Simulation of Soliton Propagation in a Directional Coupler

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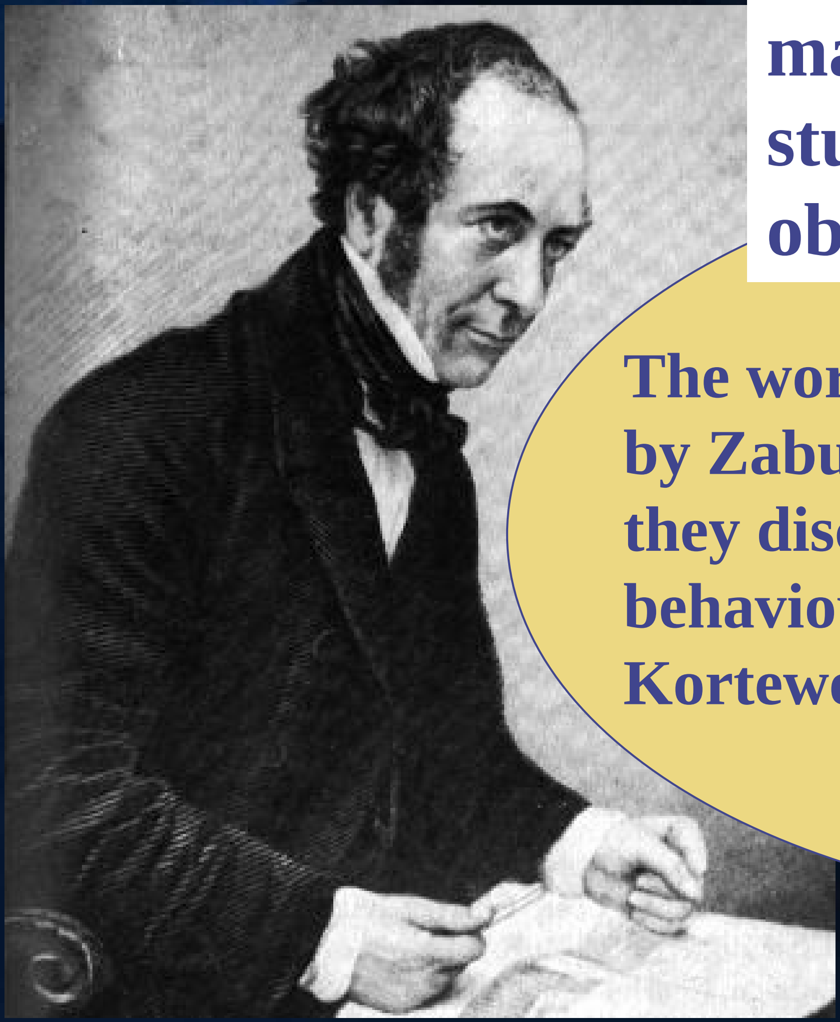
Outline

Introduction

Analysis

Results and Discussion

Solitons



In 1895, Korteweg and deVries developed a mathematical theory to study the solitary waves observed by Scott Russell.

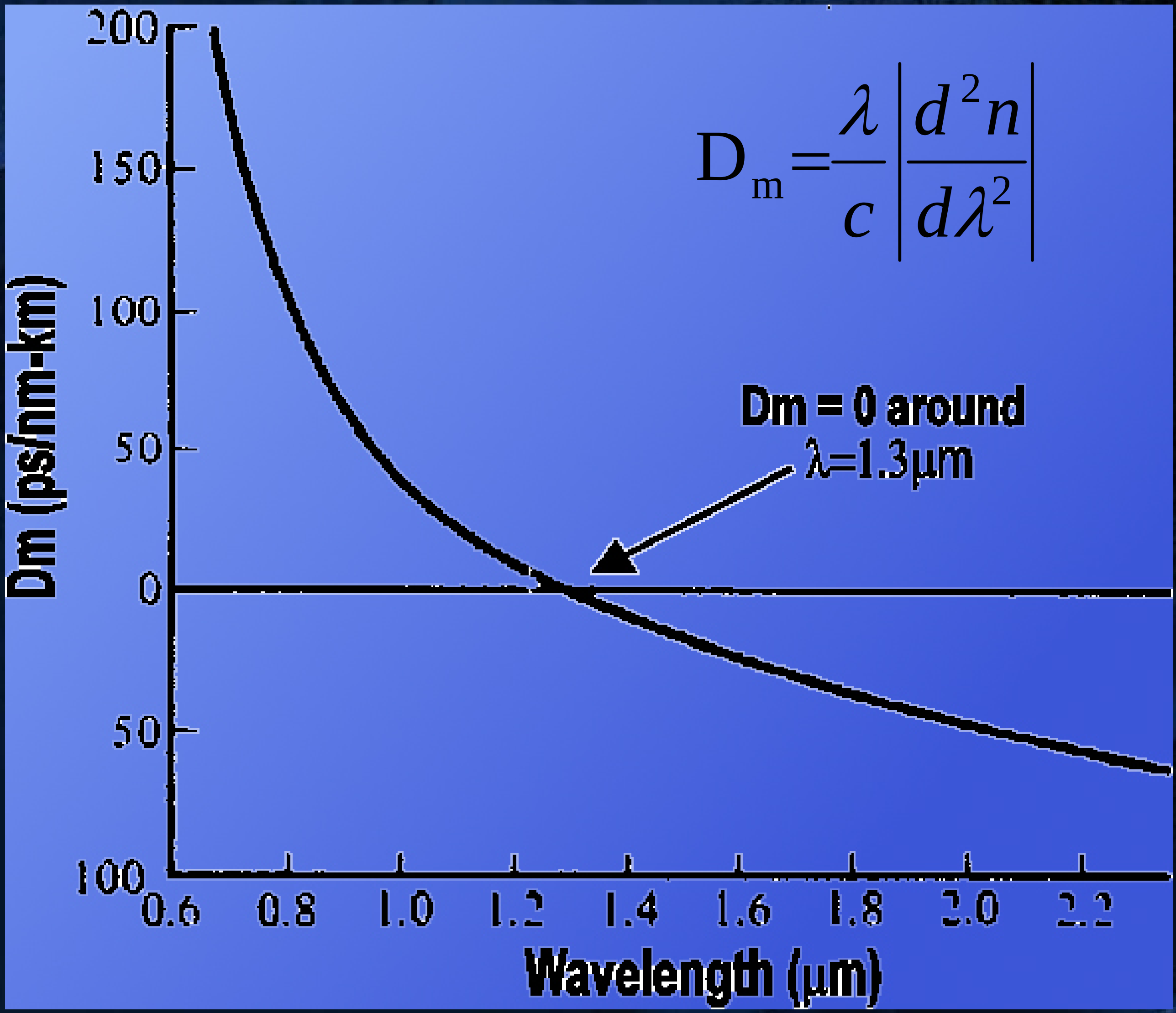
The word '*soliton*' was first introduced by Zabusky and Kruskal in 1964 when they discussed the particle-like behaviour of numerical solutions of the Korteweg deVries equation.

Solitons :

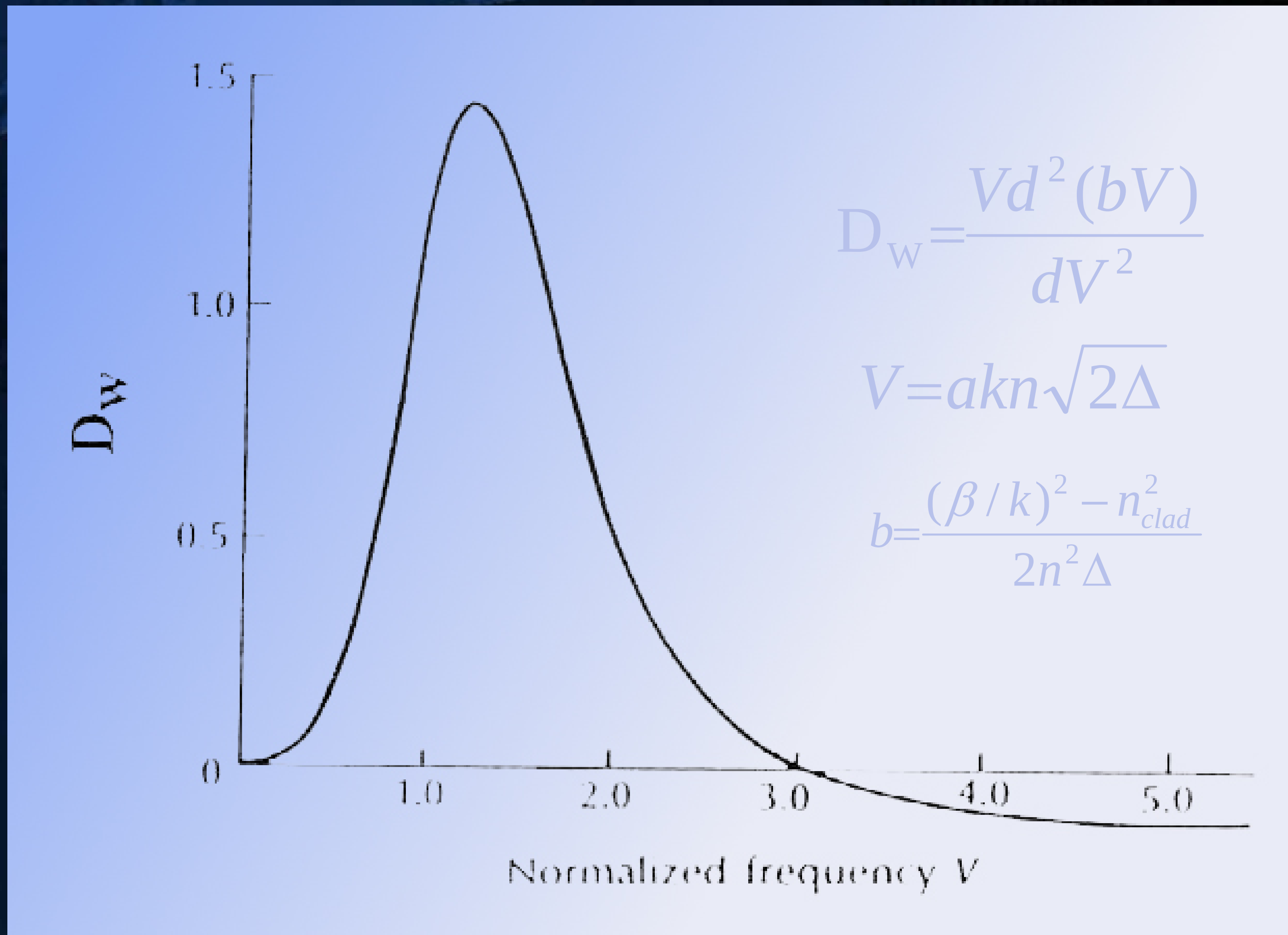
- **ion-acoustic waves in plasma,**
- **magneto hydrodynamic waves in plasma,**
- **anharmonic lattice,**
- **longitudinal dispersive waves in elastic rods,**
- **pressure waves in liquid-gas bubble mixtures,**
- **thermally excited photon packets in low-temperature nonlinear crystals,**
- **propagation of magnetic flux on a Josephson line,**
- **electrical signals in nonlinear transmission lines**
- **light waves in optical fibres**

Dispersion Effect

As an optical pulse travels along a fibre, the shorter wavelength components travel faster and the longer wavelength ones tend to fall behind. This produces a difference in frequencies between the leading and trailing edges of the pulse. This is the so-called "optical dispersion" which causes conventional optical pulses to <u>broaden</u> along the fibre thus <u>limiting the maximum bit rate</u>.



The material dispersion parameter for silica as a function of wavelength (Payne & Gambling 1975)

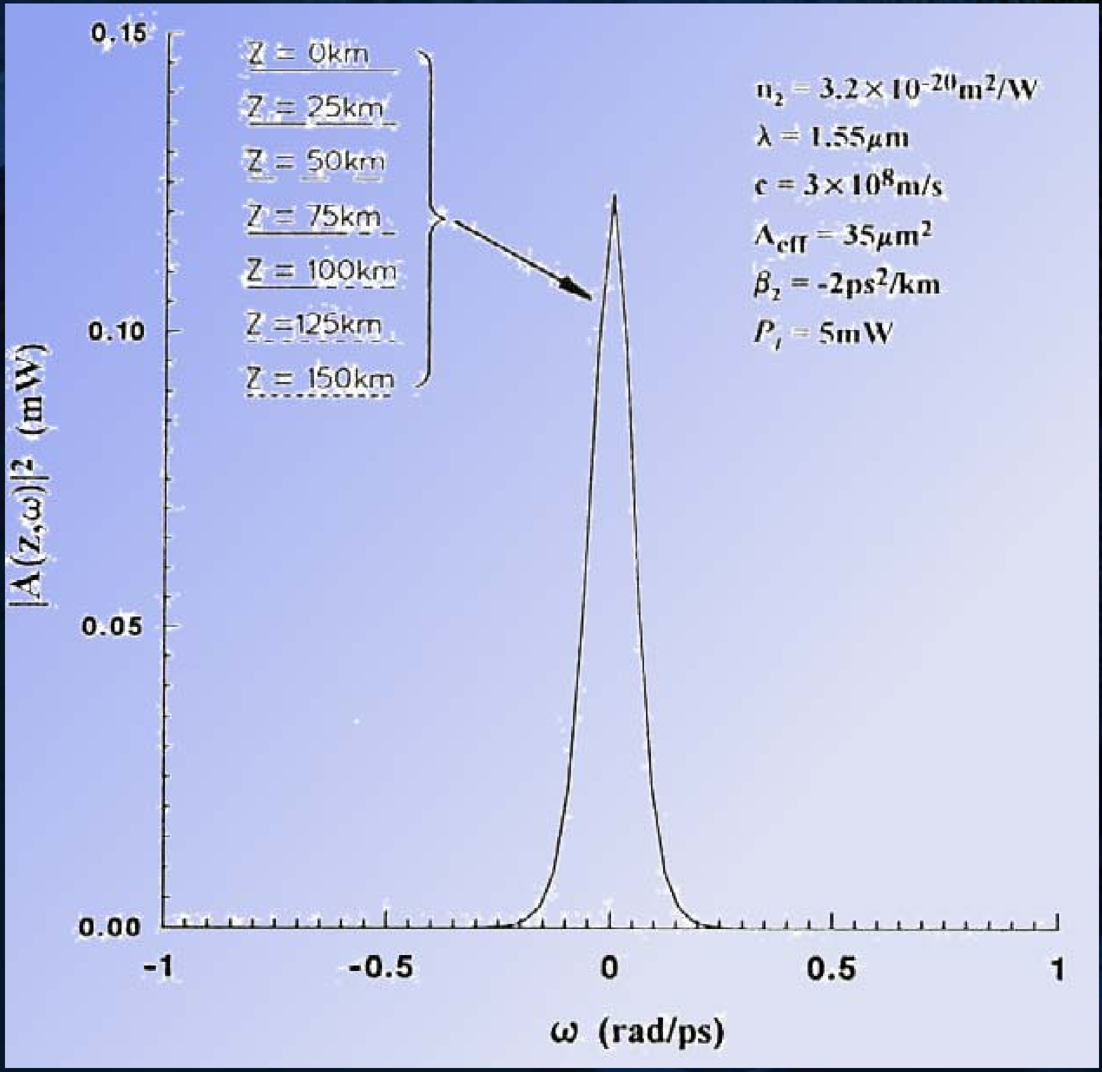


$$D_W = \frac{V d^2 (bV)}{dV^2}$$

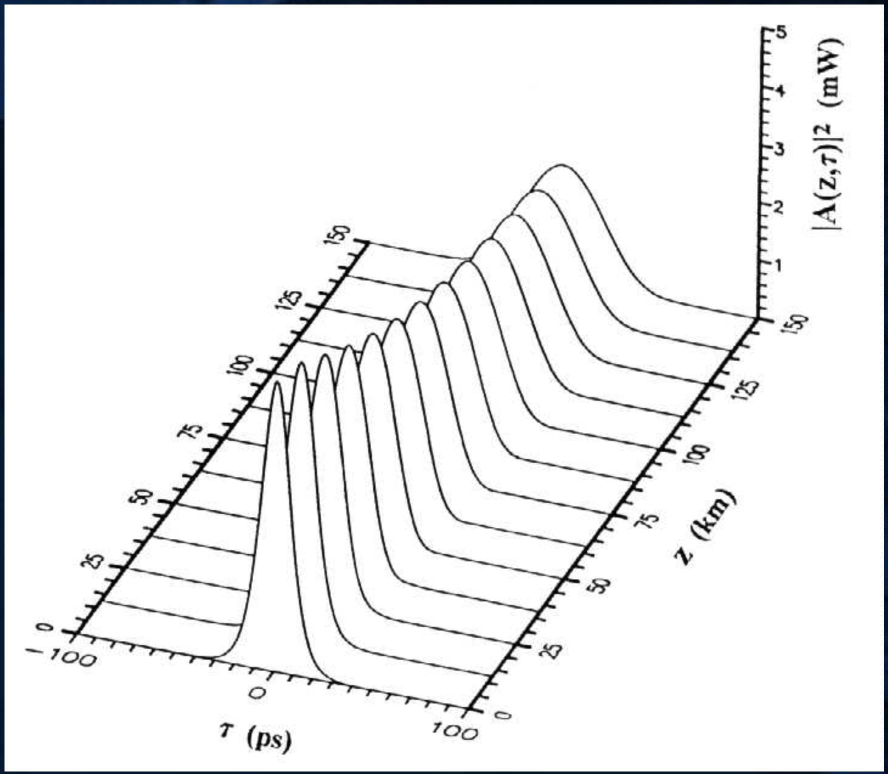
$$V = akn\sqrt{2\Delta}$$

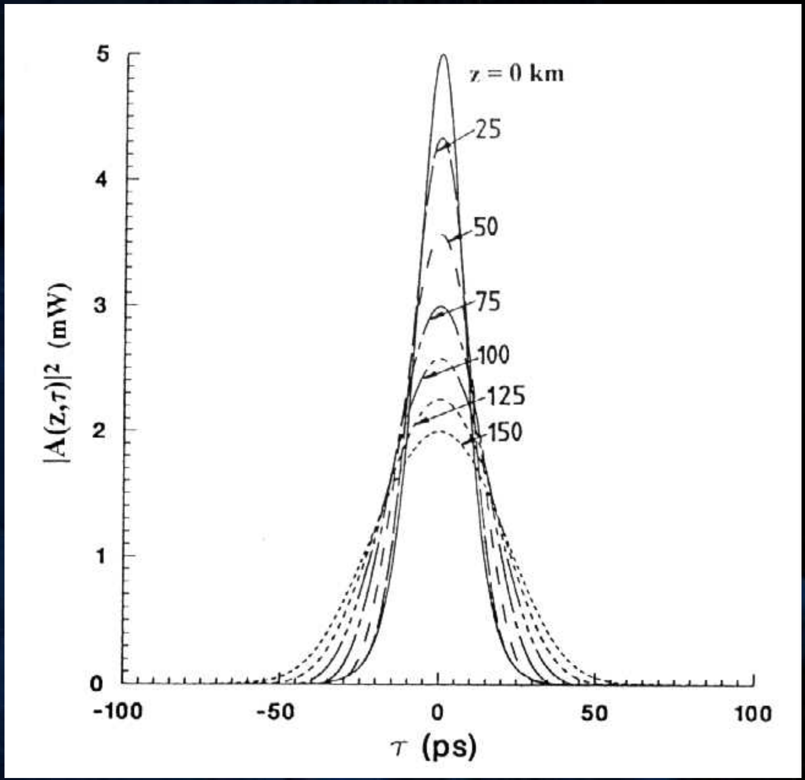
$$b = \frac{(\beta / k)^2 - n_{clad}^2}{2n^2\Delta}$$

The waveguide dispersion coefficient versus normalized frequency (Gambling *et al*. 1981)



Propagation of a fundamental soliton pulse in a purely dispersive optical fibre

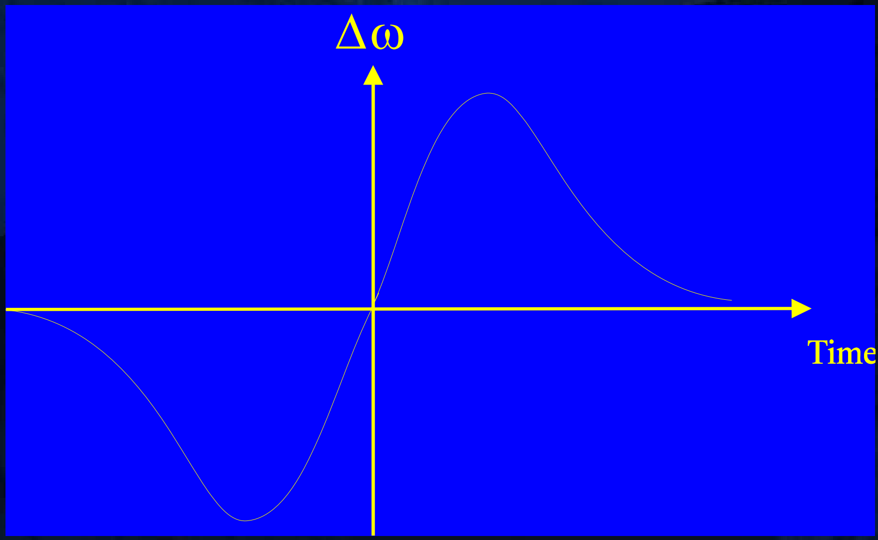


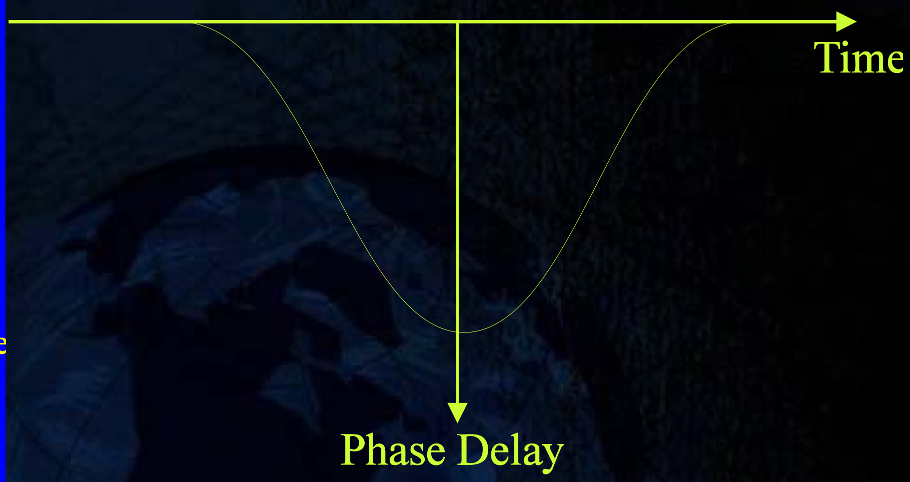


Propagation of a fundamental soliton pulse in a purely dispersive optical fibre

Nonlinear Effect

For a very intense optical pulse, the fibre refractive index increases as the intensity of the light increases. Named after a 19th century Scotsman, John Kerr, this nonlinear effect has come to be known as the Kerr optical effect (or the Kerr nonlinearity).

