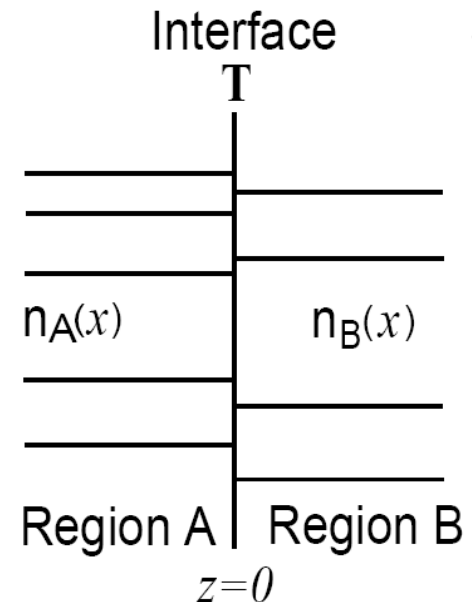
PARM-Q

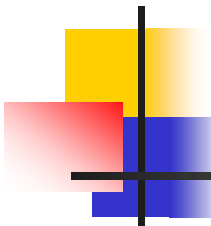
- Negative eigenvalues become positive if they are squared. Therefore, singularities disappear even with real reference index.
- Evanescent waves are modeled as propagating waves.

$$\sqrt{A} \approx \sqrt[4]{A^2} \approx Q_{nd}$$

$$E_A^- = \frac{\sqrt{A_A} - \sqrt{A_B}}{\sqrt{A_A} + \sqrt{A_B}} E_A^+ \approx \frac{Q_A - Q_B}{Q_A + Q_B} E_A^+$$

$$E_B^+ = \frac{2\sqrt{A_A}}{\sqrt{A_A} + \sqrt{A_B}} E_A^+ \approx \frac{2Q_A}{Q_A + Q_B} E_A^+$$





