TLM Modeling of Spatio-temporal Dynamics of TM-Waves in Dispersive Nonlinear Slab Waveguides With Sharp Discontinuities

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Outline of the Presentation

- Introduction
- TLM: a brief overview
- The Duffing Model
- Jacobian solution for coupled nonlinear TM mode equations
- TLM implementation
- Results
- Conclusions

Introduction

- Dielectric waveguides are widely used now in optical networks and optoelectronic devices.

- High-power, ultrashort laser pulses are available.

- Combined effect of material dispersion and nonlinearity should be considered (e.g. in DWDM applications).

- Sharp discontinuities may also be present in waveguides.

- Time domain numerical methods such as the Transmission Line Modeling (TLM) provide an accurate description of the spatio-temporal dynamics of the pulse.

- In this paper, a TLM algorithm is developed and applied to investigate pulse dynamics in two-dimensional dielectric waveguides.

The Transmission Line Modeling Method, TLM

- TLM is a well-established time-domain numerical method used to simulate electromagnetic problems (similar to FDTD).
- TLM is not based on any inherent assumptions, and is capable of providing vital spatio-temporal information about the pulse dynamics.
- TLM is a transient analysis – response to an impulse function can give information about the frequency response over a range from 0 – f_{max} .
- It is based on the analogy between the field quantities and lumped circuit equivalents. The space is discretised using a mesh of transmission lines.
- The field, which is represented by wave pulses scattered in the nodes and propagating in the transmission lines, is calculated at each node at every time step.

The Duffing model

- Based on the linear Lorentz model of dispersion

- Polarisation $P(t)$ for the general case can be written as

$$\frac{\partial^2 P(t)}{\partial t^2} + 2\delta \frac{\partial P(t)}{\partial t} + \omega_0^2 f(P(t))P(t) = \varepsilon_0 \Delta\chi_e \omega_0^2 E(t)$$

δ = damping frequency, ω_0 = resonant frequency,
$\Delta\chi_e$ = susceptibility contrast of the material, i.e. (n_0^2-1),
n_0 = refractive index of the material.

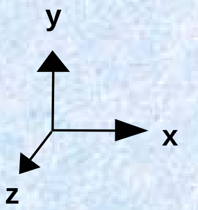
  - The function $f(P(t)) = 1 \Rightarrow$ *Linear, dispersive Lorentz model*

  - The function $f(P(t)) \neq 1 \Rightarrow$ *Nonlinear, dispersive model*

The Duffing model *cont.*

- The function $f(P(t)) = 1+\alpha P^2 \Rightarrow$ Nonlinear, dispersive Duffing model

- For the 2-D TM case, two distinct equations for $P_x(t)$ and $P_y(t)$ can be obtained.



$$\frac{\partial^2 p_y}{\partial t^2} + 2\delta \frac{\partial p_y}{\partial t} + \omega_0^2 f(p_{xy}) p_y = \Delta\chi_e \omega_0^2 V_y$$

$$\frac{\partial^2 p_x}{\partial t^2} + 2\delta \frac{\partial p_x}{\partial t} + \omega_0^2 f(p_{xy}) p_x = \Delta\chi_e \omega_0^2 V_x$$

- $f(P(t)) = f(P_{xy}) = 1+\alpha (P_x{}^2 + P_x{}^2) \Rightarrow$ *coupled nonlinear equations*

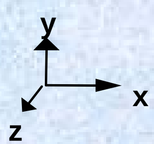
Jacobian Solution and TLM Implementation

- The two coupled equations are discretised using Z-transform, and solved iteratively using a Jacobian method:

$$\begin{bmatrix} p_x \\ p_y \end{bmatrix}_{next} \leftarrow \begin{bmatrix} p_x \\ p_y \end{bmatrix}_{prev} - \underline{\underline{J}}^{-1} \begin{bmatrix} f_{rx} \\ f_{ry} \end{bmatrix}$$

$$where \qquad \underline{\underline{J}} = \begin{bmatrix} \dfrac{\partial f_{rx}}{\partial p_x} & \dfrac{\partial f_{rx}}{\partial p_y} \\ \dfrac{\partial f_{ry}}{\partial p_x} & \dfrac{\partial f_{ry}}{\partial p_y} \end{bmatrix}$$