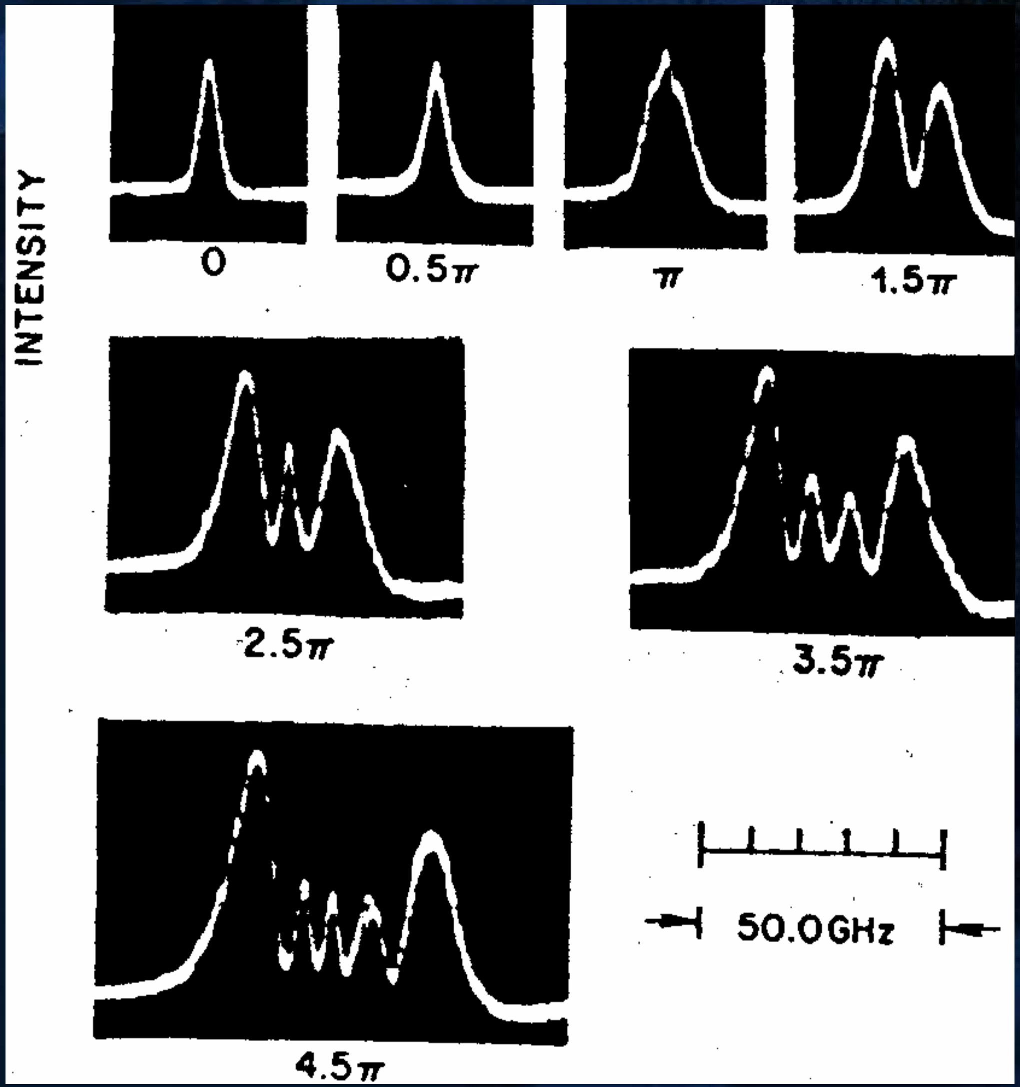




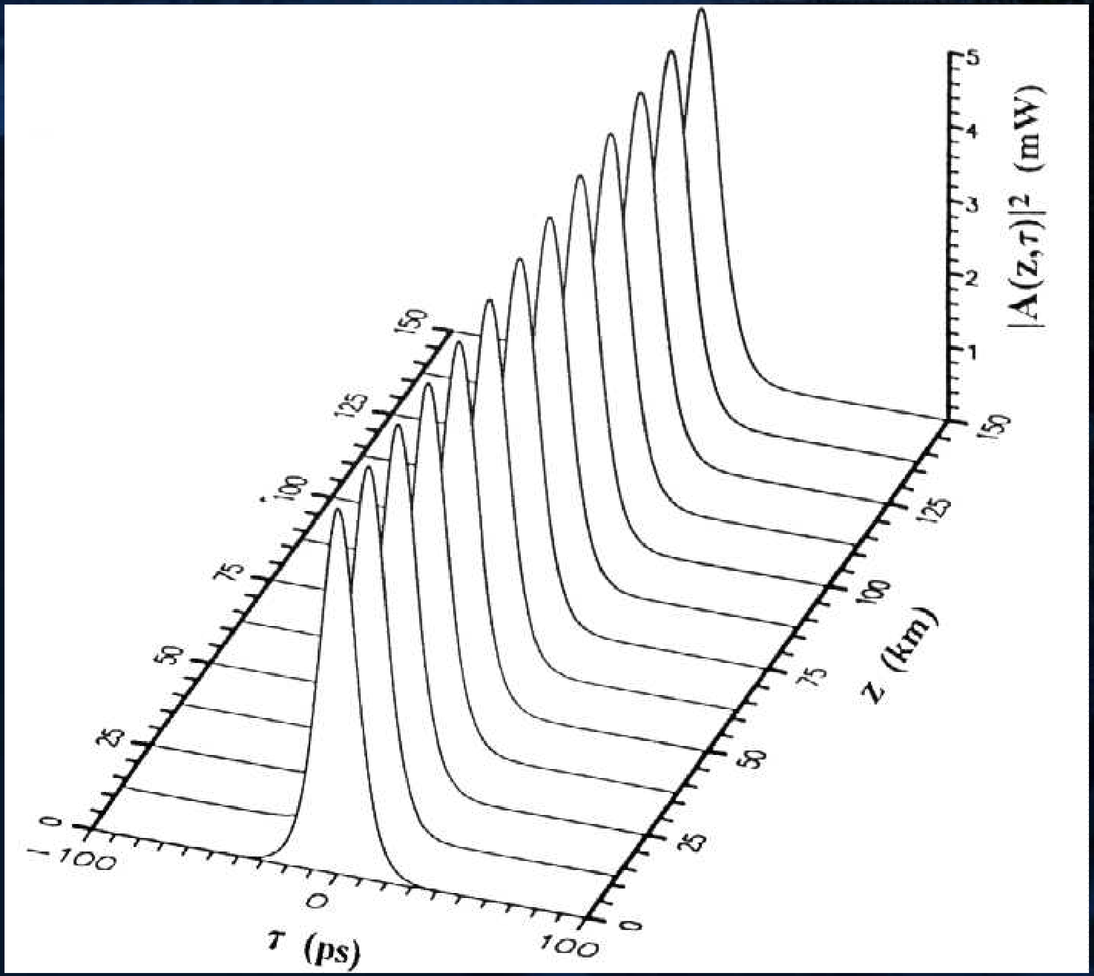
$$\Delta\omega = \frac{\partial \phi}{\partial T}$$

The time dependence of Δω can be treated as a frequency chirp which is due to the *Self-Phase Modulation*.

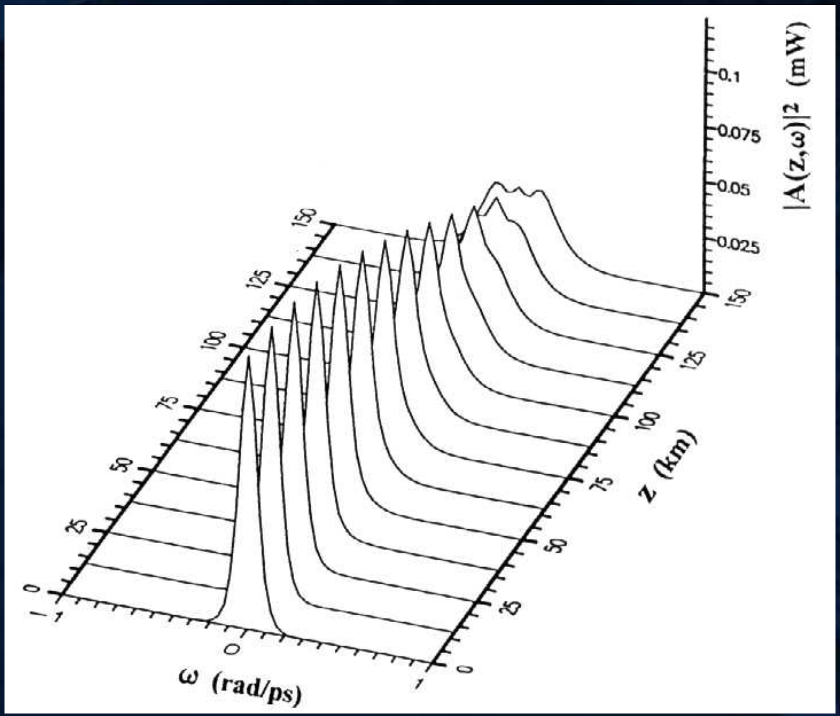
Self-phase modulation is the modulation of a pulse's own phase as the result of exciting the Kerr nonlinearity.

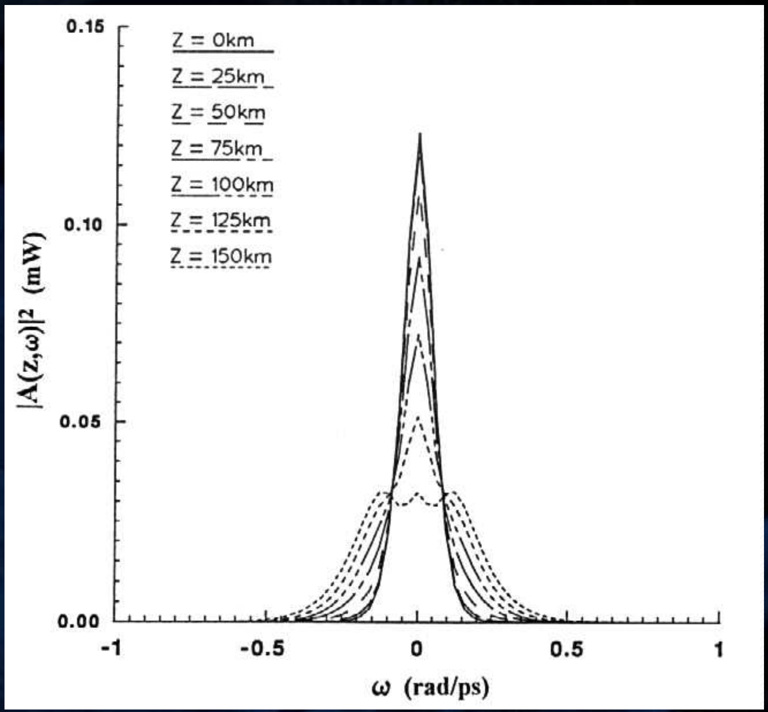


The frequency output spectra of a 3.35μm core diameter silica fibre (Stolen & Lin 1978)



Time domain solution for the propagation of soliton in an optical fibre with only nonlinear effect





Frequency domain solution for the propagation of soliton in an optical fibre with only nonlinear effect

Solitons can propagate without distortion for a very long distance if the propagating medium's nonlinear effect and its dispersion effect cancel each other.

A Brief History of Fibre Soliton

1834, John Scott Russell observe the solitary wave on water

1955-1975, theory of solitary wave propagation is developed

1973, Hasegawa and Tappert (optical soliton theory)

1980, Mollenauer (first experimental demonstration)

1987, P. Emplit made the first experimental observation of the propagation of a dark soliton

A Brief History of Fibre Soliton

1991, Bell Lab transmitted solitons error-free at 2.5 gigabits over more than 14,000 kilometers

1991, Ellis proposed the idea of dispersion manage (DM) soliton (help to reduce sideband instability and dispersive wave)

1993, Nakazawa (stable soliton transmission at 10Gbit/s over 180 million km)

A Brief History of Fibre Soliton

1995, Nakazawa successfully used part of the Tokyo metropolitan optical loop network to demonstrate the error free transmission of soliton at 20Gbit/s over 2000km

1996, Mollenauer (8x10Gbit/s transoceanic error free soliton WDM transmission)

1999, LeGuen (1.02Tbit/s soliton DWDM transmission over 1000km of standard fibre with 100km amplifier span)

A Brief History of Fibre Soliton

2000, Corvis Corp. filed for an IPO, and spent $100 million to acquire France-based Algety Telecom, which develops soliton- based DWDM transmission systems

2001, Algety Telecom. Practical soliton transmission system carrying real traffic data was deployed

2004, Koji Igarashi (shortest fiber soliton pulse, 15.6 fs)

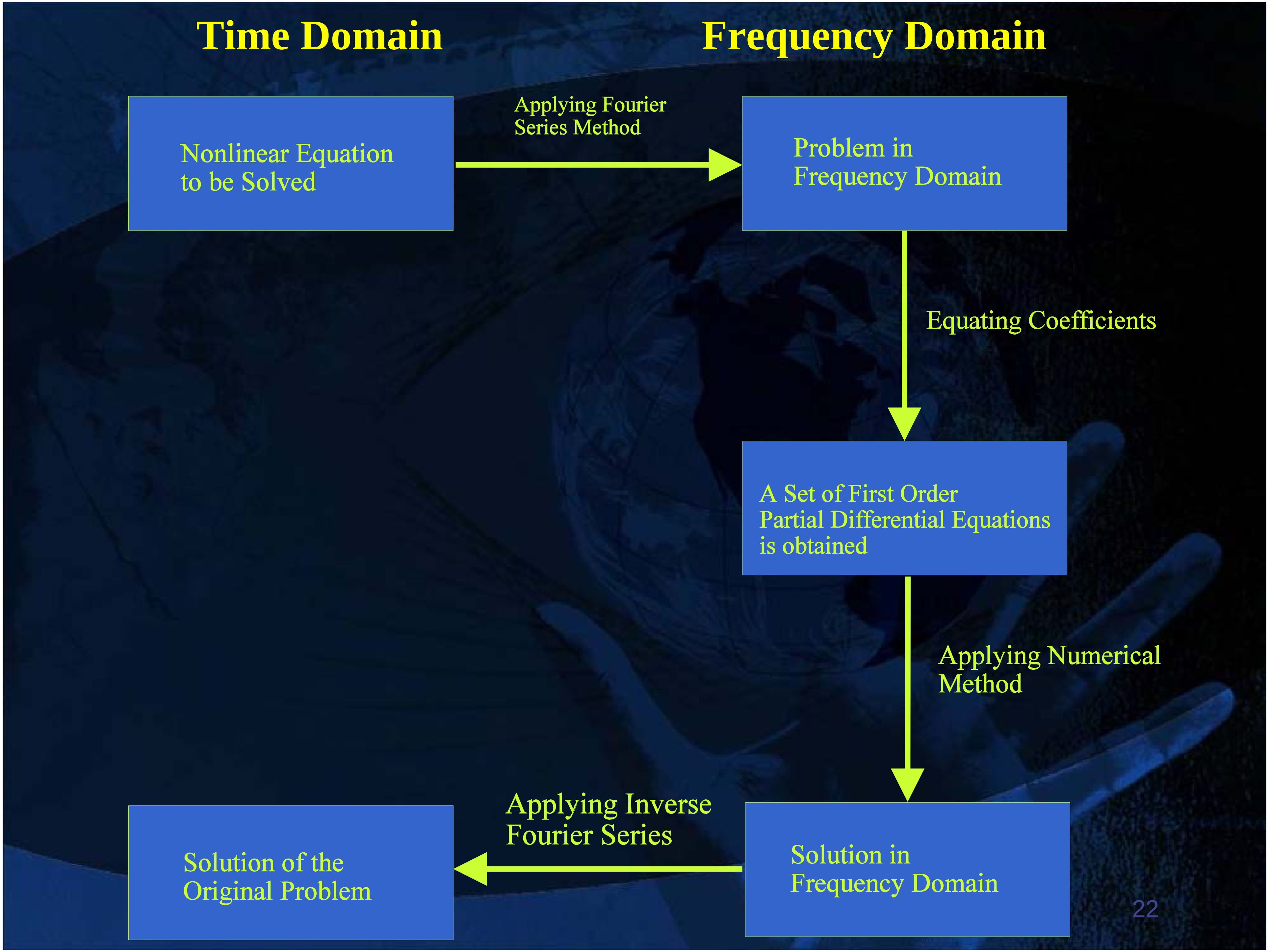
Soliton Propagation Equation

The general soliton propagation equation in an optical fibre can be expressed as

$$\frac{\partial u}{\partial x} = j\frac{1}{2}\frac{\partial^2 u}{\partial T^2} + B\frac{\partial^3 u}{\partial T^3} + j|u|^2 u - \Gamma u$$

Methods for Solving Nonlinear Soliton Propagation Equation

- Fourier Series Analysis Technique
- Inverse Scattering Method
- Split-Step Fourier Method
- Perturbation Method
- Finite Element Method

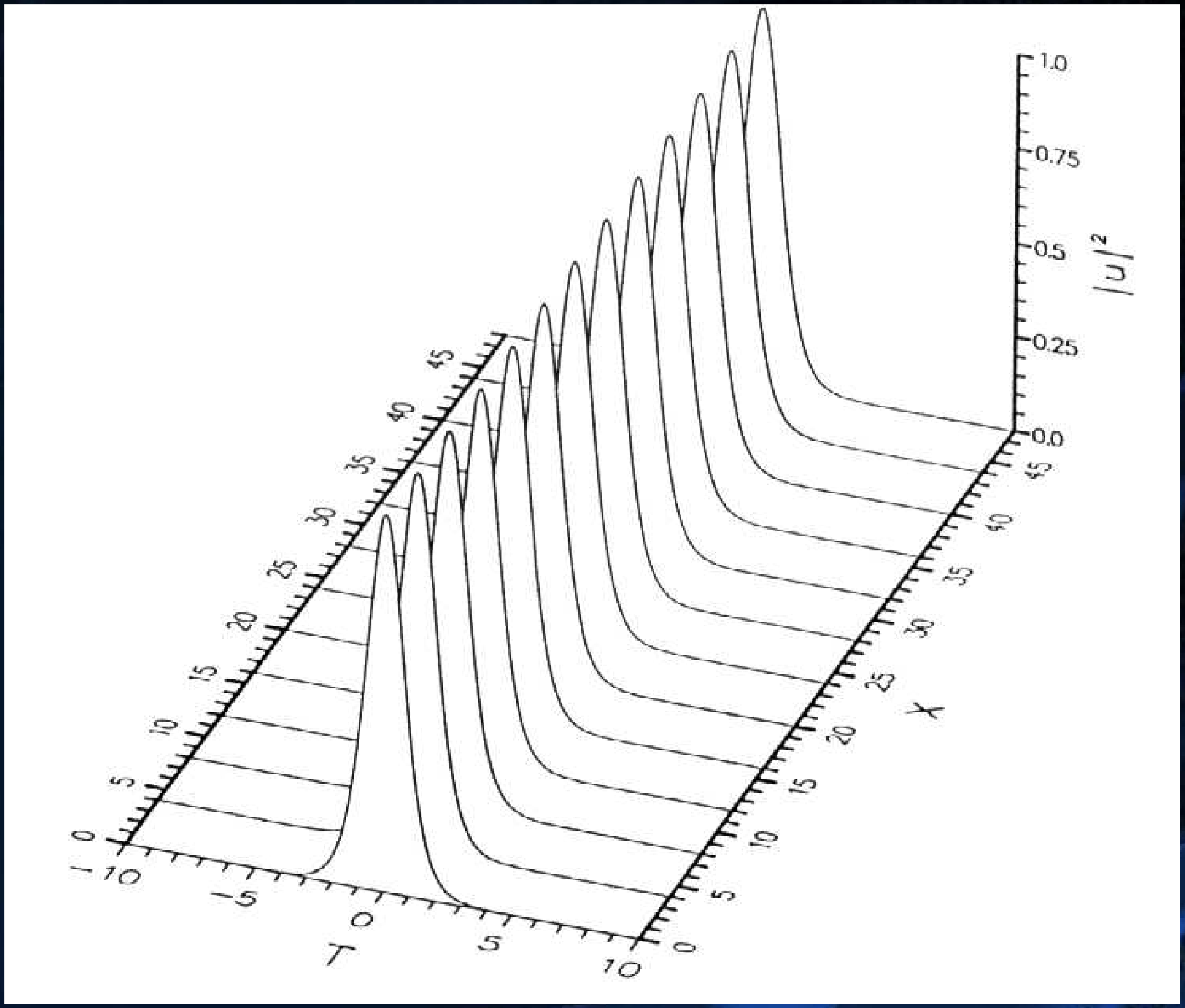

Time Domain
Frequency Domain
Nonlinear Equation to be Solved
Applying Fourier Series Method
Problem in Frequency Domain
Equating Coefficients
A Set of First Order Partial Differential Equations is obtained
Applying Numerical Method
Solution in Frequency Domain
Applying Inverse Fourier Series
Solution of the Original Problem

Inverse Scattering Method

$$\frac{\partial u}{\partial x} = j\frac{1}{2}\frac{\partial^2 u}{\partial T^2} + j|u|^2 u$$

The inverse scattering method (ISM) was introduced by Gardner *et al*. and used by Zakharov and Shabat, which can be used to solve the above nonlinear Schrodinger equation. The ISM <u>maps the solution of the nonlinear partial differential equation on solutions of linear differential equations</u>. The linear equations can be solved by standard methods. The transformation back yields the solution of the nonlinear equation for arbitrary initial conditions.