$$Q_{10} = k_0 \bar{n} \left(1 + \frac{[\Omega]}{4} \right)$$

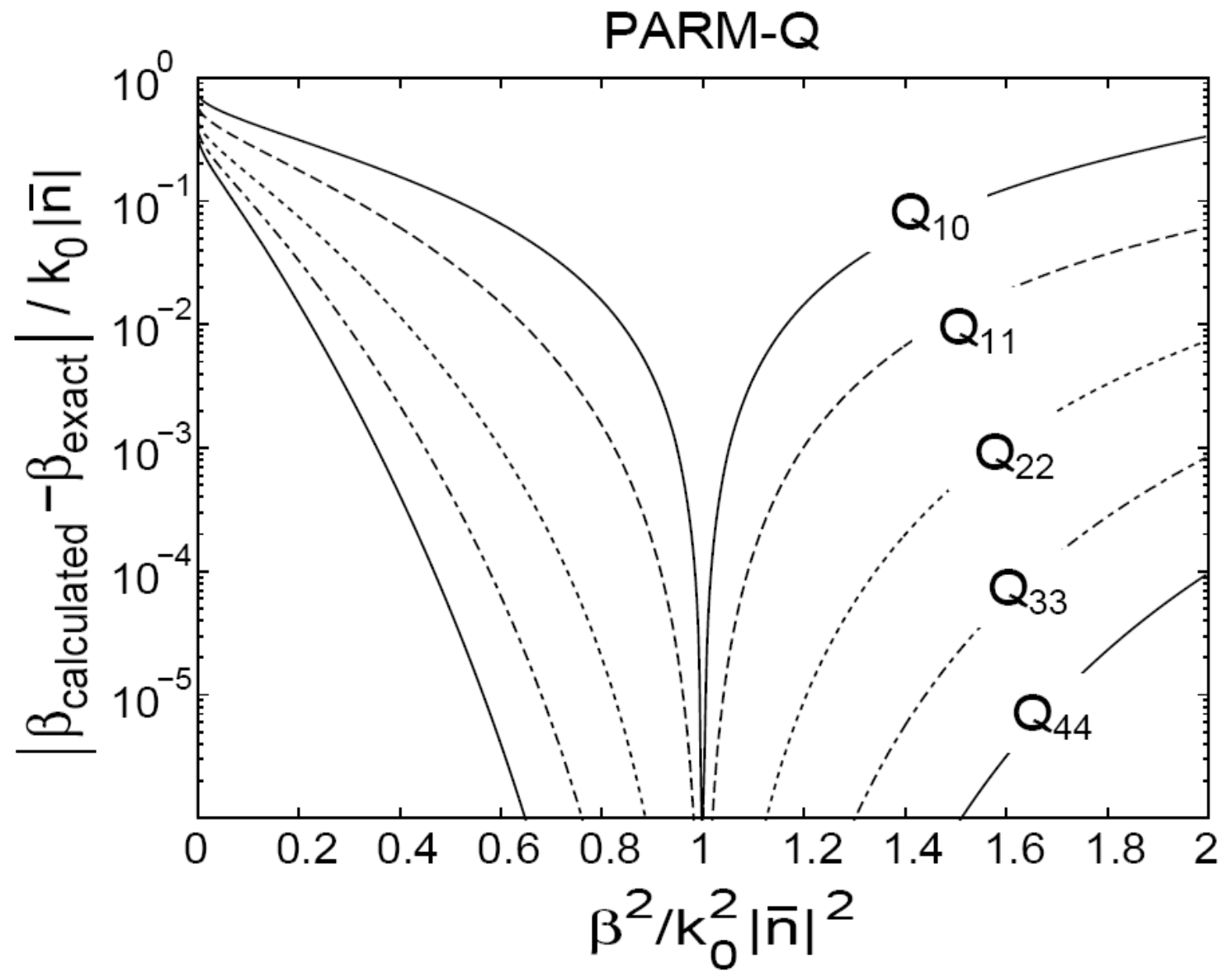
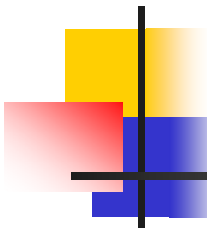
$$Q_{11} = k_0 \bar{n} \frac{1 + \frac{5}{8}[\Omega]}{1 + \frac{3}{8}[\Omega]}$$