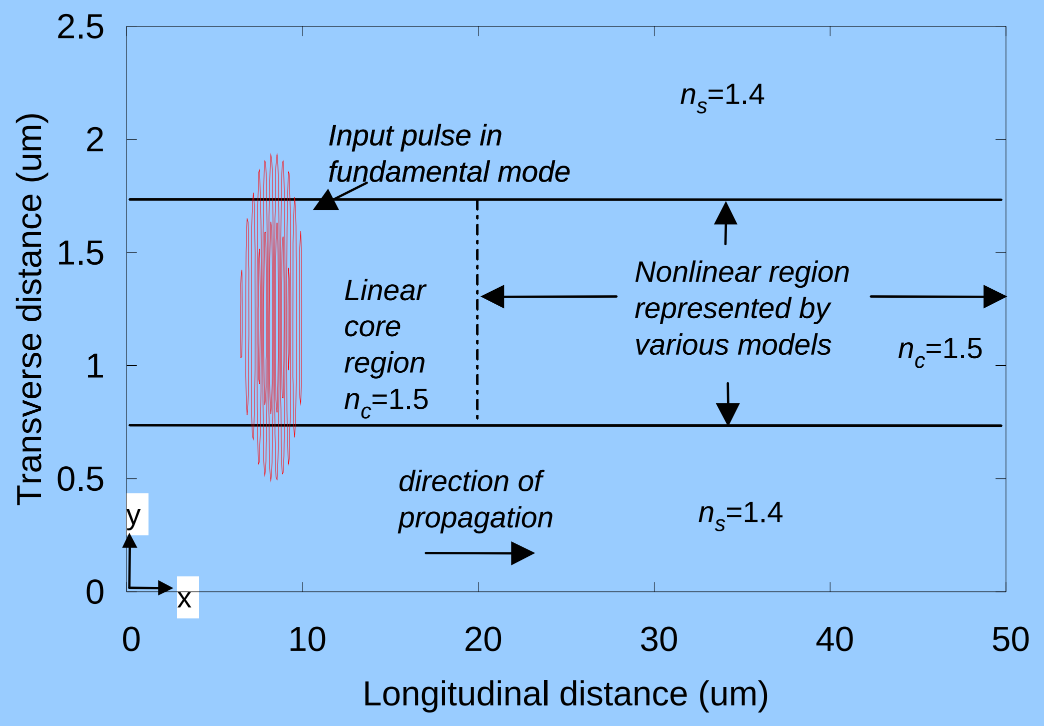
- These equations are solved within TLM along with the Maxwell's equation (which, for 2-D TM case, are):



$$-\frac{\partial H_z}{\partial x} = \frac{\partial(\varepsilon_0 E_y + P_y)}{\partial t}, \qquad \frac{\partial H_z}{\partial y} = \frac{\partial(\varepsilon_0 E_x + P_x)}{\partial t}, \qquad \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\mu_0 \frac{\partial H_z}{\partial t}$$