Fundamental soliton solution

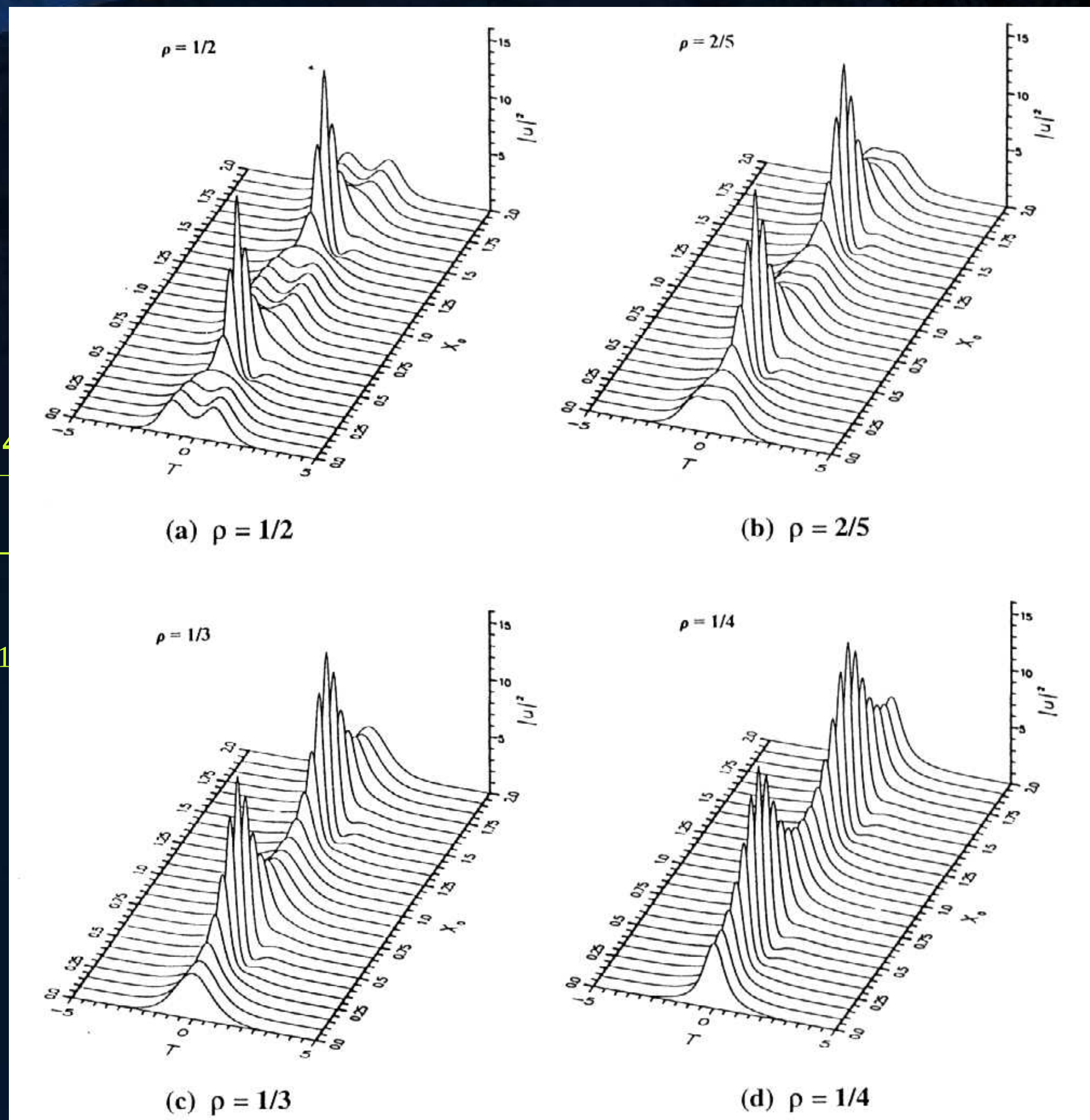


Second order soliton solution

$-\eta_1^2)x\,]$

$u(x,T)=$

$\cosh[2(\eta_1$ … $s[2(\eta_2^2-\eta_1^2)x]$



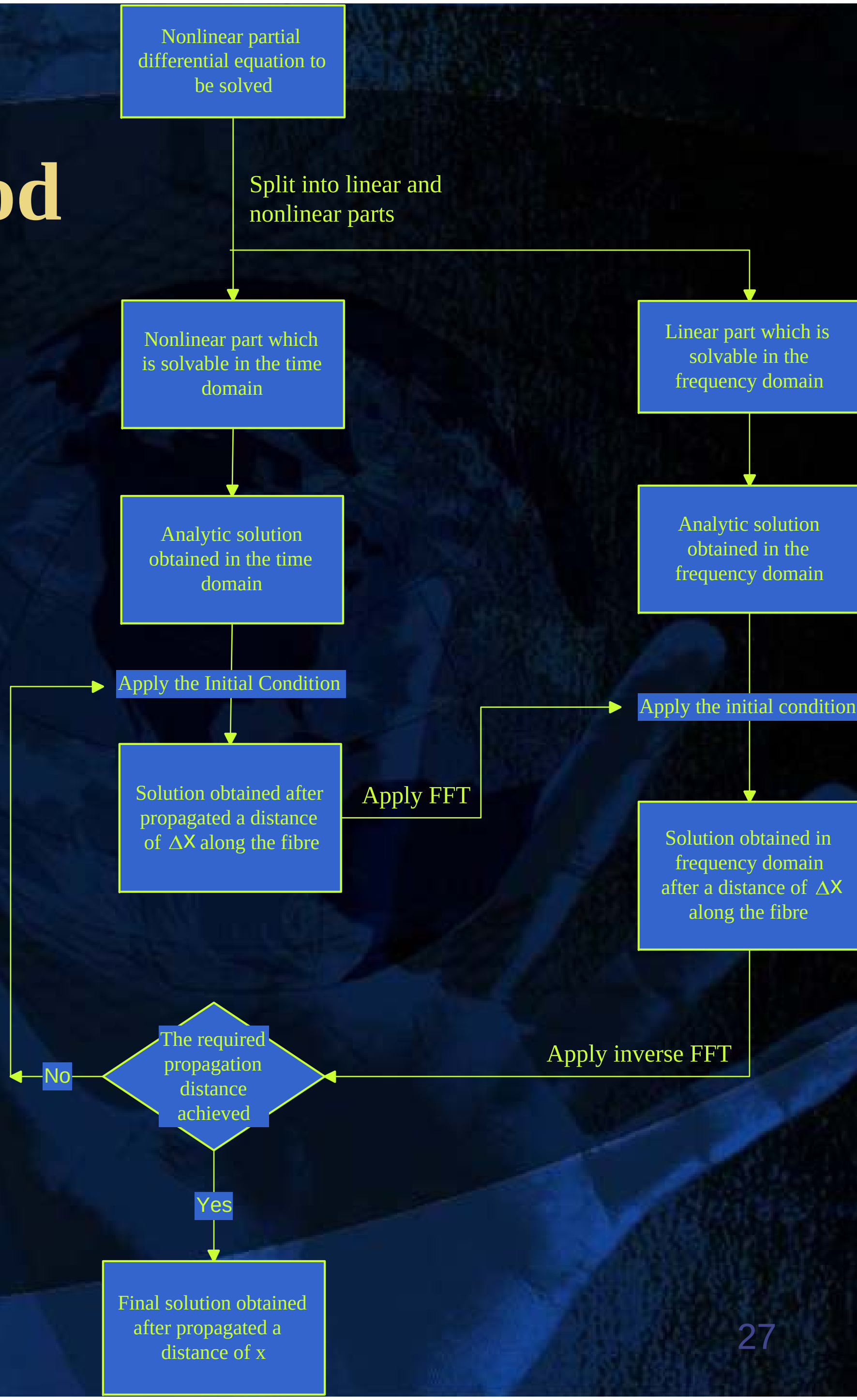
(a) ρ = 1/2 (b) ρ = 2/5 (c) ρ = 1/3 (d) ρ = 1/4

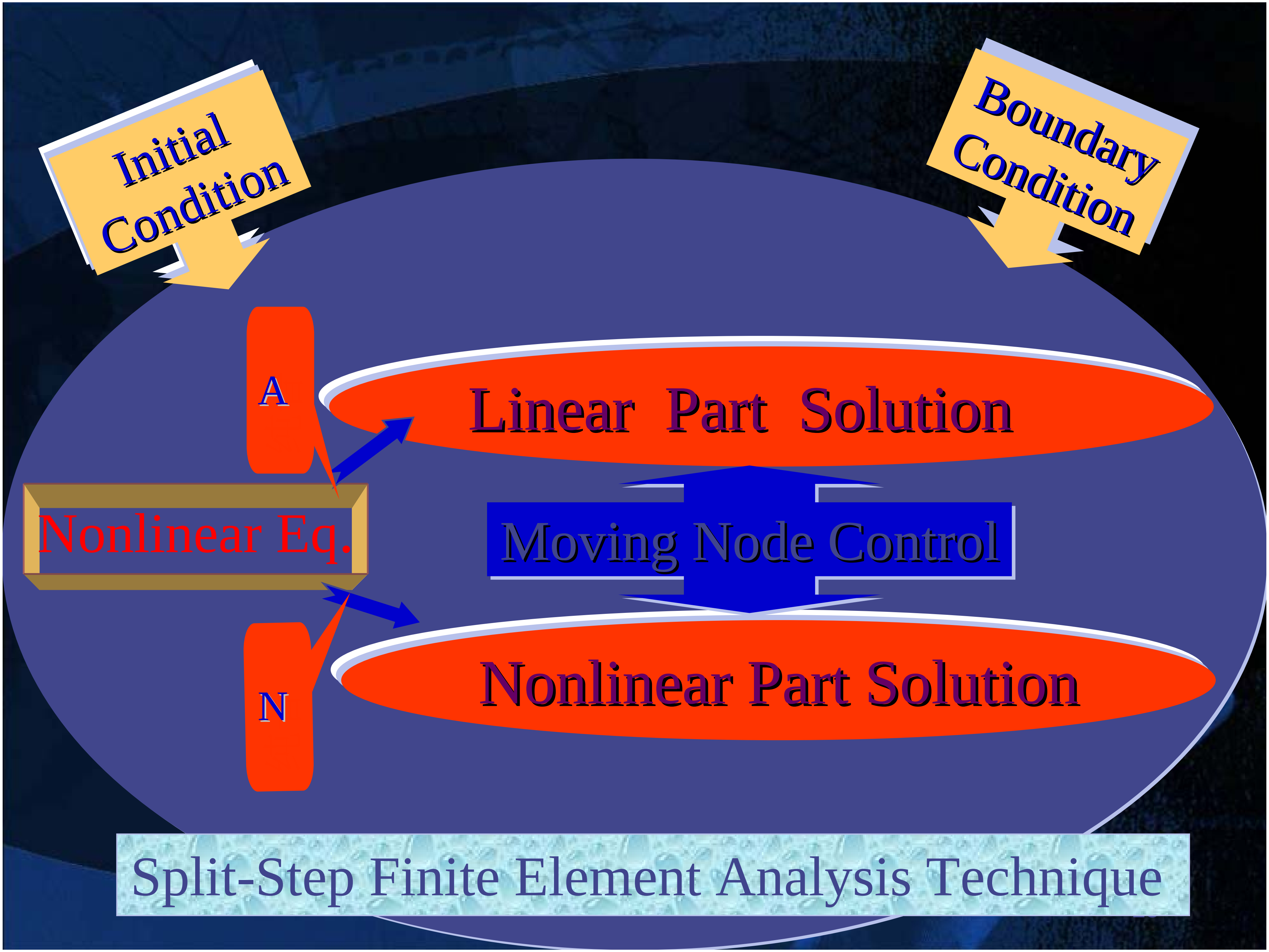
Perturbation Method

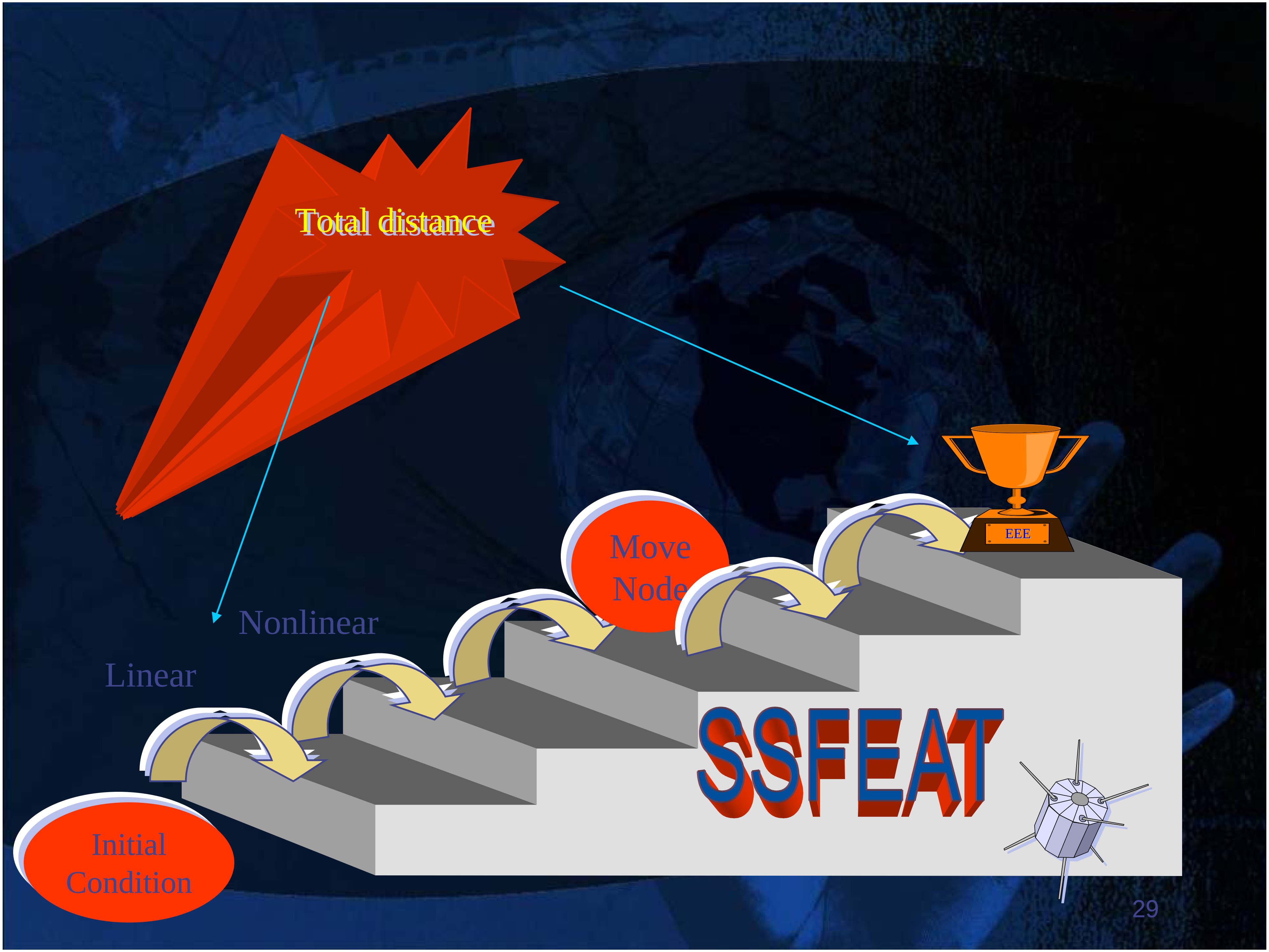
In some cases, an exact analytic solution cannot be obtained by ISM, hence the PM should be used to take the effect of higher order terms and fibre loss into account. Due to the limitation of the PM, the value of the loss factor Γ cannot be greater than a certain value (typically 〈0.015) otherwise inaccurate results will be obtained. For a conventional fibre with fibre loss 0.2dB/km and an input soliton pulse width of 6ps, the value of Γ is calculated to be 0.04. This value of Γ is obviously much larger than 0.015, hence the PM cannot be applied in this practical case.

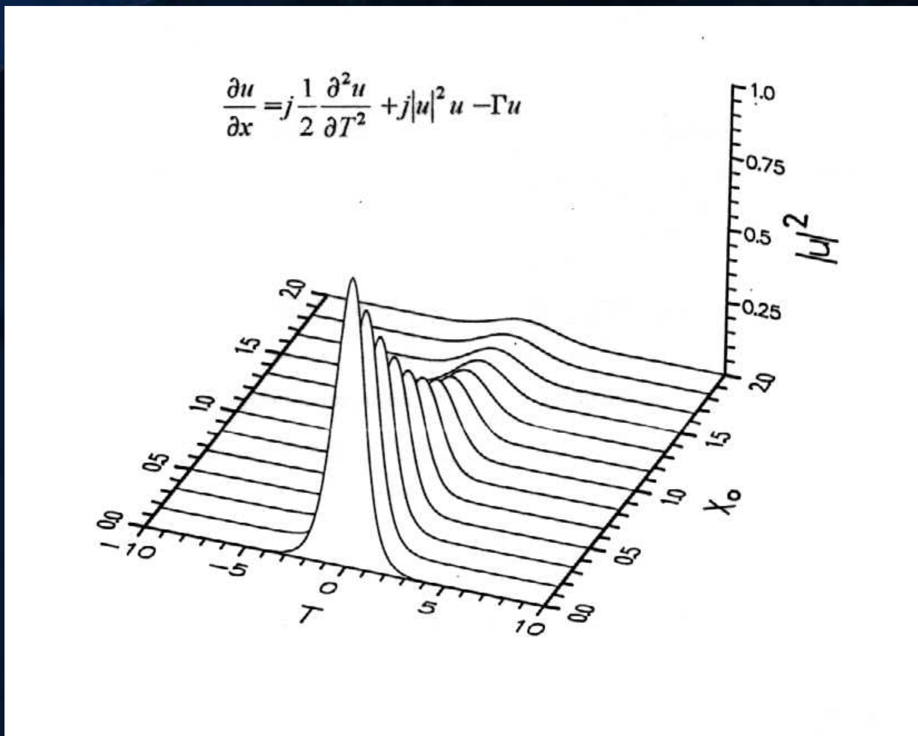
Split-Step Fourier Method

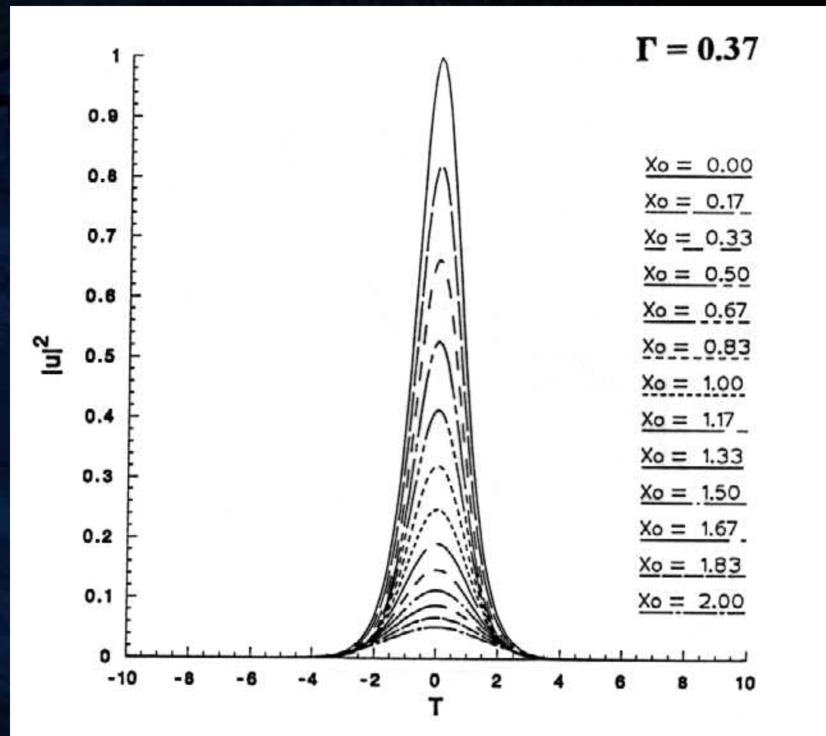
A large number of sampling points is required by the SSFM (〉200) and this method is highly numerical. The FFT is heavily used to transform solutions between time and frequency domains after every small propagation distance of Δx (since error is proportional to Δx^2, which means Δx should be smaller then 10^{-3} in order to maintain an accuracy of 10^{-6}). For a propagation distance of one soliton period, we may well need to use the FFT as many as 3000 times and cumulative errors due to the heavy use of FFT may occur




Initial Condition
Boundary Condition
A
Linear Part Solution
Nonlinear Eq.
Moving Node Control
N
Nonlinear Part Solution
Split-Step Finite Element Analysis Technique


Total distance
Nonlinear
Linear
Move
Node
EEE
SSFEAT
Initial
Condition



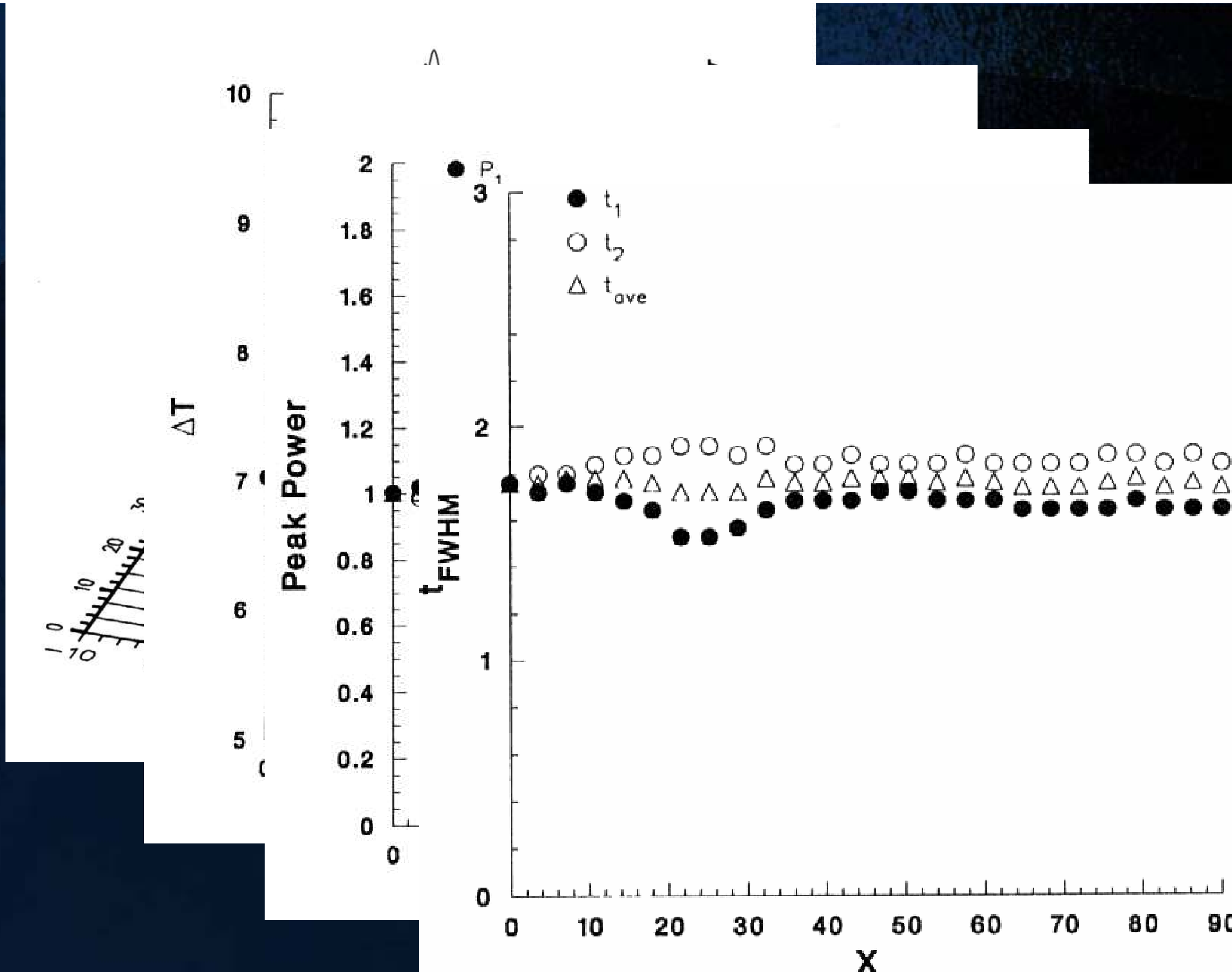


Propagation of Soliton in a Dispersion-Shifted Fibre (Γ=0.37)