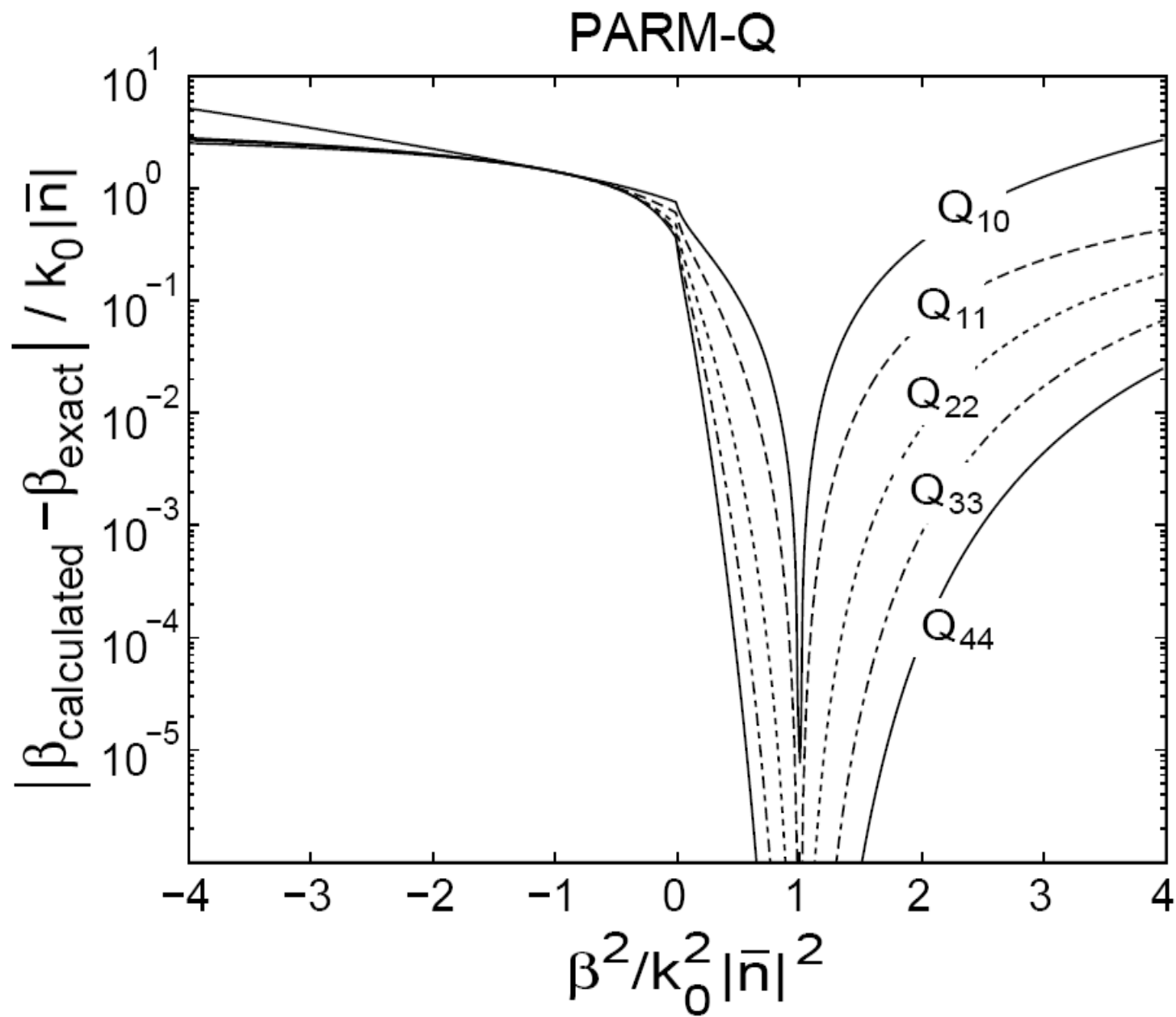
$$Q_{22} = k_0 \bar{n} \frac{1 + \frac{9}{8}[\Omega] + \frac{15}{64}[\Omega]^2}{1 + \frac{7}{8}[\Omega] + \frac{7}{64}[\Omega]^2}$$