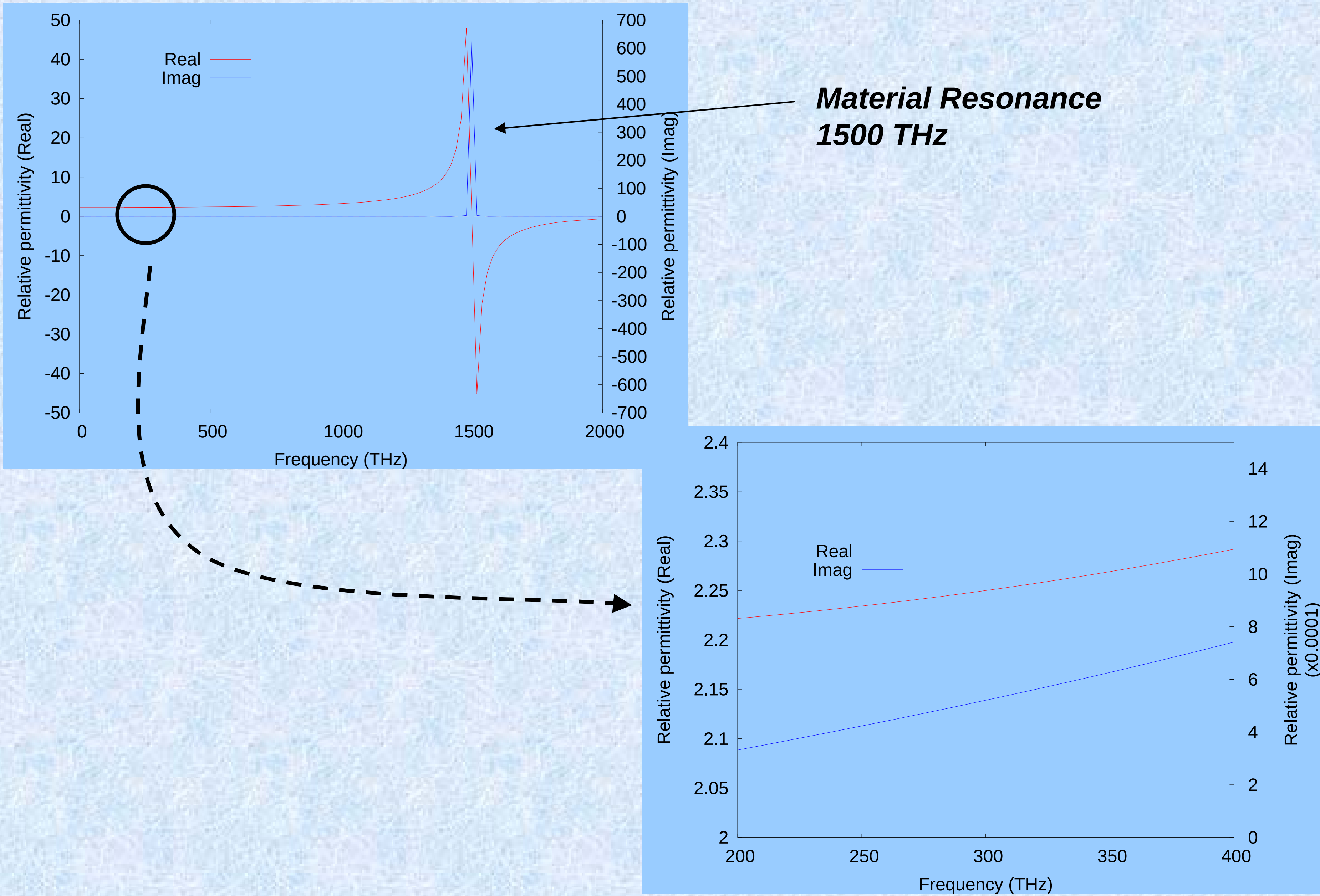
Results

Structure of the waveguide, and the input pulse

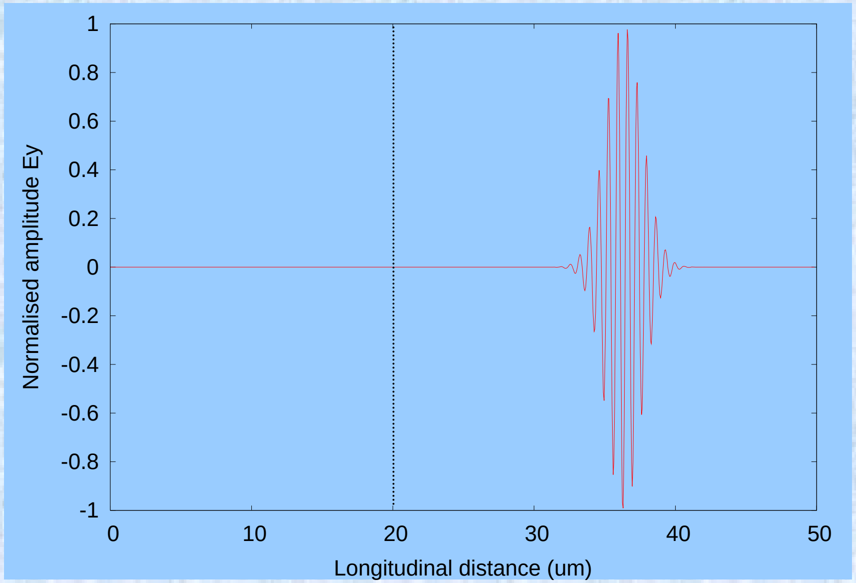


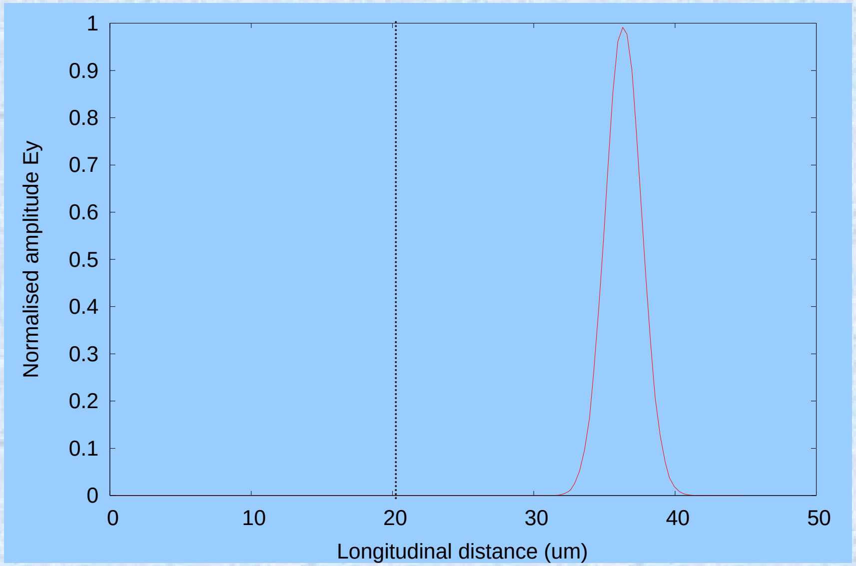