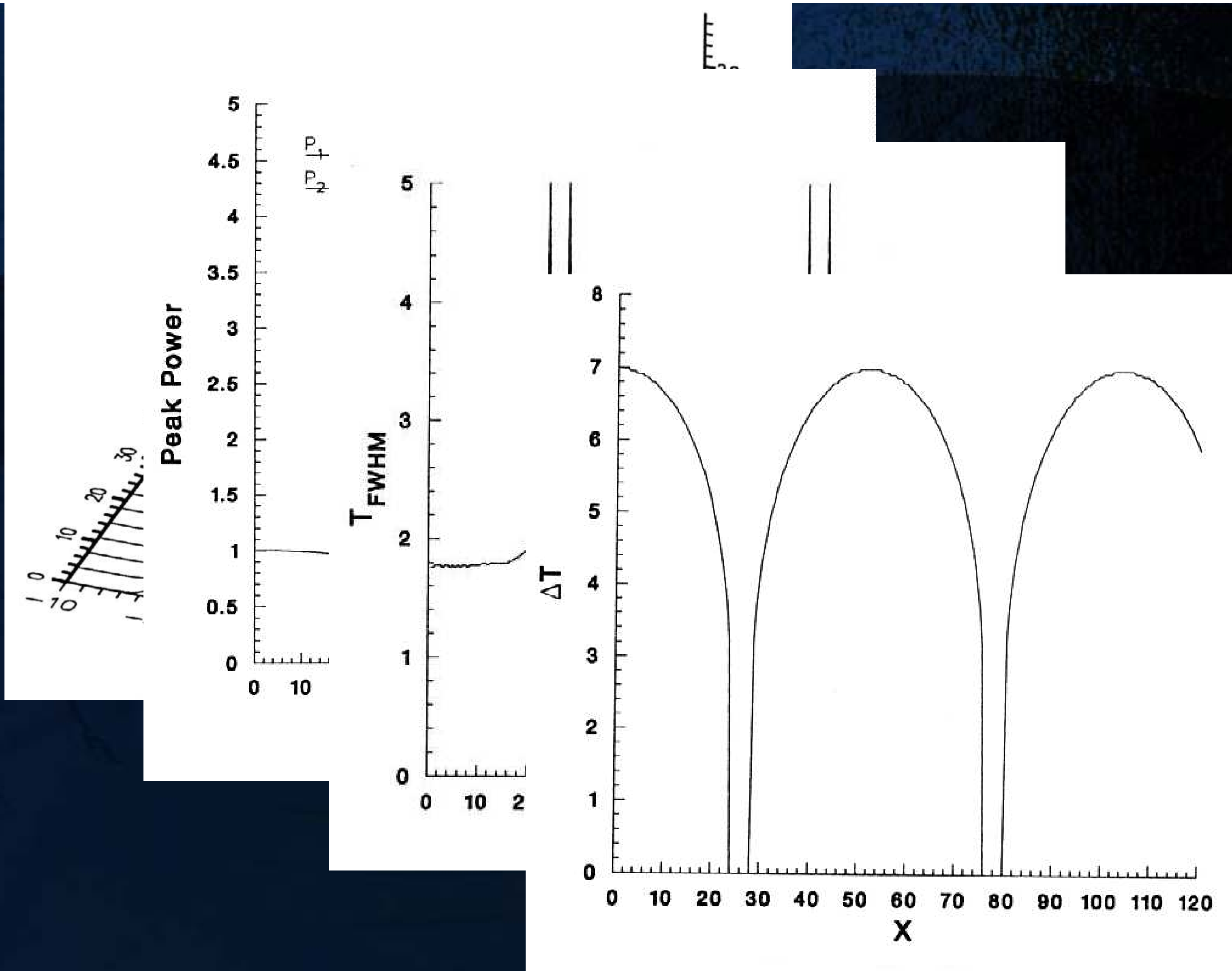
Soliton Interaction

Optical solitons have been considered as potential information carriers in high bandwidth optical fibre communication systems. In a soliton digital communication system, it is necessary to determine the optimum separation between adjacent solitons.

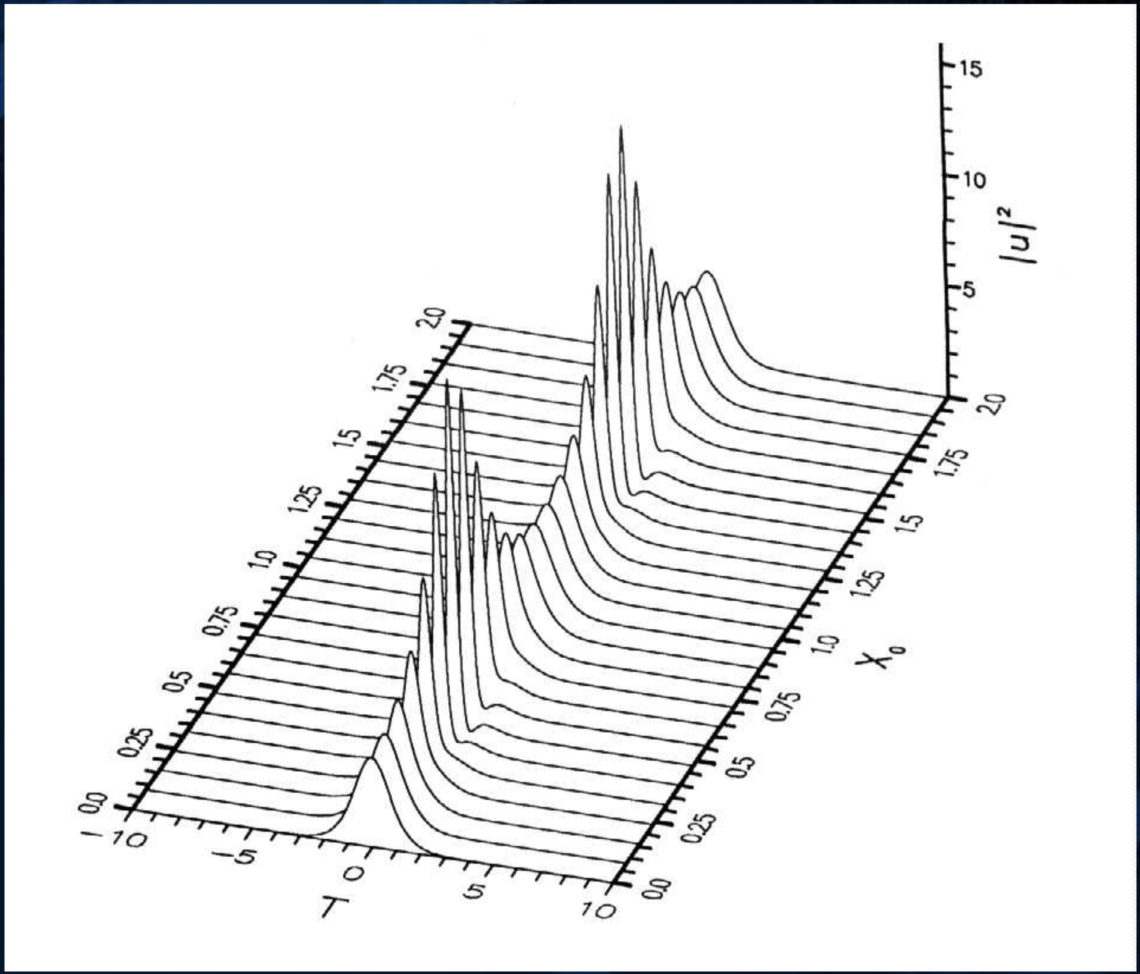
$$u(0,T) = A_1 \operatorname{sech}\left(T - \frac{\Delta T}{2}\right) + A_1 \operatorname{sech}\left(T + \frac{\Delta T}{2}\right) \exp(j\phi)$$



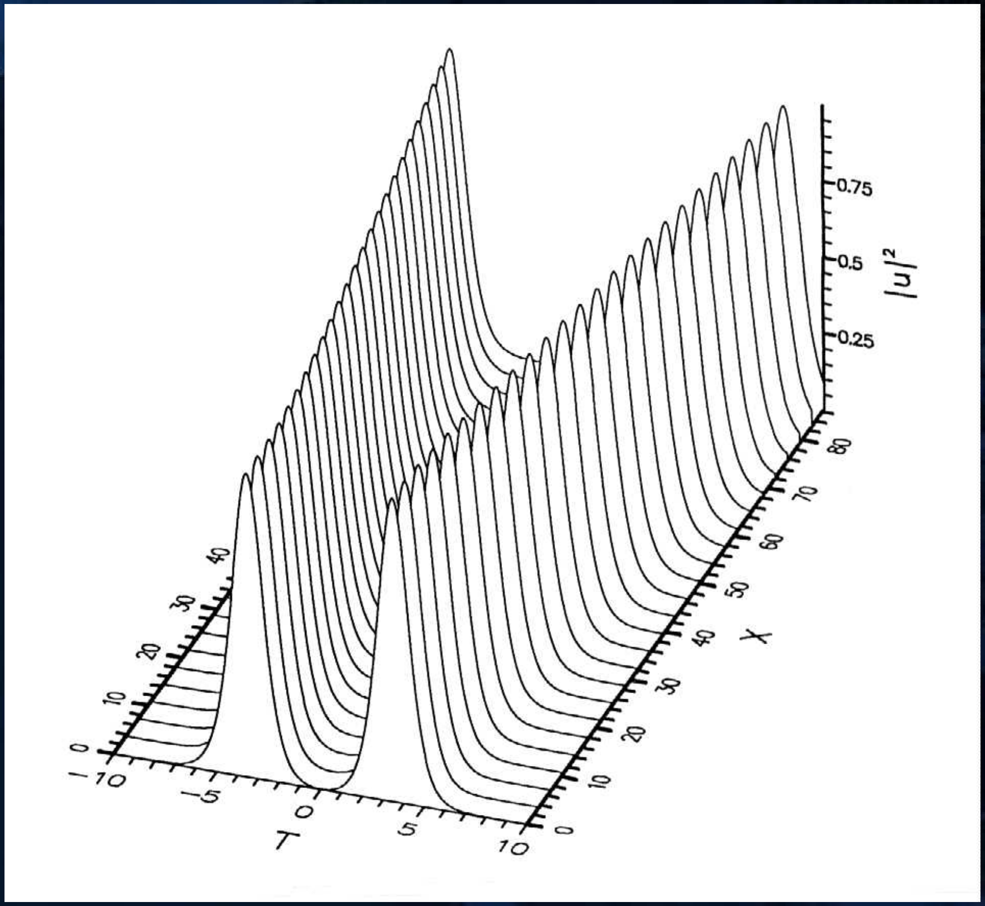
Soliton Interaction ($\phi=\pi/4$, $\Delta T=7$, $B=\Gamma=0$)



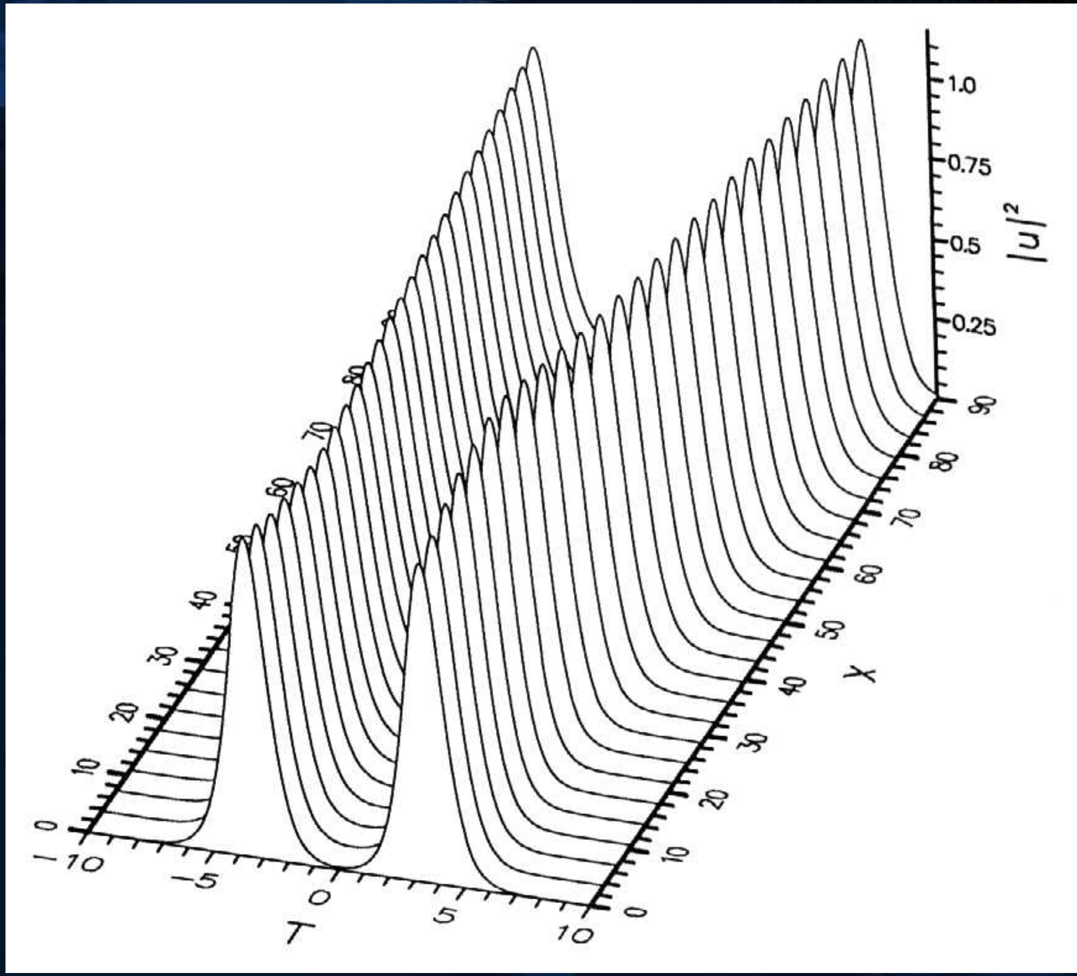
Soliton Interaction (ϕ=0, ΔT=7, B=Γ=0)



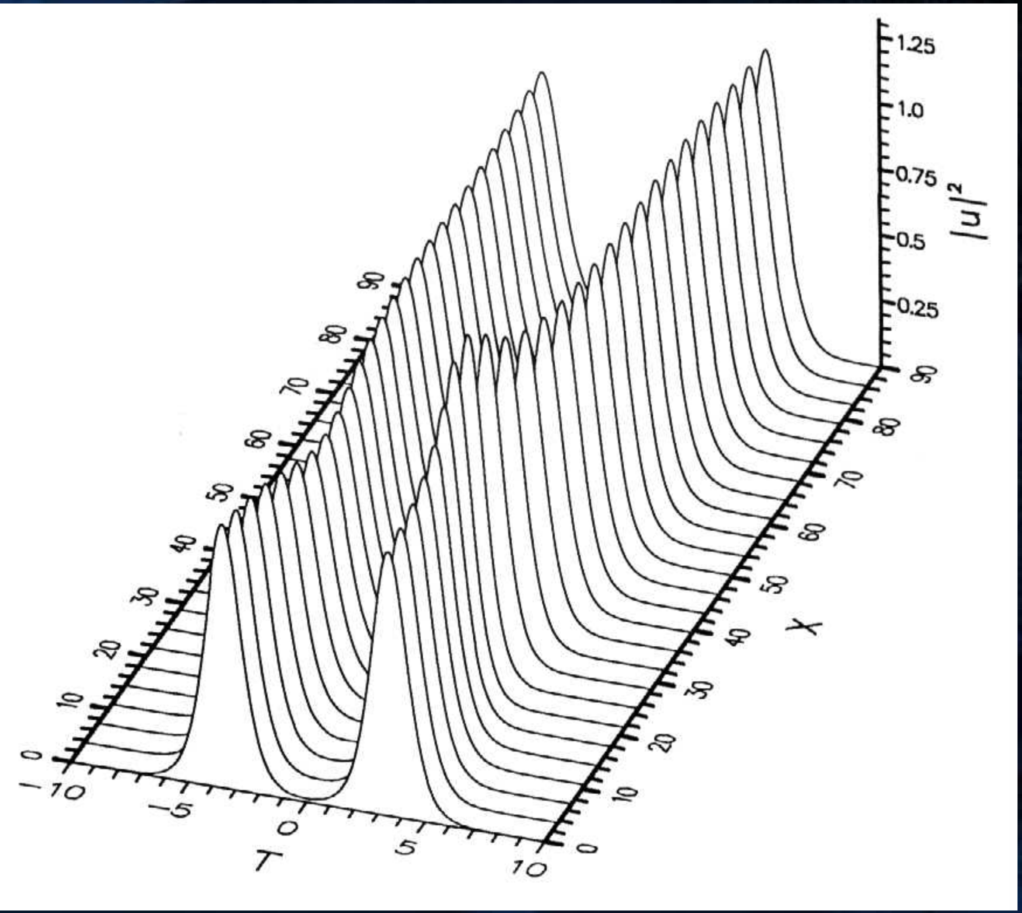
Propagation of two fundamental solitons in a lossless fibre
(ϕ=0, ΔT=0, B=Γ=0)



Propagation of two fundamental solitons in a lossless fibre (ϕ=-π, ΔT=7, B=Γ=0)



Propagation of two fundamental solitons in a lossless fibre (ϕ=-π/2, ΔT=7, B=Γ=0)

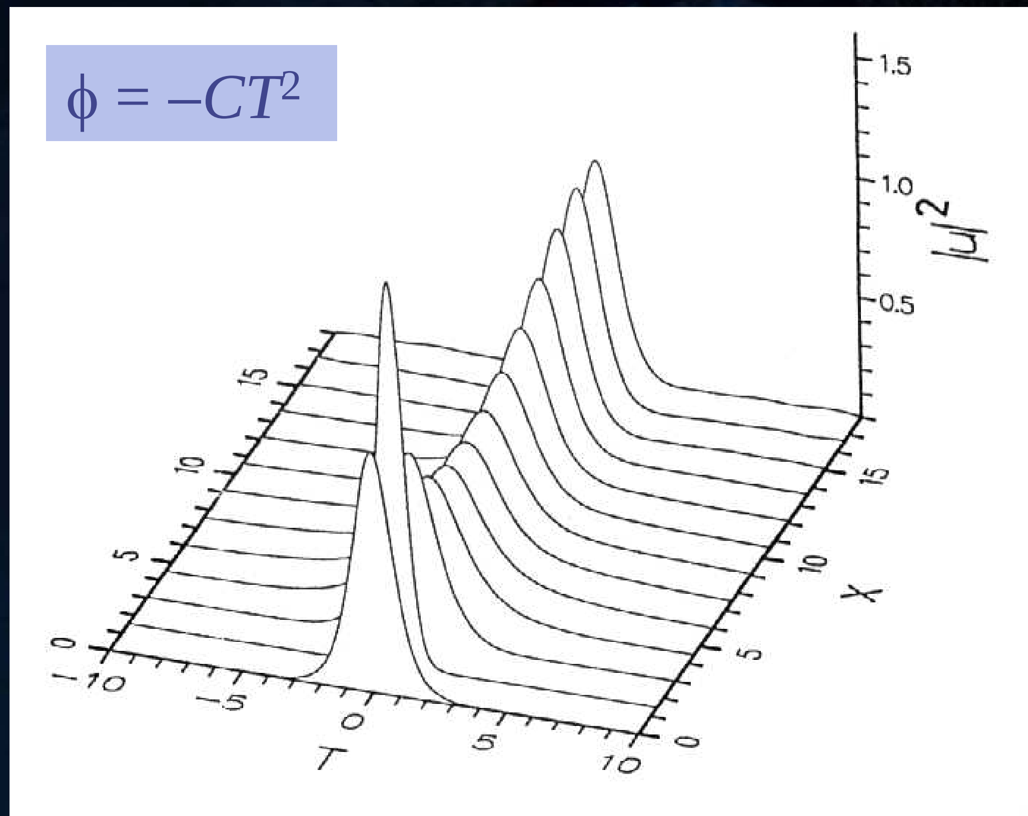


Propagation of two fundamental solitons in a lossless fibre (ϕ=-π/4, ΔT=7, B=Γ=0)