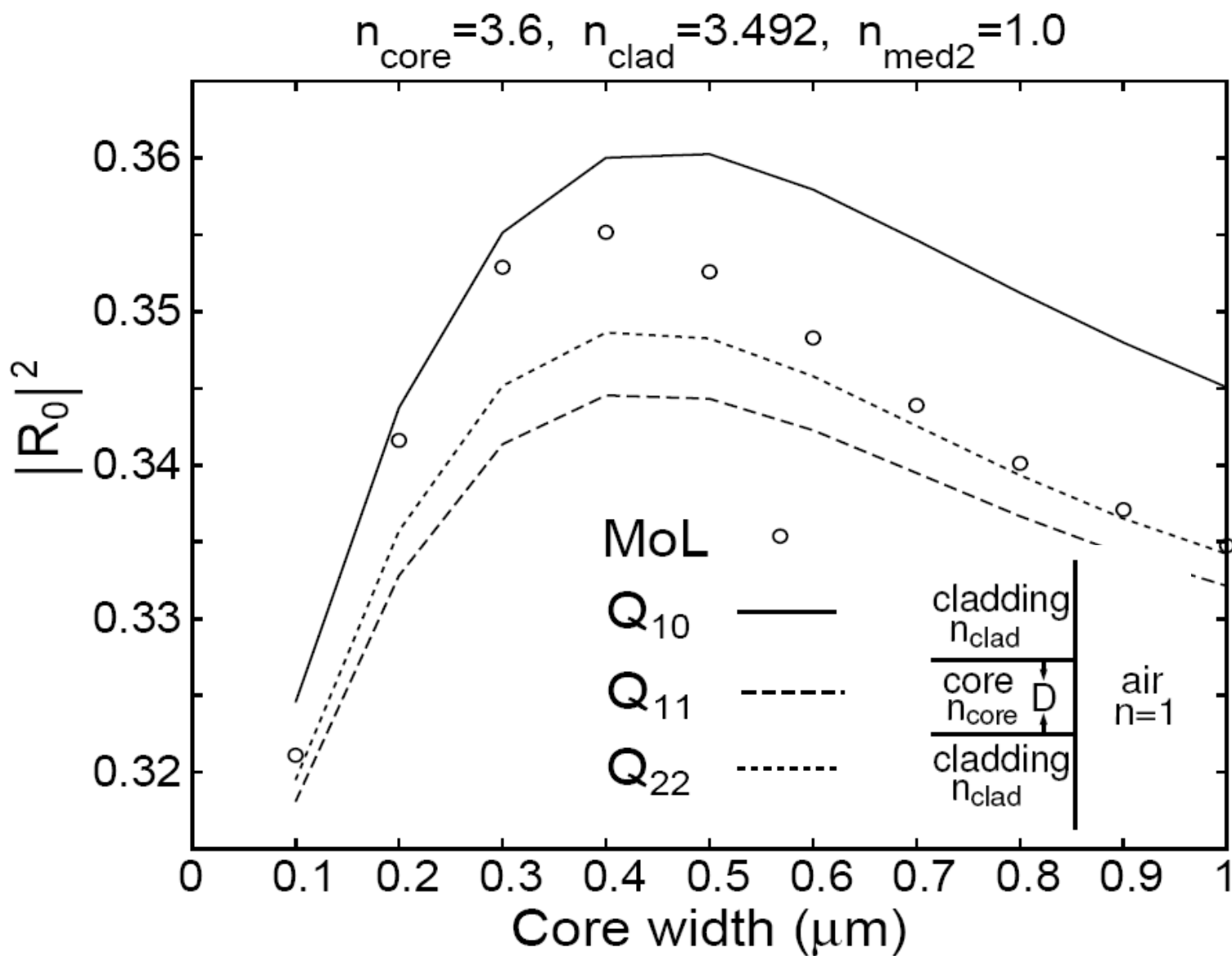
$$Q_{33} = k_0 \bar{n} \frac{1 + \frac{13}{8}[\Omega] + \frac{117}{160}[\Omega]^2 + \frac{39}{512}[\Omega]^3}{1 + \frac{11}{8}[\Omega] + \frac{77}{160}[\Omega]^2 + \frac{77}{2560}[\Omega]^3}$$