Temporal width of Gaussian pulse = 47 fs (≅ 5 μm FWHM), Core width = 1.2 μm, modulated frequency = 300 THz, TLM cell size = 50 nm (λ / Δx = 20)

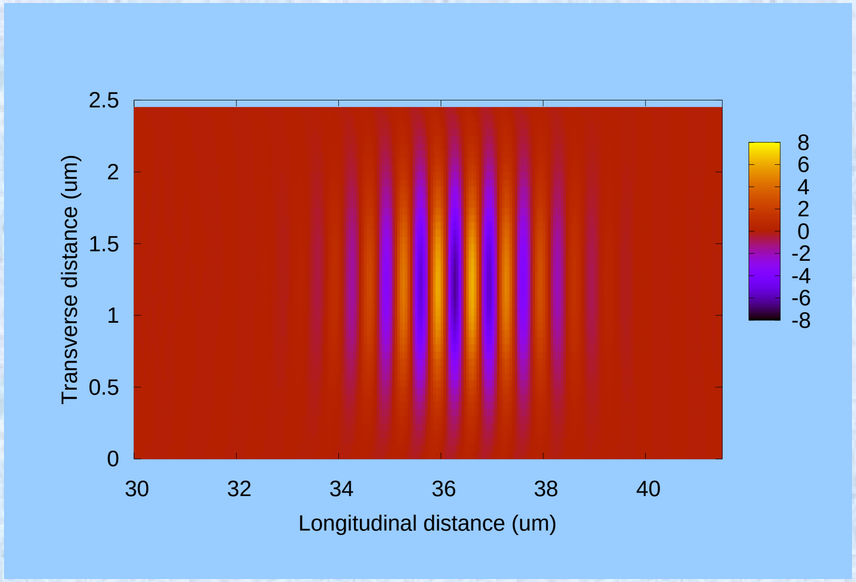
Dispersive index profile of the material simulated