Initial frequency chirp

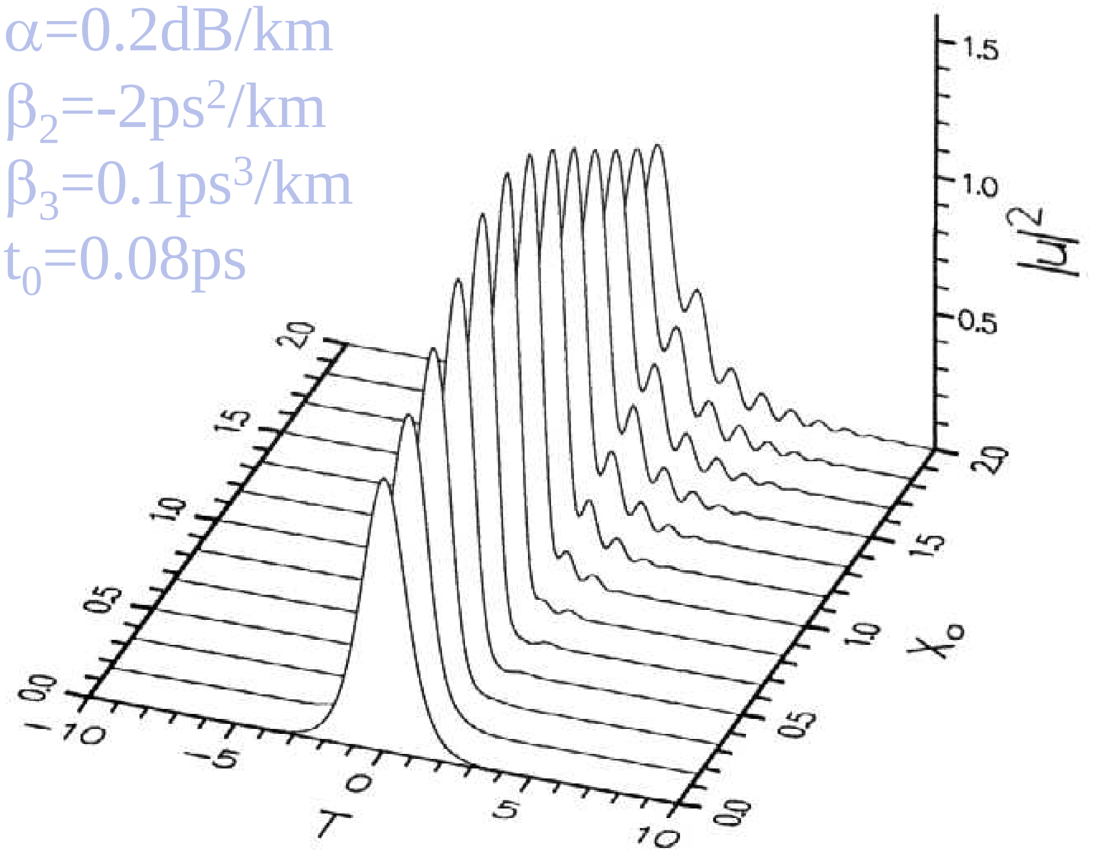
$$u_c(0,T) = u(0,T)\exp(j\phi) = A\,\mathrm{sech}(T)\exp(j\phi)$$

$$\phi = -CT^2$$

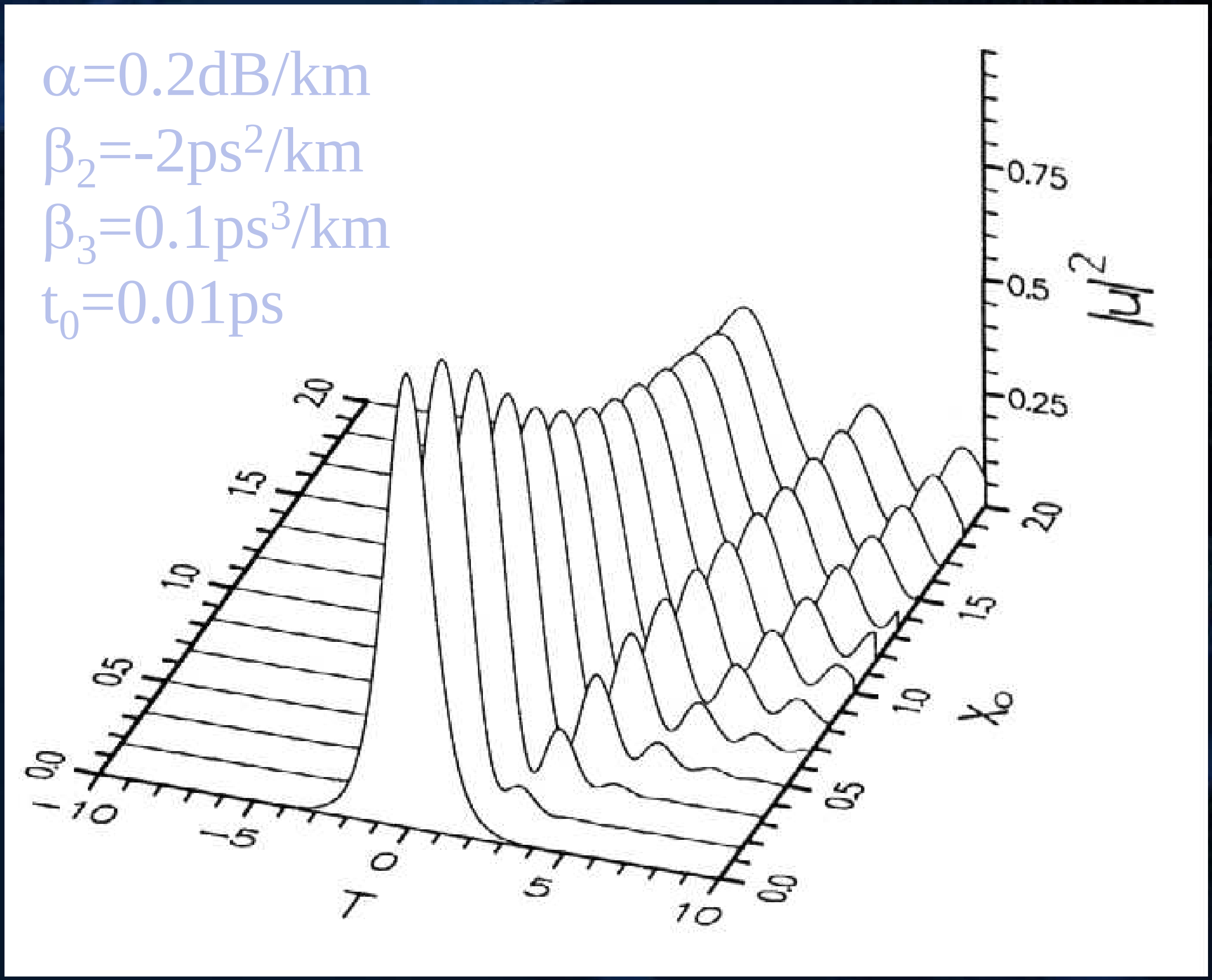


Propagation of a fundamental solitons in a lossless fibre with an initial requency chirp of C=0.25

α=0.2dB/km
β_2=-2ps^2/km
β_3=0.1ps^3/km
t_0=0.08ps



Propagation of a fundamental solitons in a dispersion-shifted fibre with an initial requency chirp of C=0.25



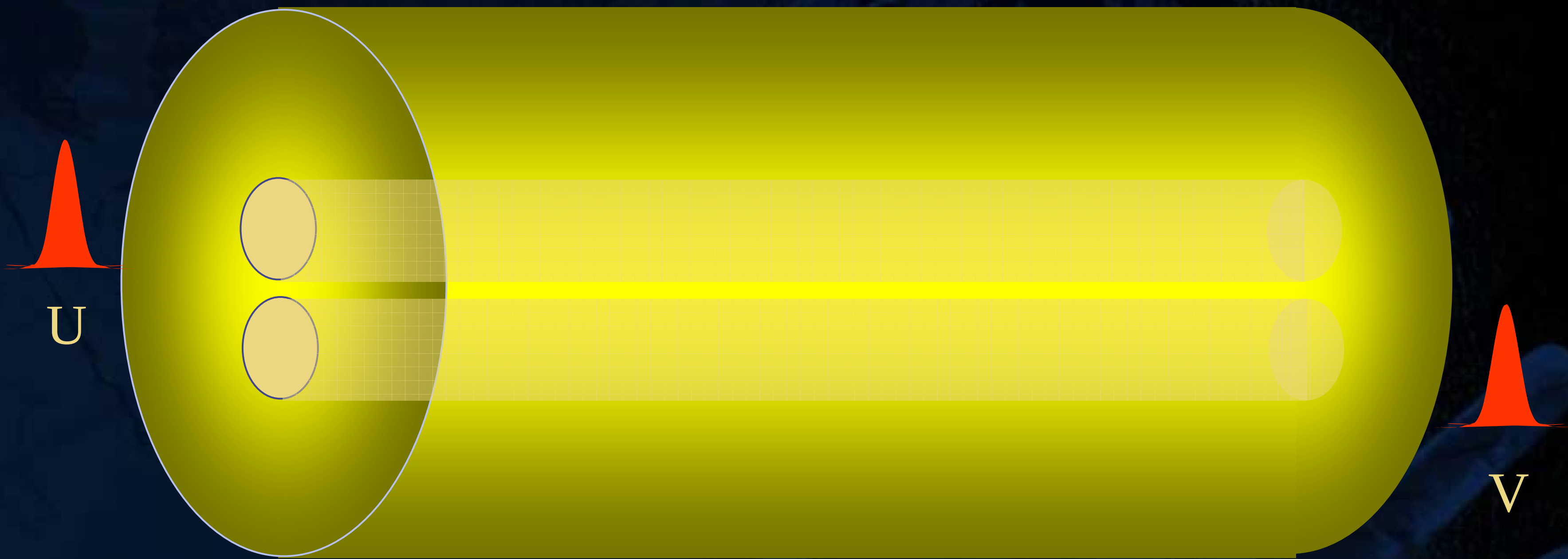
Propagation of a fundamental solitons in a dispersion-shifted fibre with an initial requency chirp of C=0.25

T*wo-core fiber couplers*

Two-core fiber couplers have been the subjects of theoretical and experimental research for their potential in optical signal processing and switching.

Many useful devices can be constructed from the two-core fiber couplers, such as optical switches, wavelength-selectors and two-core fiber amplifiers.

Nonlinear Directional Couplers



Coupled-mode theory

A widely used theory for the study of devices based on evanescent-field coupling.

$$\frac{dE_1(z)}{dz} + i\beta_1 E_1 = iC_{12}E_2$$

$$\frac{dE_2(z)}{dz} + i\beta_2 E_2 = iC_{21}E_1$$

- $E_{1(2)}$: Single-mode field amplitudes
- $\beta_{1(2)}$: Single-mode propagation constants in cores 1 and 2
- $C_{12(21)}$: Coupling coefficients between the two cores

Nonlinear Directional Couplers

Directional couplers are commonly used passive devices in optical communication systems. The general coupled-mode equations are given by

$$j\left(\frac{\partial a_1}{\partial Z}+R'\frac{\partial a_2}{\partial T}\right)+\frac{1}{2}\frac{\partial^2 a_1}{\partial T^2}+Ra_2-\frac{R''}{2}\frac{\partial^2 a_2}{\partial T^2}+\left|a_1\right|^2 a_1=0$$

$$j\left(\frac{\partial a_2}{\partial Z}+R'\frac{\partial a_1}{\partial T}\right)+\frac{1}{2}\frac{\partial^2 a_2}{\partial T^2}+Ra_1-\frac{R''}{2}\frac{\partial^2 a_1}{\partial T^2}+\left|a_2\right|^2 a_2=0$$

R **is the normalized coupling coefficient;** ***R'*** **and** ***R"*** **are the normalized first-order and second-order coupling-coefficient dispersions. The light intensities in the two waveguides are given by** $U=|a_1|^2$ **and** $V=|a_2|^2$**, respectively.**

Coupled equations for a twin-core fiber

For twin-core couplers, $\mathbf{C_{12} = C_{21}}$

$$i\left(\frac{\partial a_1}{\partial Z} + R'\frac{\partial a_2}{\partial T}\right) + \frac{1}{2}\frac{\partial^2 a_1}{\partial T^2} + Ra_2 - \frac{R''}{2}\frac{\partial^2 a_2}{\partial T^2} + |a_1|^2 a_1 = 0$$

$$i\left(\frac{\partial a_2}{\partial Z} + R'\frac{\partial a_1}{\partial T}\right) + \frac{1}{2}\frac{\partial^2 a_2}{\partial T^2} + Ra_1 - \frac{R''}{2}\frac{\partial^2 a_1}{\partial T^2} + |a_2|^2 a_2 = 0$$

First order coupling coefficient dispersion

Intermodal dispersion

Second order coupling coefficient dispersion

Coupled-Mode Equations

$$j\left(\frac{\partial a_1}{\partial Z}+R'\frac{\partial a_2}{\partial T}\right)+\frac{1}{2}\frac{\partial^2 a_1}{\partial T^2}+Ra_2+|a_1|^2 a_1=0$$

$$j\left(\frac{\partial a_2}{\partial Z}+R'\frac{\partial a_1}{\partial T}\right)+\frac{1}{2}\frac{\partial^2 a_2}{\partial T^2}+Ra_1+|a_2|^2 a_2=0$$

Parameters

$a_1(Z,T)$, $a_2(Z,T)$: Normalized Amplitude

Z : Normalized Distance

R : Normalized Coupling Coefficient

R’: Normalized Coupling Coefficient Dispersion

R”: Group Velocity Dispersion

Normalized Coupling Coefficient

- From the coupled equation,

$$R = \frac{-Ct_0^2}{k''}$$

t_o : width of the pulse
C : coupling coefficient
k”: dispersion property of group velocity

Normalized Coupling Coefficient

$$R = -\frac{\left(\dfrac{(n_1^2 - n_1^4)\,k_0\left(\dfrac{s}{p}\right)}{k_1^2\,kn_1\,p^2\,(n_1^2 - n_2^2)}\right) t_0^2}{\dfrac{1}{c}\,\dfrac{\left(2\dfrac{\partial n}{\partial \omega} + \omega\dfrac{\partial n^2}{\partial \omega^2}\right)}{t_0^2}}$$

R is affected by

- refractive index of the fiber
- wavelength of the waveguide
- core radius
- center to center separation between 2 cores.

Coupling Coefficient

$$C = \frac{(n_1^{\,2} - n_1^{\,4})k_0(\frac{s}{p})}{k_1^{\,2} k n_1 p^2 (n_1^{\,2} - n_2^{\,2})}$$

k_o & k_1 : Normalized modified Bessel function

n_1 : refractive index of the core

n_2 : refractive index of the cladding

p : core radius

s : center to center separation between 2 cores

Normalized Distance

- From the coupled equation,

$$Z = \frac{k''}{t_0^{\ 2}} z$$

$$Z = -\frac{1}{c} \frac{(2\frac{\partial n}{\partial \omega} + \omega \frac{\partial n^2}{\partial \omega^2})}{t_0^{\ 2}} z$$

k”: group velocity dispersion

t_o: width of the pulse

z : length of the fiber.

Normalized Distance

$$Z = -\frac{1}{c}\frac{2\left(\frac{n(\lambda+h)-n(\lambda-h)}{2h}x\left(\frac{-2\pi c}{\omega^2}\right)\right)+\omega\left(\frac{n(\lambda+h)+n(\lambda-h)}{h^2}\times\left(\frac{4\pi c}{\omega^3}\right)\right)}{t_0^{\ 2}}$$

From the equation, Z is affected by

- wavelength of the waveguide
- frequency of the waveguide
- width of the input pulse

Normalized Coupling Coefficient Dispersion

$$R' = \frac{-C' t_0^2}{k''}$$

t_o : width of the pulse
C : 1st order coupling coefficient
k”: dispersion property of group velocity

Normalized Second Order Dispersion

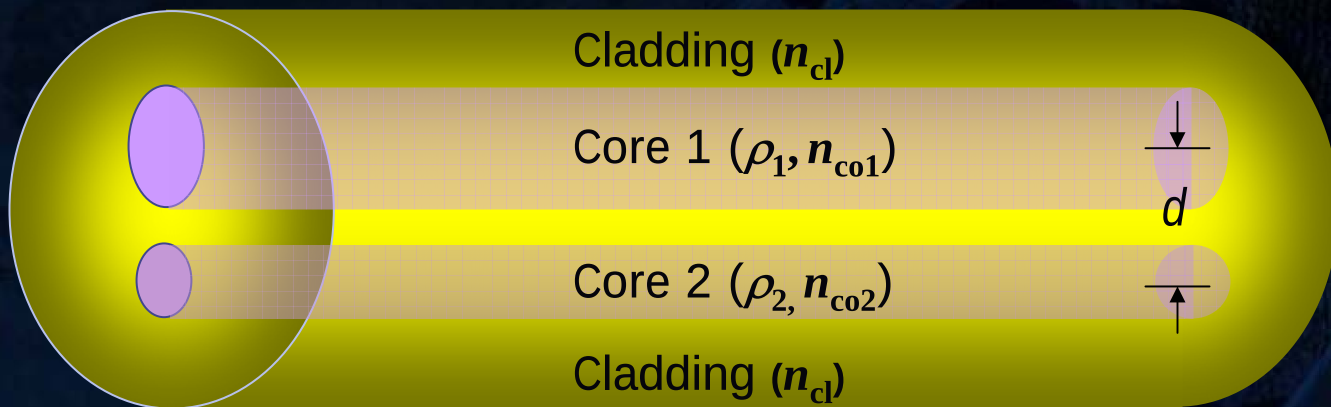
$$R'' = \frac{-C''}{k''}$$

C : 2nd order coupling coefficient
k”: dispersion property of group velocity

Most previous research on pulse propagation in a two-core fiber coupler was based on twin-core fiber couplers. Using the Fourier series analysis technique, we have studied the pulse propagation in the non-identical core fiber coupler.

Structure of a Two-core Fiber Coupler

A two-core fiber coupler contains two parallel cor



$\Delta_1 = (n_{co1}^2 - n_{cl}^2)/2n_{co1}^2$ $\Delta_2 = (n_{co2}^2 - n_{cl}^2)/2n_{co2}^2$

- Twin-core coupler: $\rho_1 = \rho_2$, $\Delta_1 = \Delta_2$
- Two-nonidentical-core coupler: $\rho_1 \neq \rho_2$, $\Delta_1 \neq \Delta_2$
- Core-to-core separation: d
- $\Delta_1, \Delta_2 << 1$

- In twin-core couplers, the linearly coupled NLSE describing pulse propagation in the coupler are **symmetric**.

- In two-nonidentical-core couplers, the commonly used <u>symmetric equations can no longer be adopted</u>.

Modeling of two-nonidentical-core coupler

$$i\left(\frac{\partial a_1}{\partial Z}-\frac{t_0}{k_2''}\left(\frac{1}{v_{g1}}-\frac{1}{v_{g2}}\right)\frac{\partial a_1}{\partial T}-t_0\frac{C_{12}'}{k_2''}\frac{\partial a_2}{\partial T}\right)+\frac{1}{2}\frac{k_1''}{k_2''}\frac{\partial^2 a_1}{\partial T^2}-t_0^2\frac{C_{12}}{k_2''}a_2+\frac{C_{12}''}{2k_2''}\frac{\partial^2 a_2}{\partial T^2}+|a_1|^2a_1=0$$

$$i\left(\frac{\partial a_2}{\partial Z}-t_0\frac{C_{21}'}{k_2''}\frac{\partial a_1}{\partial T}\right)+\frac{1}{2}\frac{\partial^2 a_2}{\partial T^2}-t_0^2\frac{C_{21}}{k_2''}a_1+\frac{C_{21}''}{2k_2''}\frac{\partial^2 a_1}{\partial T^2}+|a_2|^2a_2=0$$

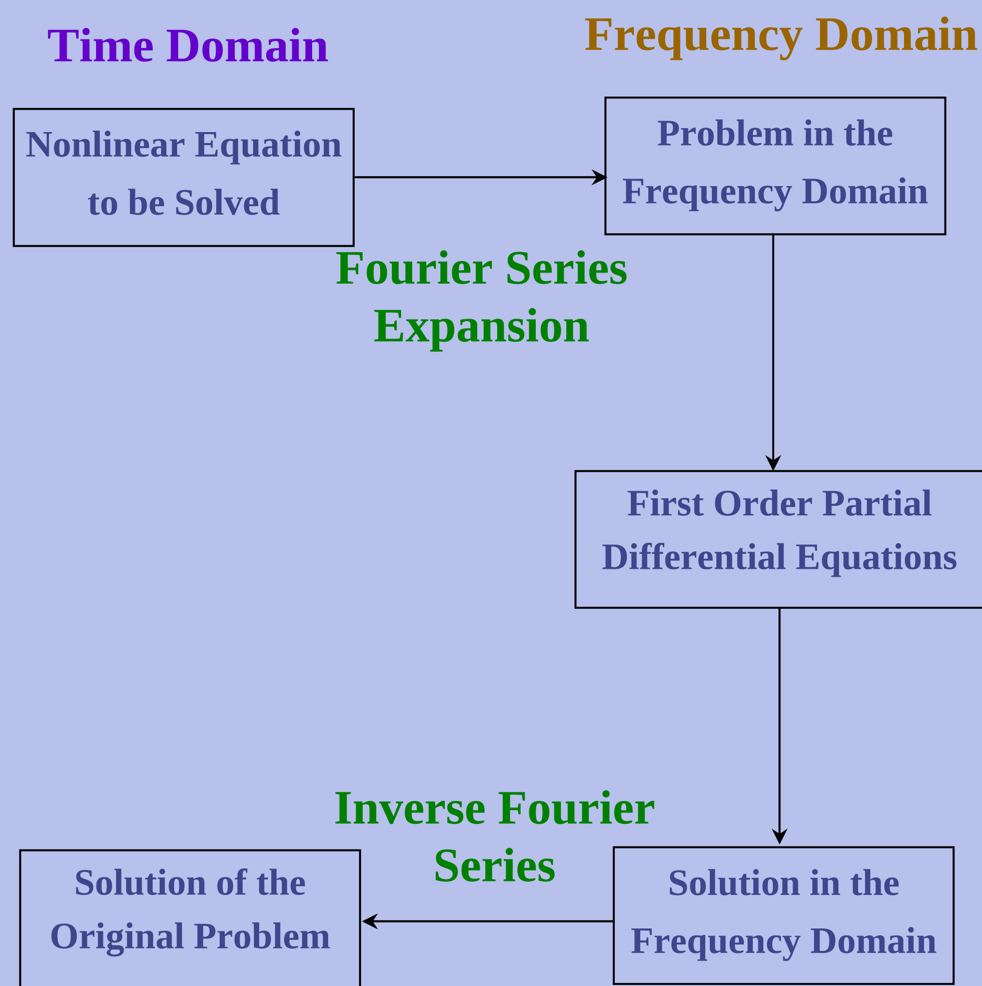
- $C_{12\,(21)}$ coupling coefficient between the two cores
- $C_{12(21)}'$ first order coupling coefficient dispersion
- $C_{12(21)}''$ second order coupling coefficient dispersion
- $v_{g1(2)}$ group velocity of the two cores
- $k_{1(2)}''$ group velocity dispersion of the two cores

$$Z=-\frac{k_2''}{T_0^2}z$$

$$T=\frac{1}{T_0}\left(t-\frac{z}{v_{g2}}\right)$$

Ref: P. Shum and M. Liu, “Effects of intermodal dispersion on two-nonidentical-core coupler with different radii,” *IEEE Photonics Technology Letters*, vol. 14, pp. 1106- 1108, 2002

Numerical technique



Fourier series analysis technique (**FSAT**)

Fourier Series Analysis Technique (FSAT)

- Simple, efficient, and easy to understand
- Less sampling points

Numerical Results Analysis

The initial conditions are given by:

$$a_1(0,T) = Asech(T)$$

$$a_2(0,T) = 0$$

A -- normalized amplitude of the input optical pulse

Switching threshold amplitude $A_{\_th}$ is defined as the value of A at which the output of each core is equal

Fourier Series Analysis Technique

$$\frac{\partial u}{\partial x} = j\frac{1}{2}\frac{\partial^2 u}{\partial T^2} + j|u|^2 u + B\frac{\partial^3 u}{\partial T^3} - \Gamma u$$

$$u(x,T) = \sum_{n=-N}^{N} \hat{u}_n(x)\exp(jn\varepsilon T)$$

$$\boxed{\sum_{n=-N}^{N} \Psi_n(x)\exp(jn\varepsilon T) = \left|\sum_{n=-N}^{N} \hat{u}_n(x)\exp(jn\varepsilon T)\right|^2 \sum_{n=-N}^{N} \hat{u}_n(x)\exp(jn\varepsilon T)}$$

$$\sum_{n=-N}^{N} \frac{\partial \hat{u}_n(x)}{\partial x}\exp(jn\varepsilon T) = -\frac{1}{2} j \sum_{n=-N}^{N} n^2\varepsilon^2 \hat{u}_n(x)\exp(jn\varepsilon T) + j\sum_{n=-N}^{N} \Psi_n(x)\exp(jn\varepsilon T)$$

$$- jB\sum_{n=-N}^{N} n^3\varepsilon^3 \hat{u}_n(x)\exp(jn\varepsilon T) - \Gamma\sum_{n=-N}^{N} \hat{u}_n(x)\exp(jn\varepsilon T)$$