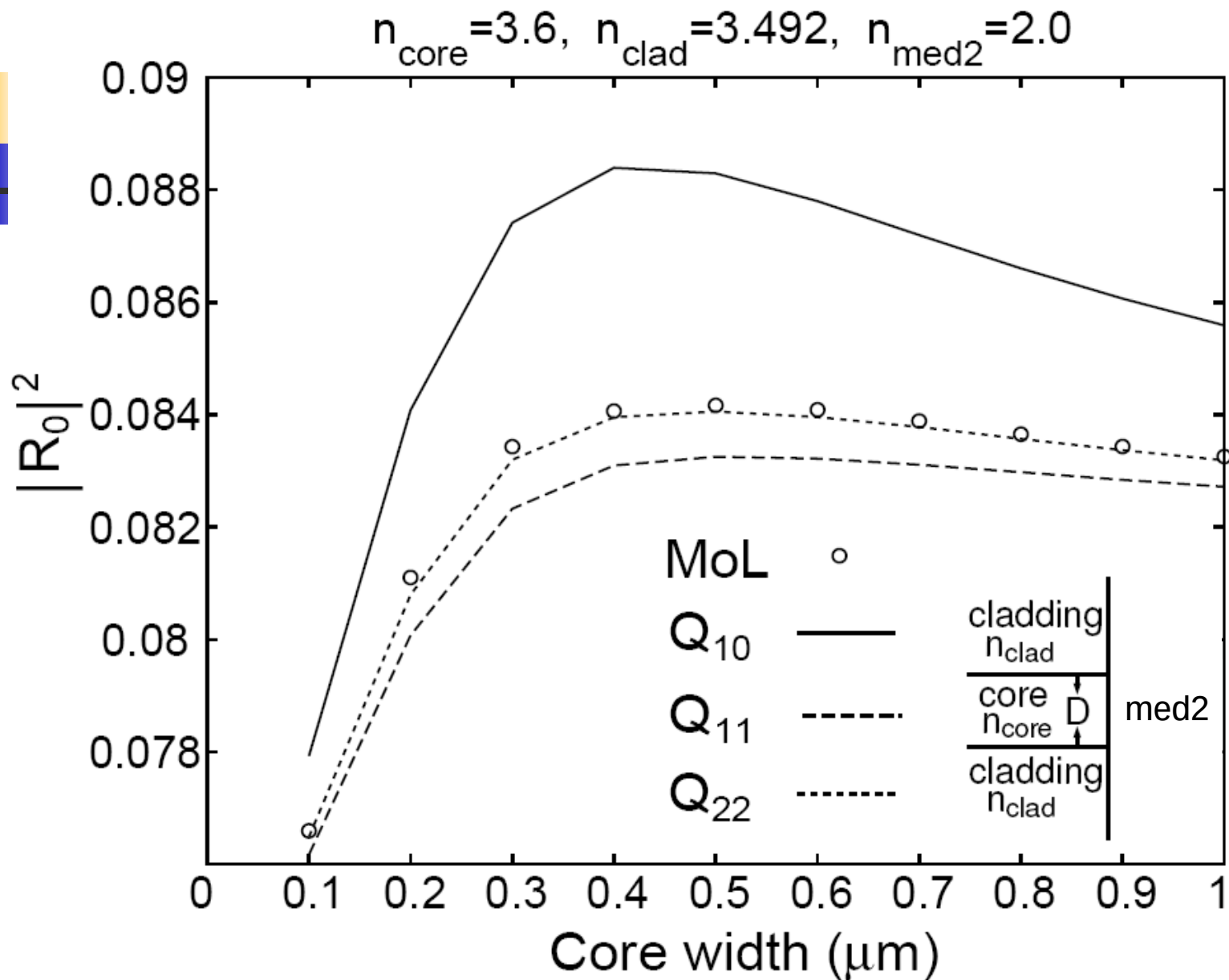
$$Q_{44} = k_0 \bar{n} \frac{1 + \frac{9}{4}[\Omega] + \frac{27}{16}[\Omega]^2 + \frac{15}{32}[\Omega]^3 + \frac{9}{256}[\Omega]^4}{1 + \frac{7}{4}[\Omega] + \frac{15}{16}[\Omega]^2 + \frac{5}{32}[\Omega]^3 + \frac{1}{256}[\Omega]^4}$$