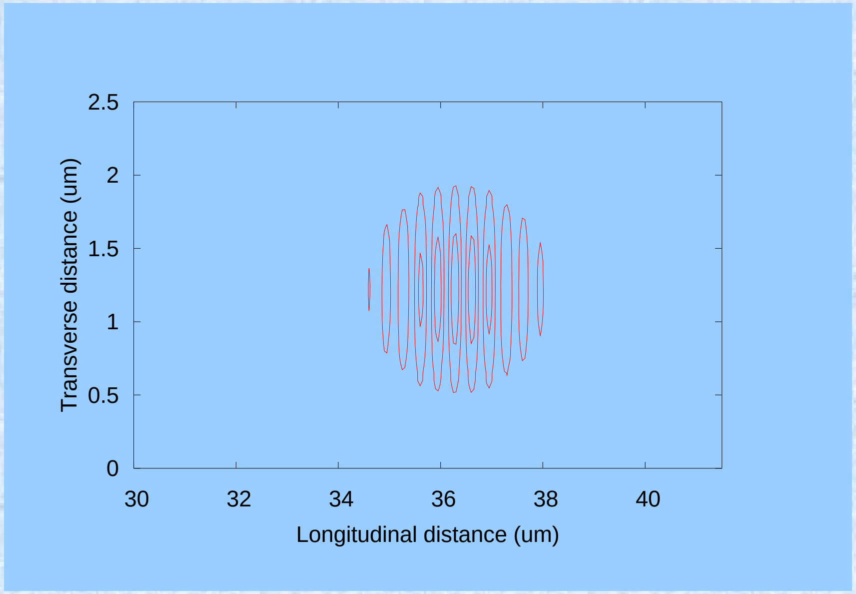


The longitudinal pulse profile inside the linear, nondispersive core after 1200 TLM time steps : The normalised E_y field, its envelope and contours