Multiply the equation by the conjugate of exp($jk\varepsilon T$) and integrate the whole equation with respect to T from $-\pi/\varepsilon$ to π/ε, an ordinary differential equation can be obtained:

$$\frac{\partial \hat{u}_k(x)}{\partial x} = -\frac{1}{2} jk^2\varepsilon^2 \hat{u}_k(x) + j\Psi_k(x) - jBk^3\varepsilon^3 \hat{u}_k(x) - \Gamma \hat{u}_k(x)$$

If $\sigma(n)$ is a function of n and defined as

$$\sigma(n) = \frac{n^2\varepsilon^2}{2} + Bn^3\varepsilon^3$$

Then for ($-N \le n \le N$), we have

$$\frac{\partial \hat{u}_n(x)}{\partial x} = -j\sigma(n)\hat{u}_n(x) + j\Psi_n(x) - \Gamma \hat{u}_n(x)$$

For the nonlinear term we have,

$$\sum_{n=-N}^{N}\Psi_n(x)\exp(jn\varepsilon T)=\left(\sum_{\mu=-N}^{N}\hat{u}_\mu(x)\exp(j\mu\varepsilon T)\right)\left(\sum_{\nu=-N}^{N}\hat{u}_\nu^*(x)\exp(-j\nu\varepsilon T)\right)\left(\sum_{\lambda=-N}^{N}\hat{u}_\lambda(x)\exp(j\lambda\varepsilon T)\right)$$

where the function u(x)* is the complex conjugate of u(x) and parameters μ, ν and λ are all integers between -*N* and *N*. The values of μ, ν and λ should satisfy the following condition

$$\mu-\nu+\lambda=n$$

where *n* is an arbitrary value. Under this circumstance the expression can be simplified as

$$\Psi_n(x)=\sum_{\forall\mu-\nu+\lambda=n}\hat{u}_\mu(x)\hat{u}_\nu^*(x)\hat{u}_\lambda(x)$$

Therefore, we have

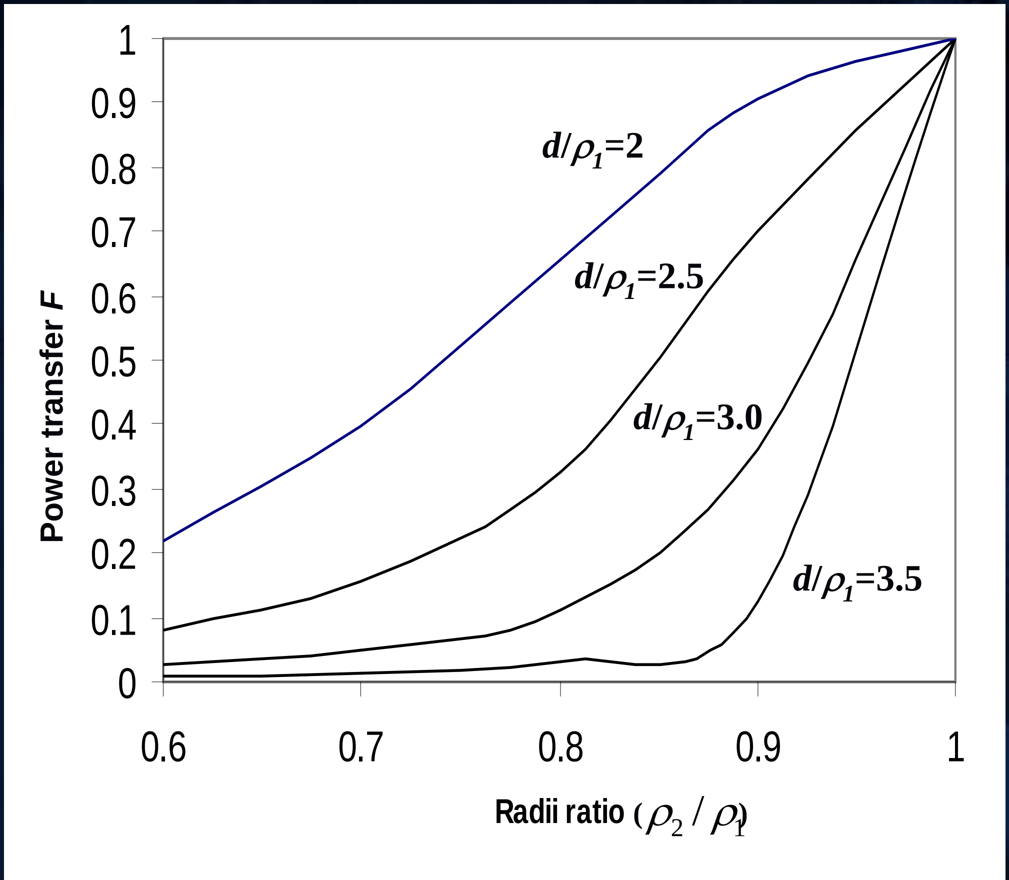
$$\frac{\partial \hat{u}_n(x)}{\partial x} = \underbrace{[-j\sigma(n) - \Gamma]\hat{u}_n(x)}_{linear\ \ term} + \underbrace{j \sum_{\forall \mu-\nu+\lambda=n} \hat{u}_\mu(x)\hat{u}^*_\nu(x)\hat{u}_\lambda(x)}_{nonlinear\ \ term}$$

Initial condition

$$\hat{u}_m(0) = \frac{\varepsilon}{2\pi}\int_{-\pi/\varepsilon}^{\pi/\varepsilon} u(0,T)\exp\left(-\ jm\varepsilon T\right)dT$$

Effects of radii ratio on the power transfer

Power transfer F should be $0.1 < F \leq 1.0$

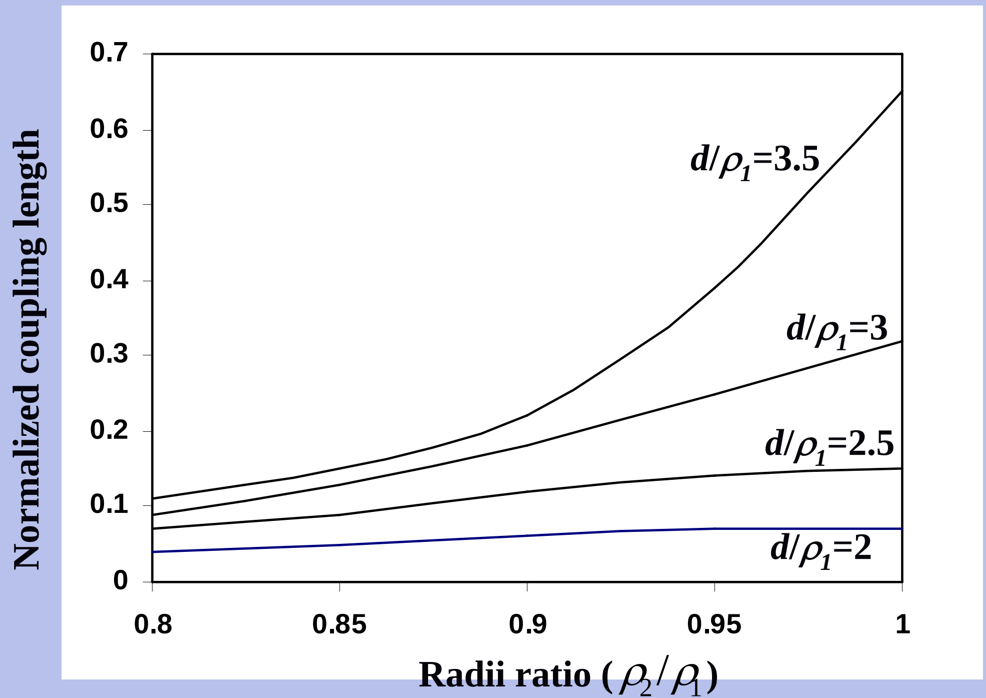


- For a given radii ratio, the power transfer decreases with the increase of the core separation
- For a certain core separation, the power transfer decreases when ρ_2/ρ_1 reduces
- Power transfer is maximum (F=1) when the two cores are identical

F as a function of ρ_2/ρ_1 (with Δ_2/Δ_1=1) for different core separations

Effects of radii ratio on the coupling length

Coupling length L_c



Normalized coupling length as a function of radii ratio ρ_2/ρ_1 for different core separations

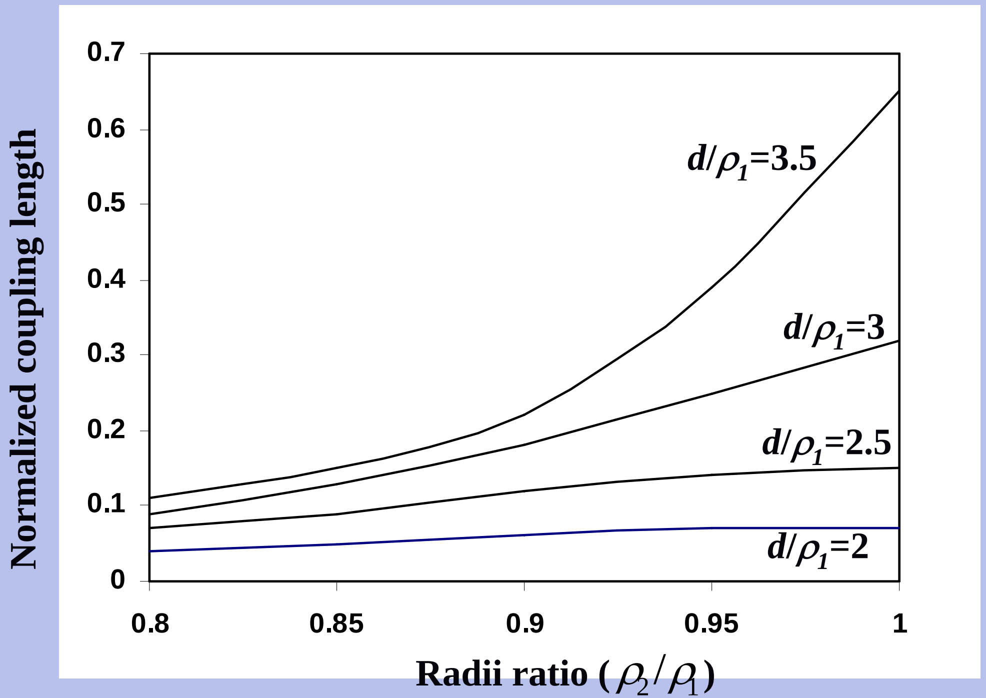
- For a certain core separation, the normalized coupling length decreases as the difference of two cores increases

- For a given radii ratio, the normalized coupling length increases with the core separation

Effects of radii ratio on the coupling length

Coupling length L_c

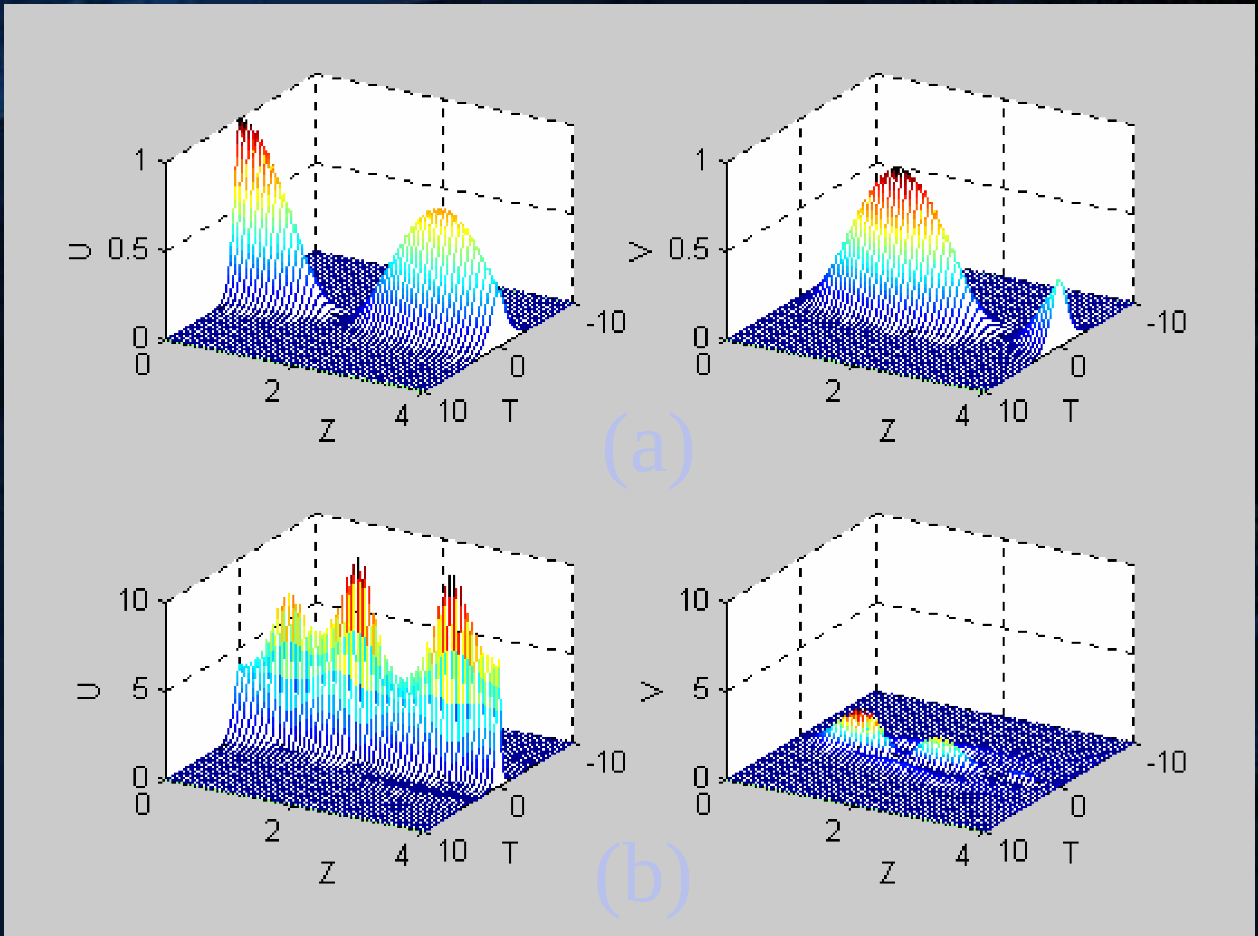
$$L_c = \frac{\pi}{2\times[(\beta_{co1}-\beta_{co2})^2/4+C_{12}C_{21}]^{1/2}}$$



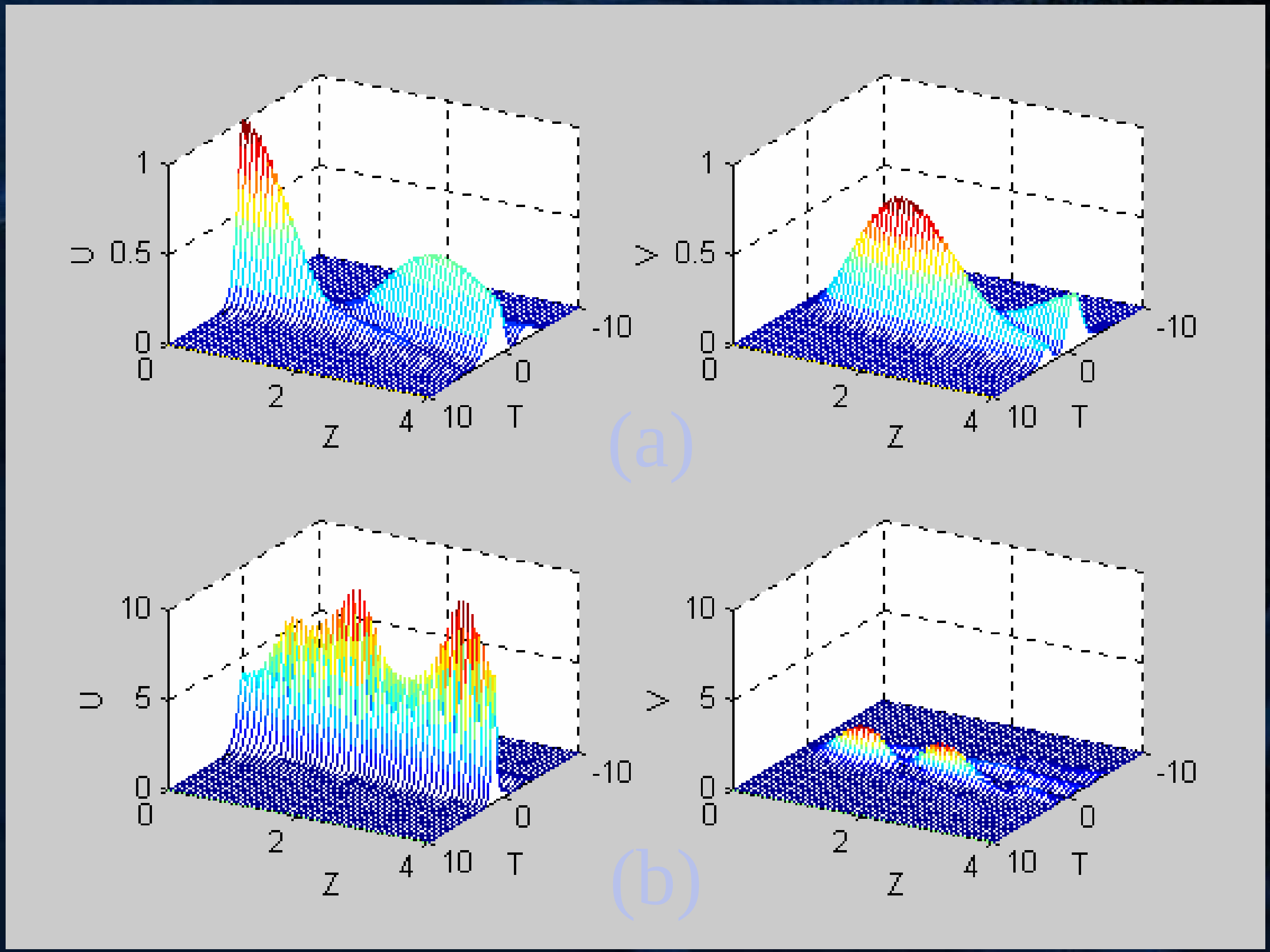
Normalized coupling length as a function of radii ratio ρ_2/ρ_1 for different core separations

- For a certain core separation, the normalized coupling length decreases as the difference of two cores increases
- For a given radii ratio, the normalized coupling length increases with the core separation

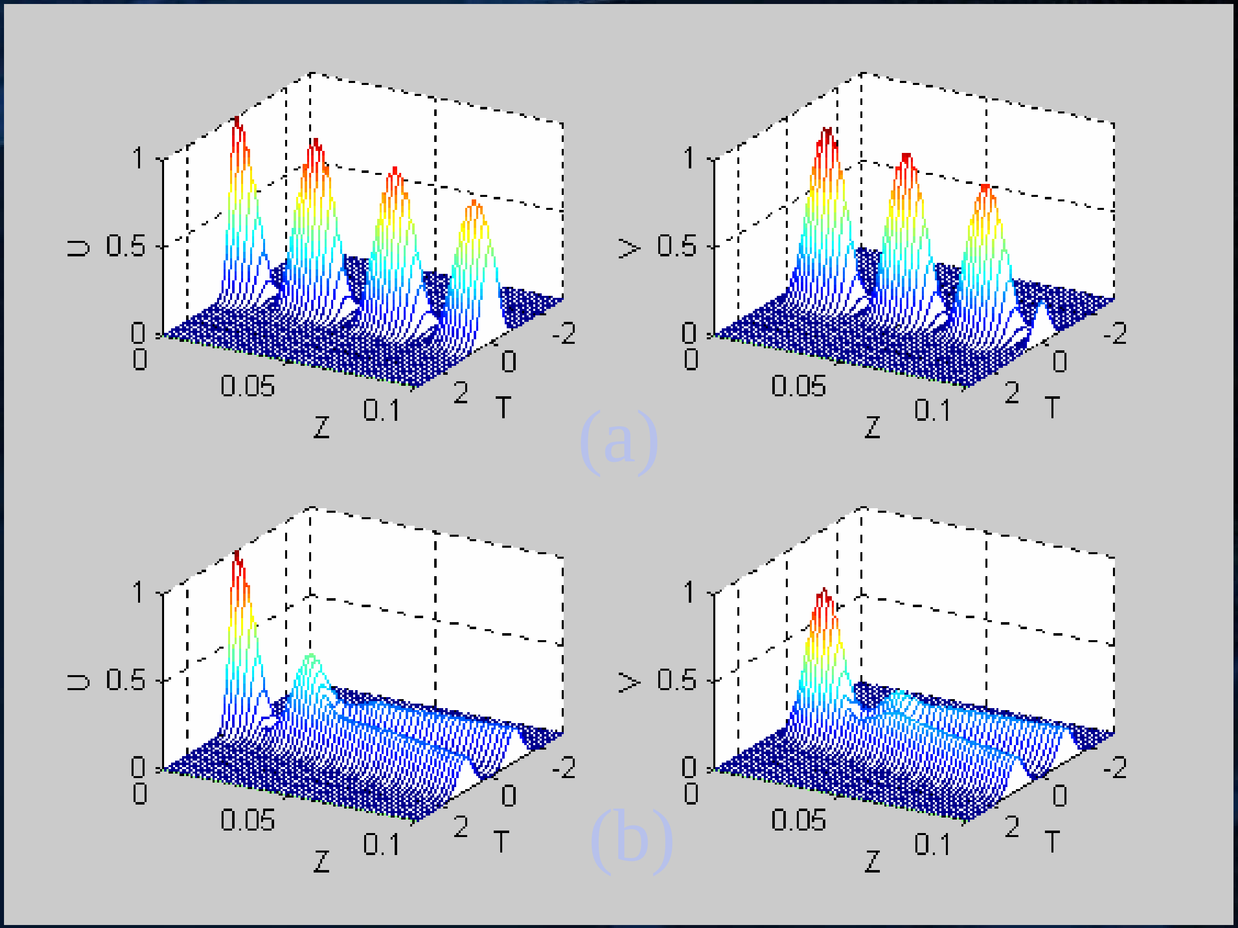
The intermodal dispersion (or coupling-coefficient dispersion) in a directional coupler can cause <u>pulse distortion or even pulse breakup</u>. This effect sets a limit on the bandwidth of a directional coupler, and hence, the <u>bit-rate</u> of a communication system that uses directional couplers.