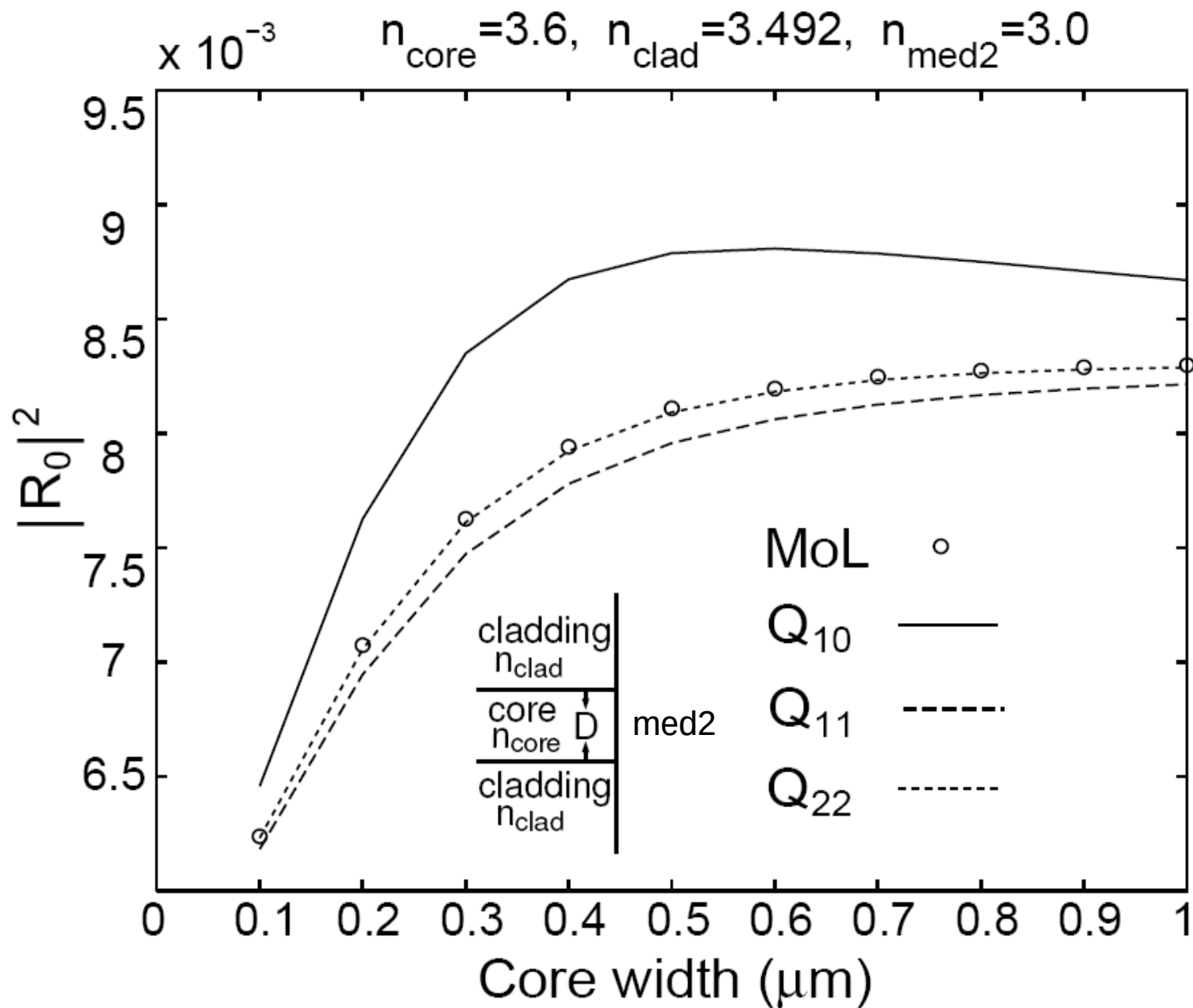
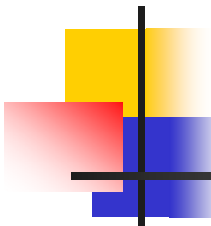
$$[\Omega] = (A^2 - k_0^4 \bar{n}^4) / k_0^4 \bar{n}^4$$

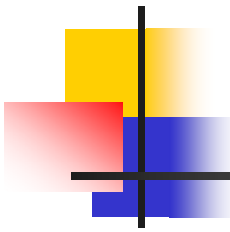












Summary

- Based on Padé approximation, an efficient and simple method for the transmission and reflection calculation of discontinuities is proposed.
- Time and memory consuming eigenvalue solving procedure is avoided.
- The reference index is real, and the evanescent waves are modeled as propagating waves.
- The proposed method works very well when the evanescent waves are negligible.