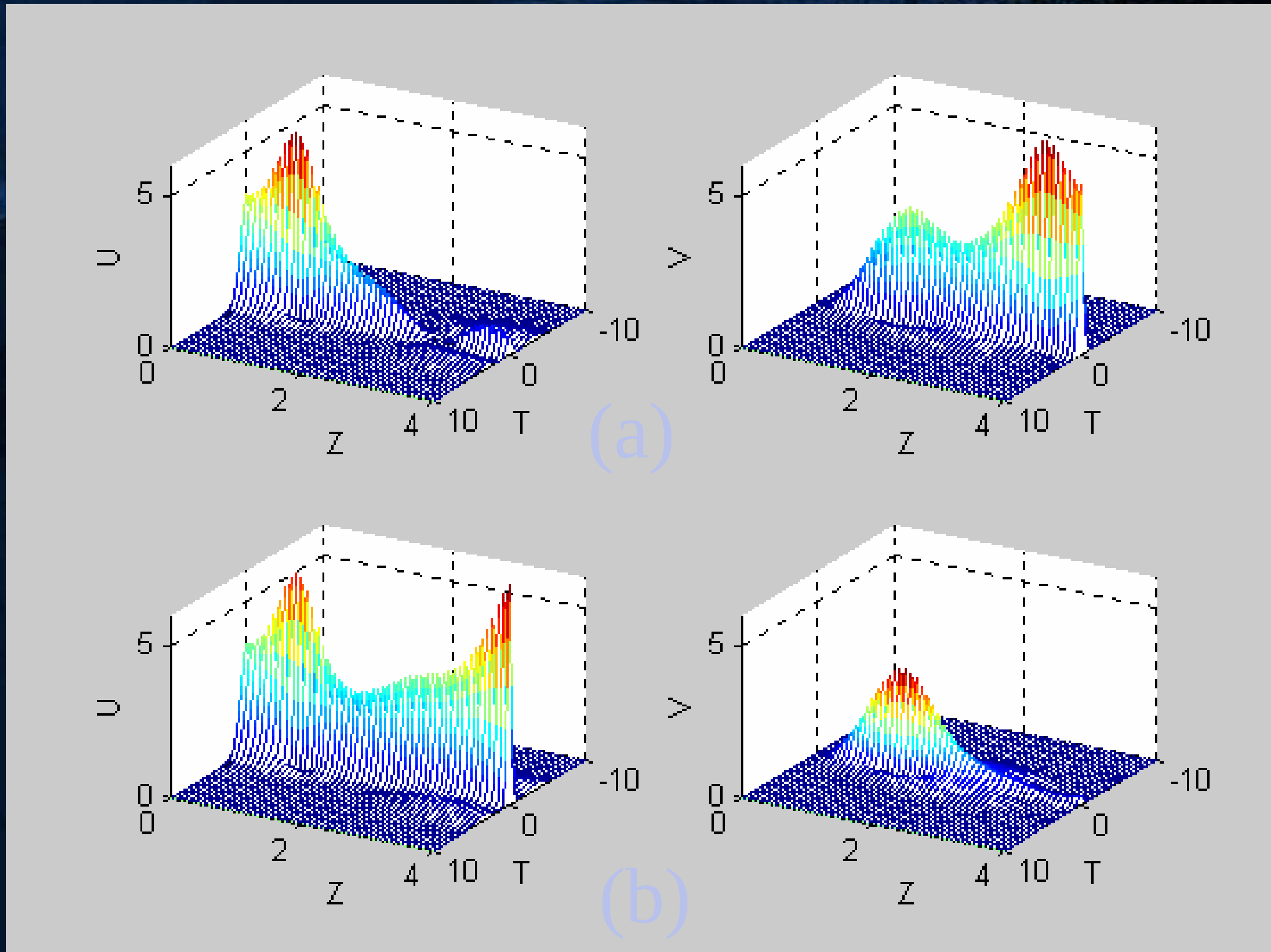
Nonlinear pulse propagation with R=1, R' = R'' =0 and an initial peak pulse amplitude of (a) 1 and (b) 2



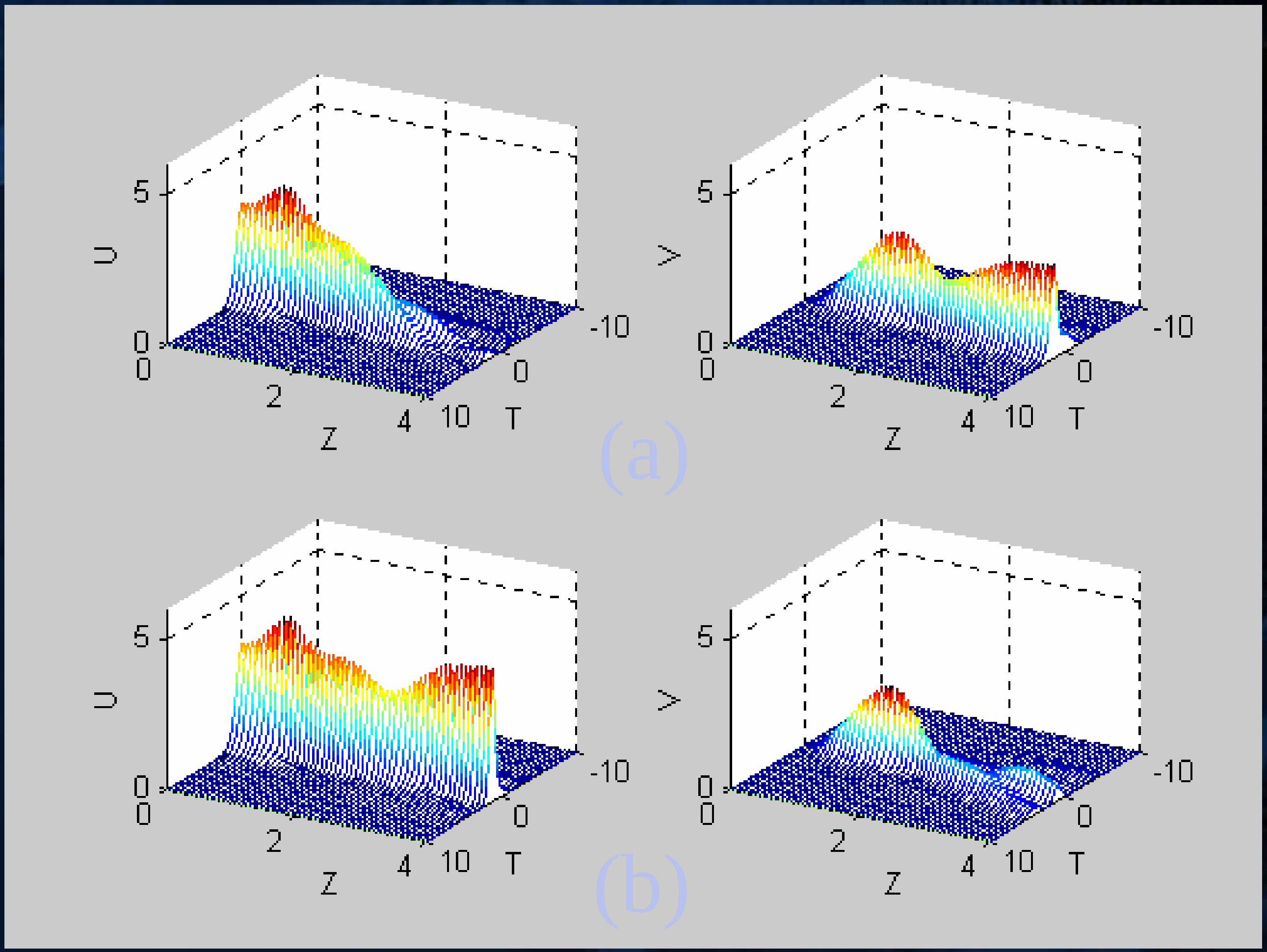
Nonlinear pulse propagation with R=1, R' = -0.5, R''=0 and an initial peak pulse amplitude of (a) 1 and (b) 2



Linear pulse propagation with *R*=100, *R''* =0, (a) *R'* =0 and (b) *R'* =-10



Nonlinear pulse propagation with *R*=1, *R'* = *R"* =0 and an initial peak pulse amplitude of (a) 1.89 and (b) 1.90



Nonlinear pulse propagation with R=1, R' = -0.5, R'' =0 and an initial peak pulse amplitude of (a) 1.80 and (b) 1.84

Comparison of power transfer and coupling length

ρ_2/ρ_1	F_{-max}	F'_{-max}	L_c	$L_c^{'}$
1	1	0.99	0.15	0.15
0.95	0.92	0.88	0.14	0.14

F_{-max} : **analytical** maximum power transfer from core 1 to core 2

F'_{-max} : **numerical** maximum power transfer from core 1 to core 2

L_c : **analytical** coupling length

L_c' : **numerical** coupling length

Analytical Expressions

Analytically, the maximum power transfer $F_{\_max}$ from core 1 to core 2 is:

$$F_{\_max} = [1 + (\frac{\beta_1 - \beta_2}{2C_{12}})^2]^{-1}$$

where $\beta_{1(2)}$ is the propagation constants of the two cores respectively

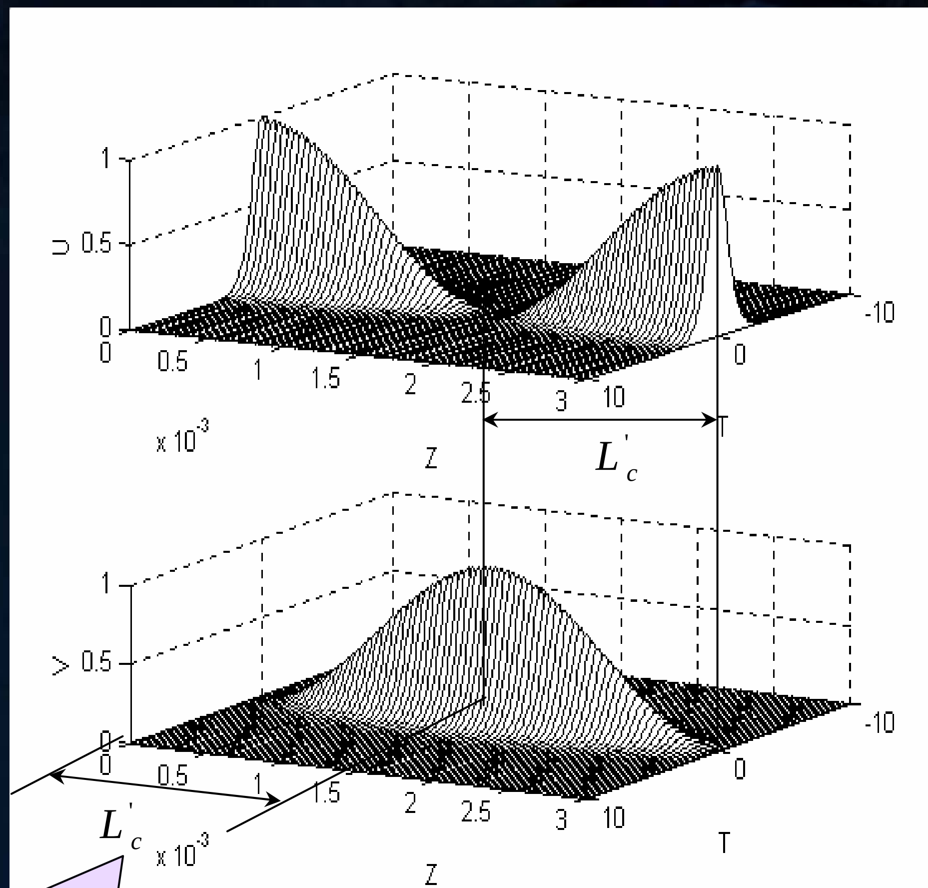
The coupling length is the distance at which $F_{\_max}$ occurs:

$$L_c = \frac{\pi}{2 \times [(\beta_{co1} - \beta_{co2})^2 / 4 + C_{12}C_{21}]^{1/2}}$$

F'_{-max}, L'_c are obtained by solving the coupled-mode equations

For example,

$\rho_2/\rho_1 = 1$



coupling length

Effects of intermodal dispersion

What's the *intermodal dispersion*？

The two-core fiber coupler, though formed with two single-mode waveguides, is actually a two-mode structure.

Intermodal dispersion: the difference between the group delays of these two modes.

Effects of intermodal dispersion

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The two-core fiber coupler, though formed with two single-mode waveguides, is actually a two-mode structure.

Intermodal dispersion: the difference between the group delays of these two modes.

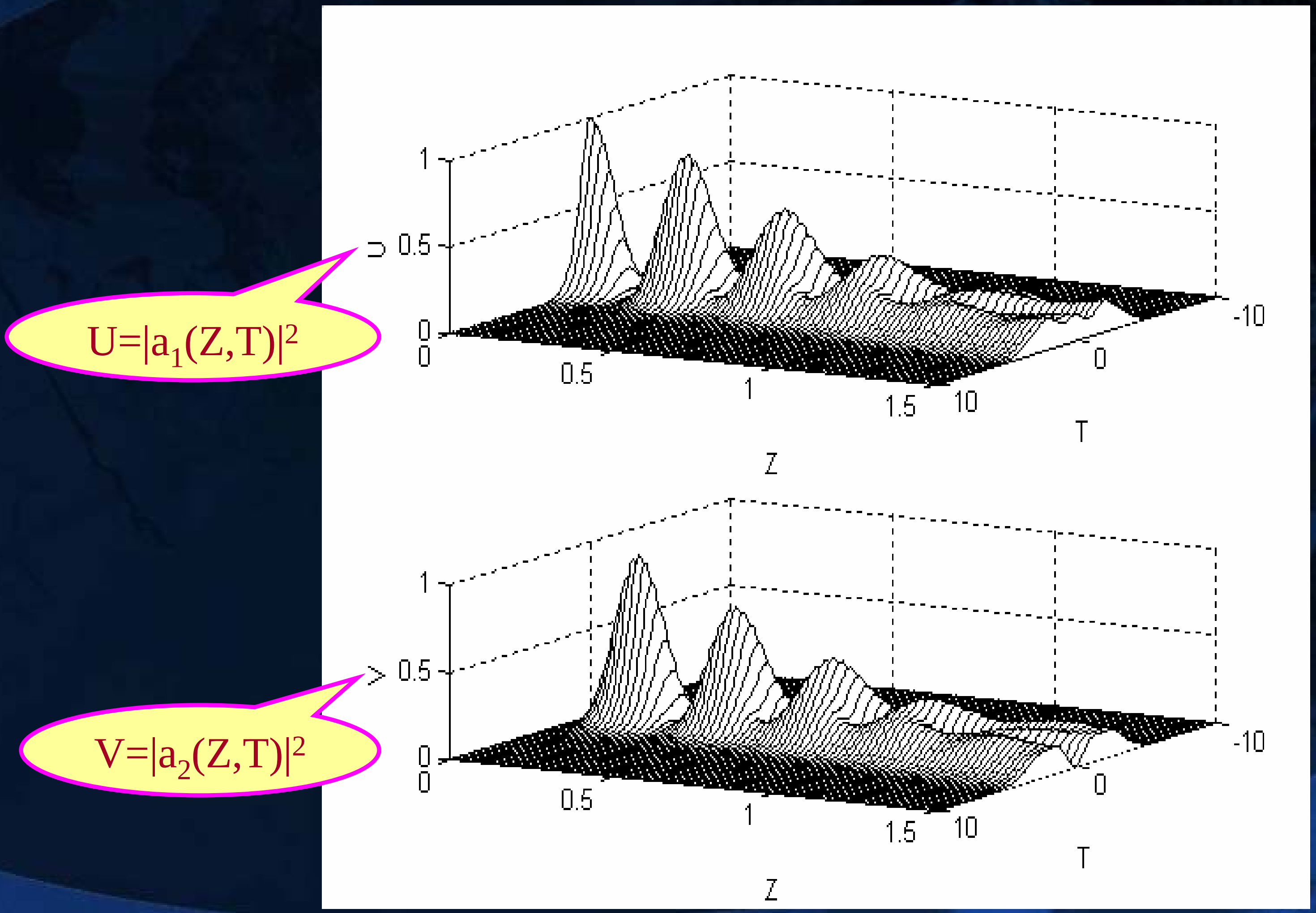
Reference Parameters:

$\rho_1 = 3.25\mu m$, $\Delta = 0.0055$, $\lambda = 1.55\ \mu m$,

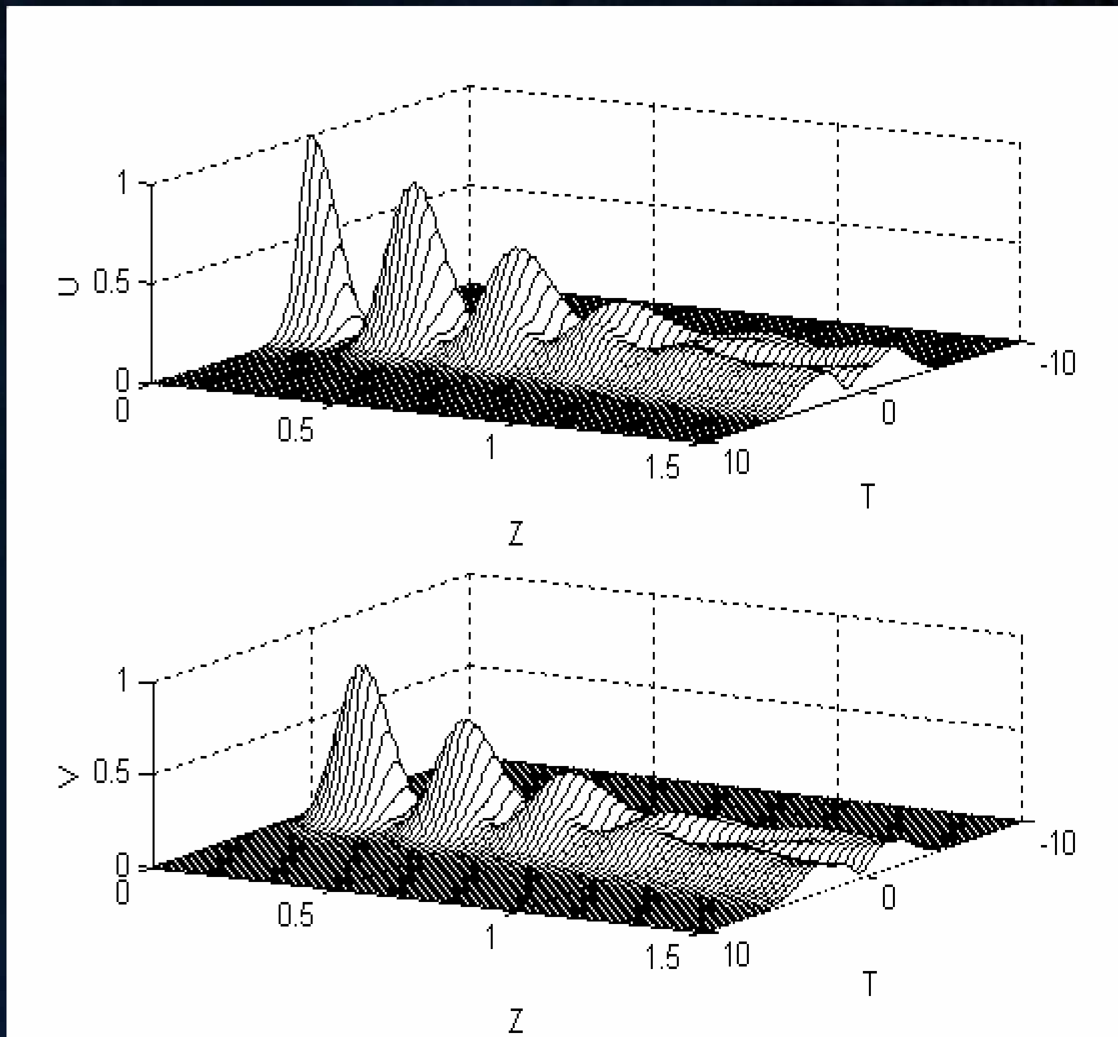
$t_0 = 10$ fs, $d/\rho_1 = 2.5$, $\rho_2/\rho_1 = 0.9$

Pulse propagation for different radii ratio

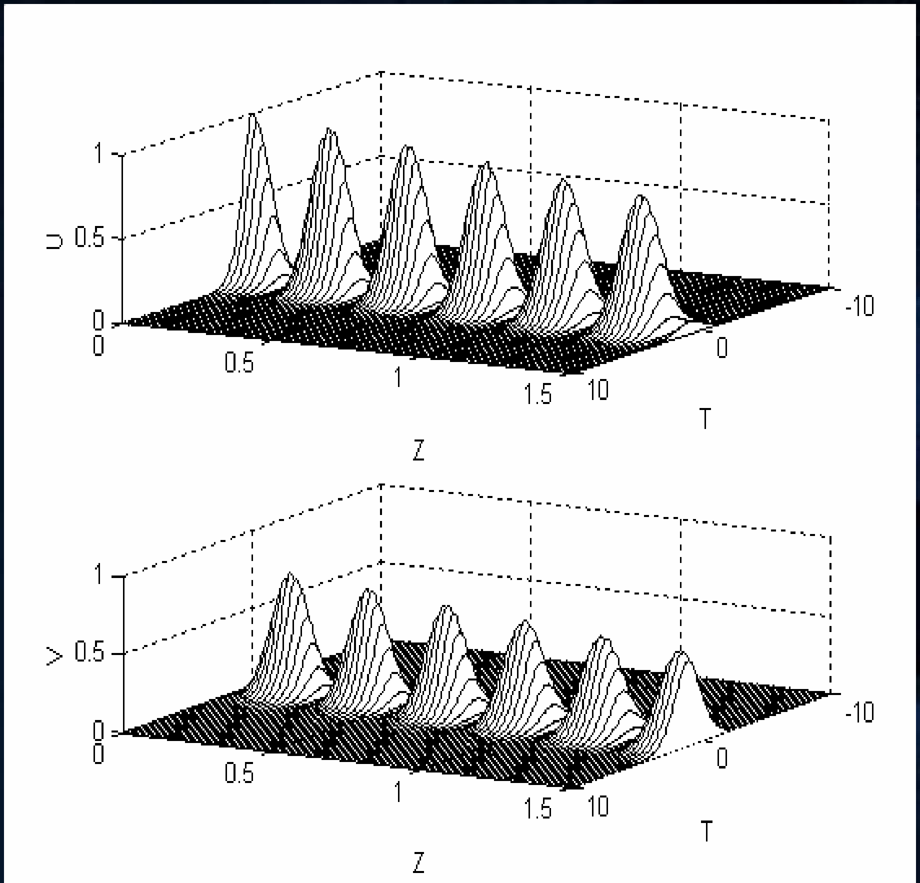
Identical core $\rho_2/\rho_1 = \mathbf{1}$



Nonidentical core ρ_2/ρ_1 = 0.95

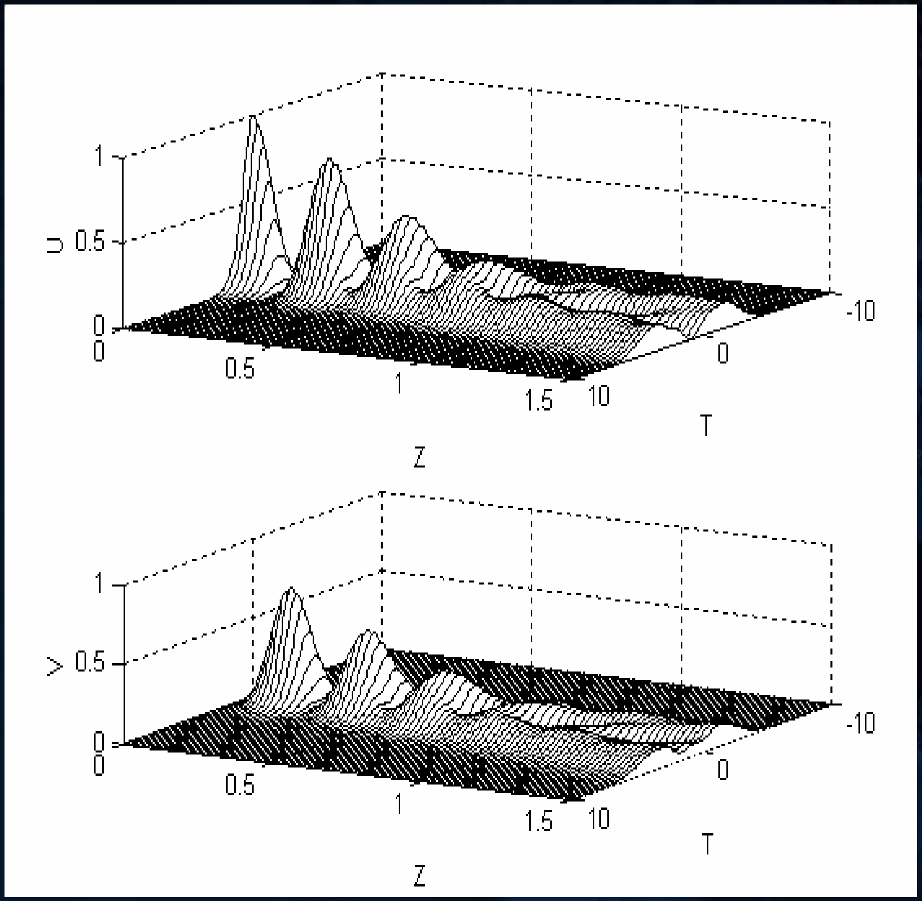


Switching dynamics



$$C'_{12} = C'_{21} = 0$$

Switching dynamics



$$C^{'}_{12}, C^{'}_{21} \neq 0$$

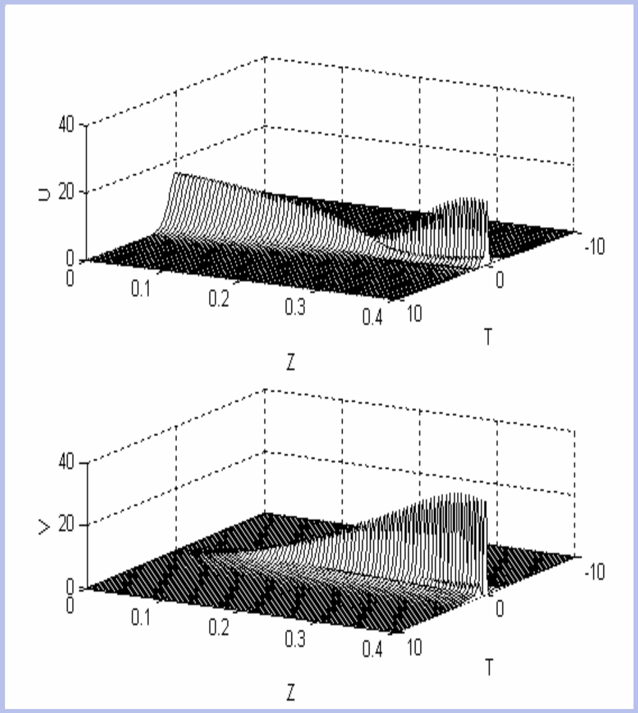
- The coupling phenomenon behaves very well with little pulse distortion without the presence of the intermodal dispersion
- The presence of intermodal dispersion leads to the breakup of the input pulse and causes severe pulse distortion
- Intermodal dispersion should be an important factor of consideration in the design of two-nonidentical-core fiber couplers

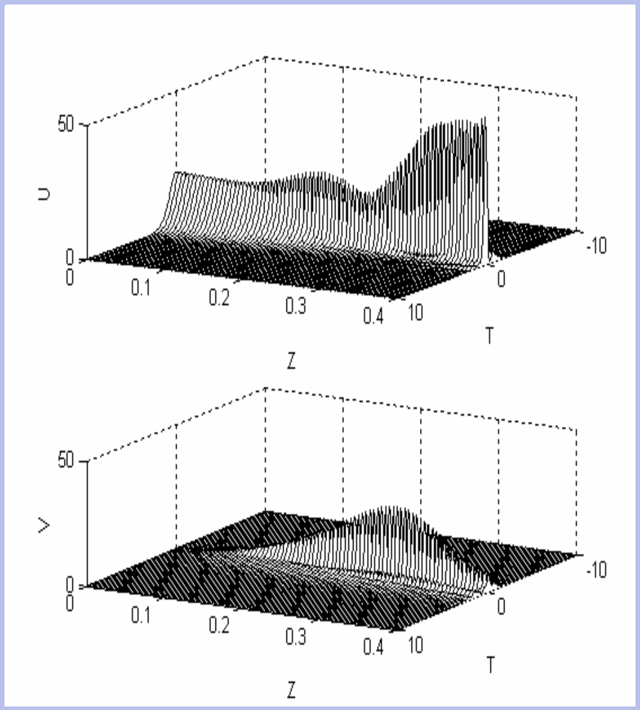
Investigation of 2nd Order Coupling Coefficient Dispersion

$C''_{12} = C''_{21} = 0$

A=4.0

A=4.5

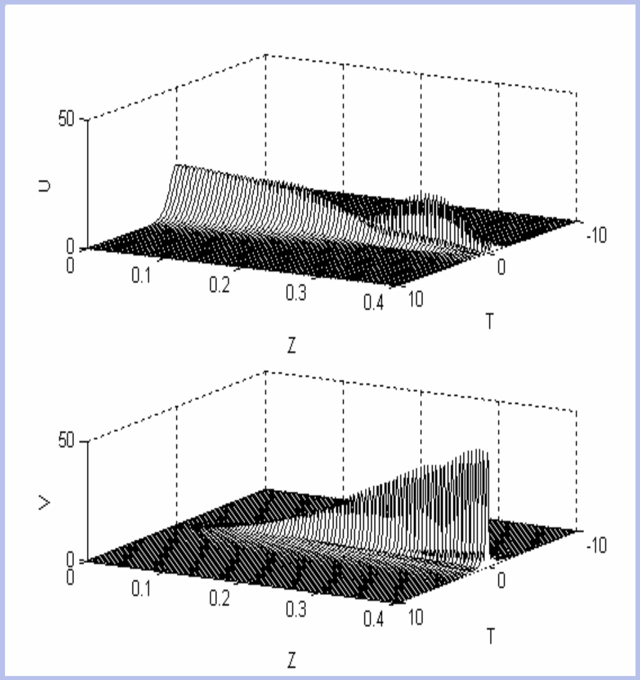




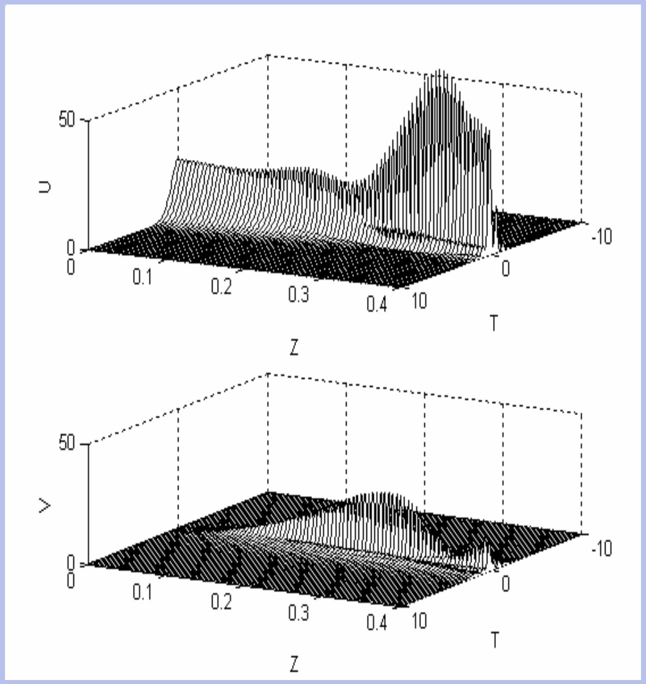
The switching threshold amplitude A_{th} is between 4.3 and 4.4

$C''_{12}\ ,\ C''_{21} \neq 0$

A=4.5



A=4.9



- The switching threshold amplitude $A_{\_th}$ is between 4.8 and 4.9
- C'' has the effect of raising the switching threshold amplitude $A_{\_th}$

Active Nonlinear Twin-Core Fiber Couplers

- Coupled-mode equations
- Effects of coupling coefficient dispersion
- Effects of gain bandwidth
- Effects of gain saturation

Coupled equations

The normalized coupled-mode equations that include the effects of gain

$$i\left(\frac{\partial a_1}{\partial Z}+R'\frac{\partial a_2}{\partial T}\right)+\frac{1}{2}\frac{\partial^2 a_1}{\partial T^2}+Ra_2+|a_1|^2 a_1=i\exp\left[-s\int_{-\infty}^{T}|a_1(T)|^2 dT\right]\left(\Gamma a_1+\mu\frac{\partial^2 a_1}{\partial T^2}\right)$$

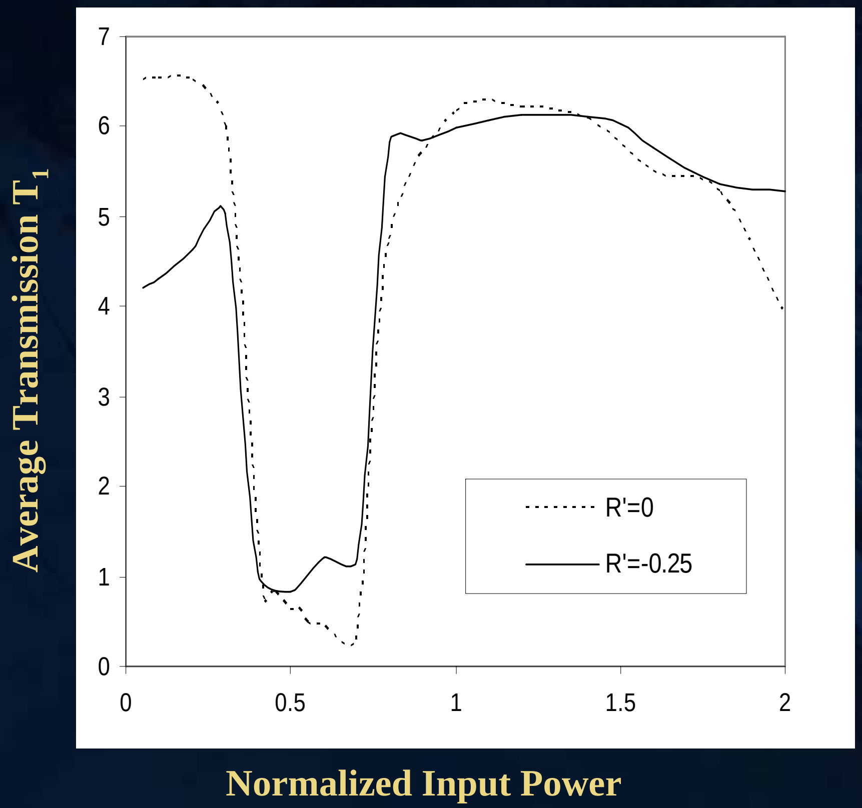
$$i\left(\frac{\partial a_2}{\partial Z}+R'\frac{\partial a_1}{\partial T}\right)+\frac{1}{2}\frac{\partial^2 a_2}{\partial T^2}+Ra_1+|a_2|^2 a_2=i\exp\left[-s\int_{-\infty}^{T}|a_2(T)|^2 dT\right]\left(\Gamma a_2+\mu\frac{\partial^2 a_2}{\partial T^2}\right)$$

The terms on the right sides of the equations represent the gain characteristics of the fiber.

- s —— gain saturation parameter
- Γ —— linear gain coefficient
- μ —— gain bandwidth

Effects of Coupling Coefficient Dispersion

Influences on switching characteristics

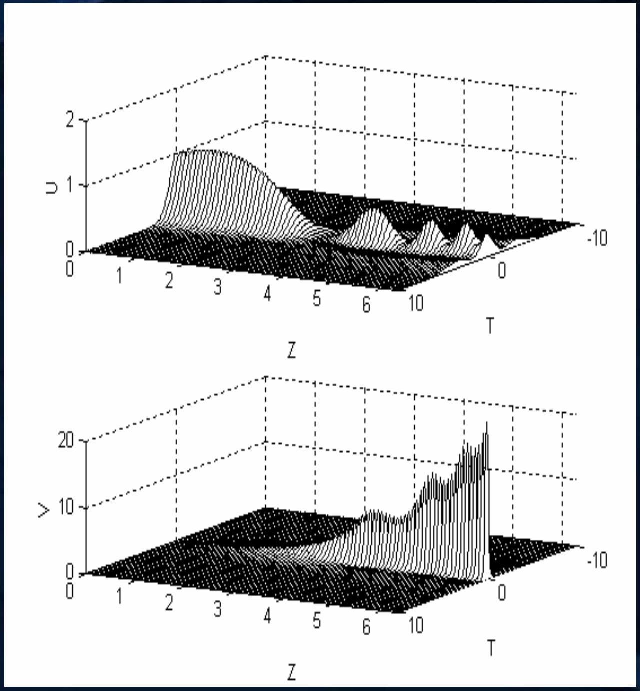


Average transmission T_1

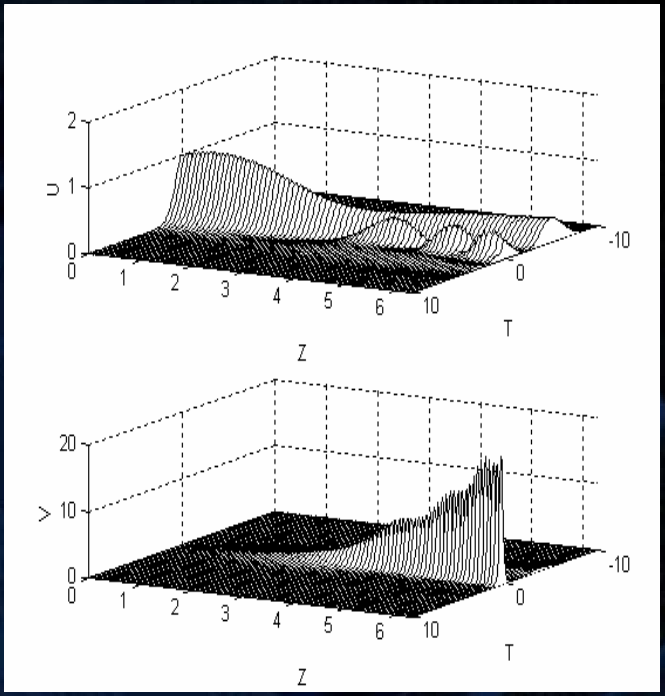
$$T_1 = \frac{\int_{-\infty}^{+\infty} |a_1(L_b, T)|^2 \, dT}{\int_{-\infty}^{+\infty} |a_1(0, T)|^2 \, dT}$$

- The switching curve with $R' = 0$ agree closely with reported results, which confirms the accuracy of our numerical analysis
- In the presence of ***R'***, the contrast between the high and low transmission levels becomes smaller

Influences on pulse shape



$R' = 0, \mu = 0$

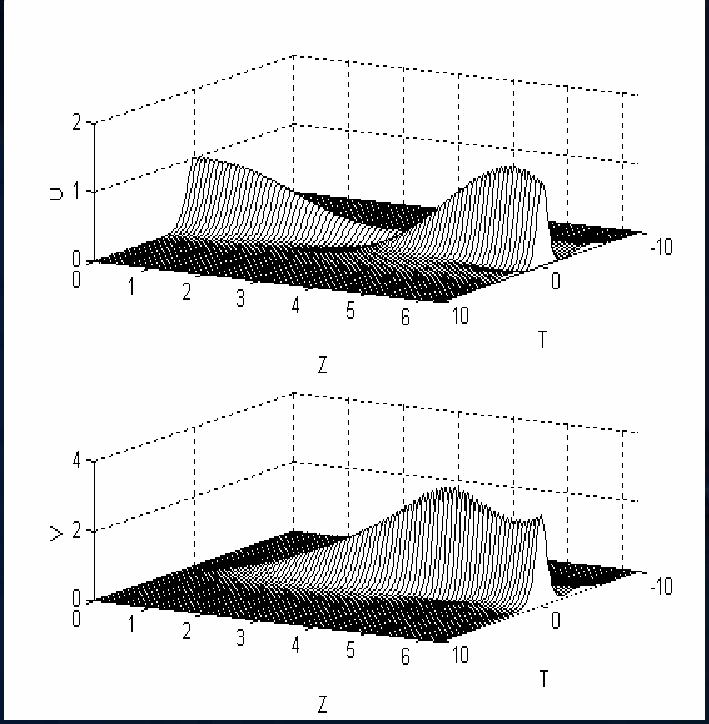


$R' = -0.25, \mu = 0$

- Reveals how the input pulse becomes distorted and eventually breaks up
- In addition to pulse amplification, pulse compression in the core V can also be observed

Effects of gain bandwidth

Balance of pulse breakup effect



$R' = -0.25$, $\mu = 0.1$

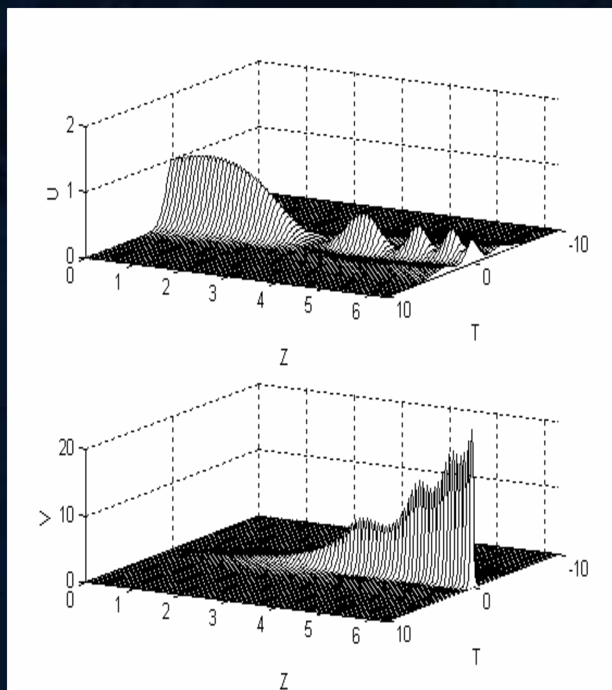
A qualitatively new feature is the suppression of pulse breakup and distortion effects

Why?

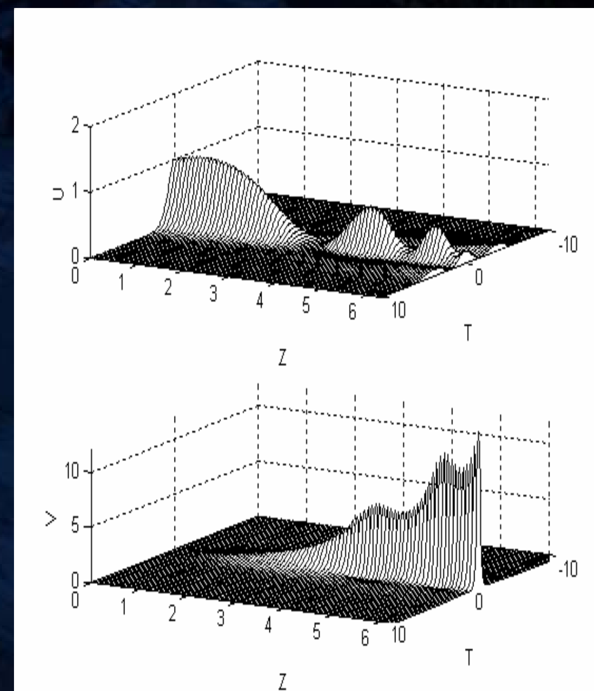
The physical reason is *that the coupling coefficient dispersion tends to split the input pulse and hence broaden its spectral width. Whereas, the finite gain bandwidth limits the spectral width of the pulse. The result of these two effects is that the pulse preserves its spectrum and thus pulse shape.*

Effects of gain saturation

Influences on pulse shape



s=0



s=0.1

❖ A significant difference is that the output power is greatly reduced

Conclusions

- Passive nonlinear two-core fiber couplers
  - New coupled-mode equations including the dispersion properties of coupling coefficient have been derived
  - The effects of dissimilarity of two cores on the characteristics of the two-nonidentical-core fiber coupler have been studied
  - The effects of intermodal dispersion and 2nd order coupling coefficient dispersion have been investigated
  - A two-nonidentical-core fiber coupler can perform as a twin-core fiber coupler
- Active nonlinear two-core fiber couplers
  - New coupled-mode equations with the consideration of coupling coefficient dispersion, gain bandwidth and gain saturation have been derived
  - The coupling-coefficient dispersion can cause significant pulse distortion or even pulse breakup as in a passive nonlinear two-core fiber
  - The gain bandwidth can suppress the adverse effect caused by the coupling coefficient dispersion, i.e., pulse breakup effect
  - The gain saturation can limit the growth of the pulse power, however, it has no adverse effect on the pulse shape

Conclusions

- New coupled-mode equations have been derived

  This equation describes pulse propagation in a two-nonidentical-core fiber coupler with the inclusion of the dispersion properties of the coupling coefficient

- The effects of dissimilarity of two cores on the characteristics of the two-nonidentical-core fiber coupler have been studied

- The effects of dispersive coupling coefficients have been investigated

  The intermodal dispersion can cause pulse breakup and the 2nd order coupling coefficient has the effect of increasing the switching threshold amplitude

Q&A

Some of our recent projects in NTRC

Thank You Very Much for Your Attention