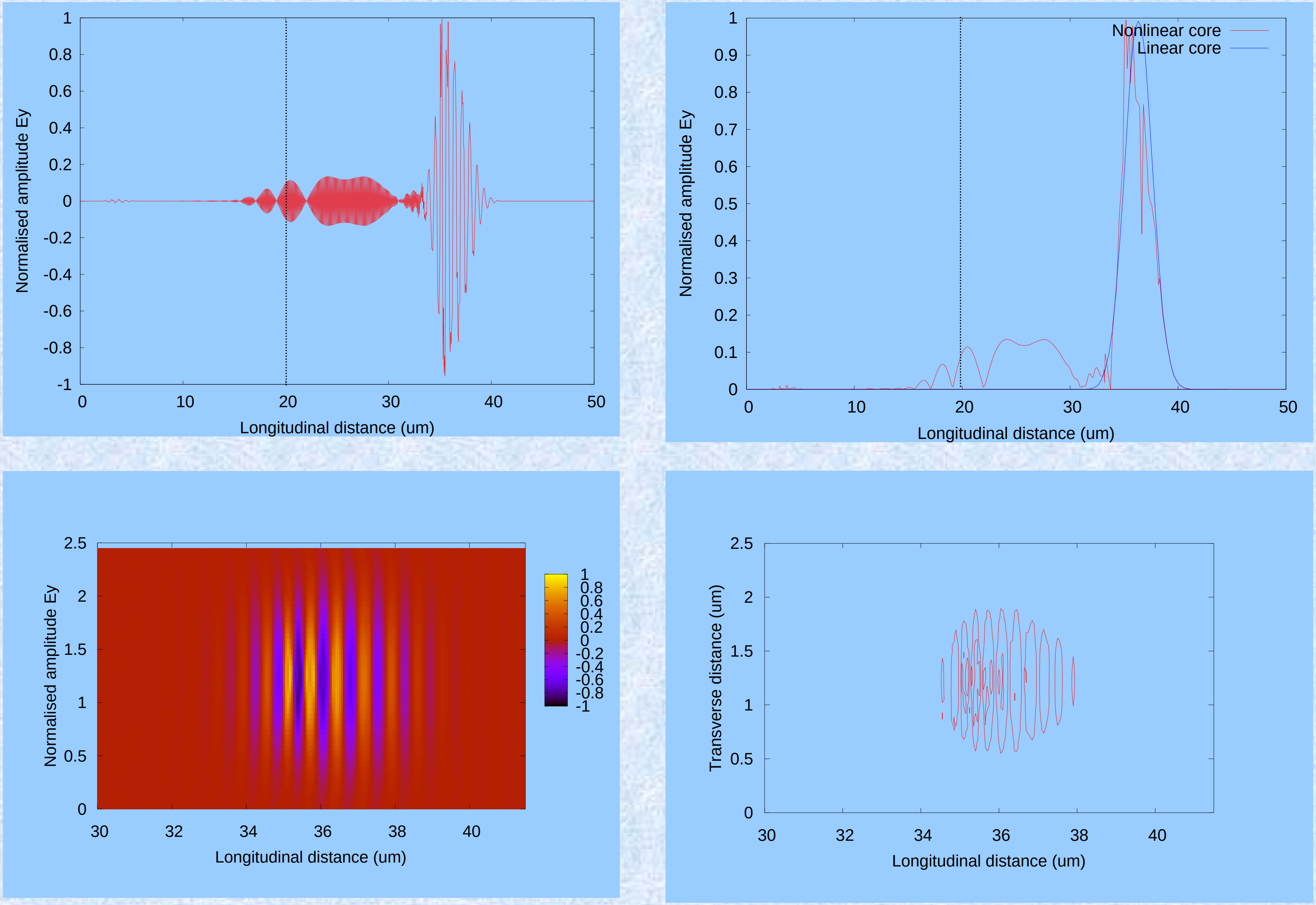




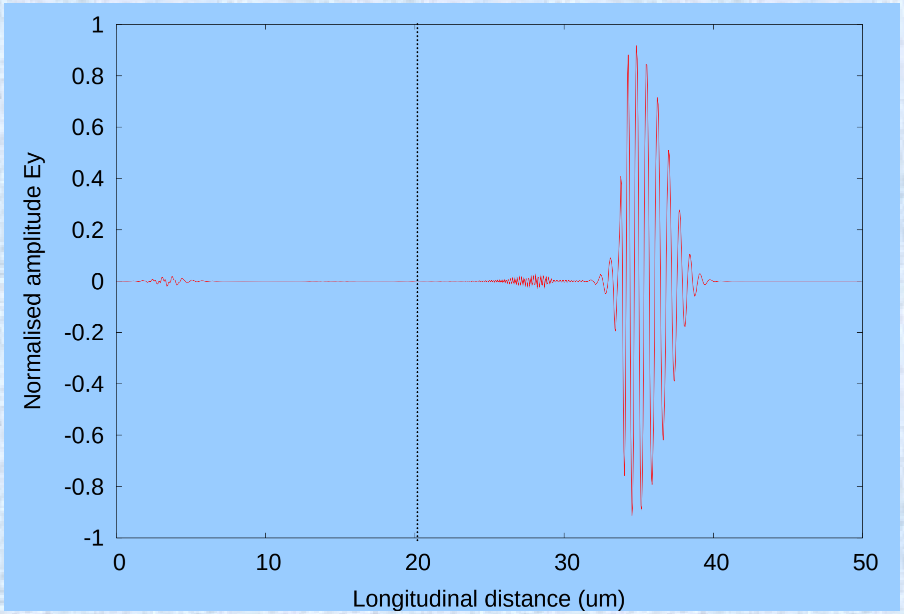


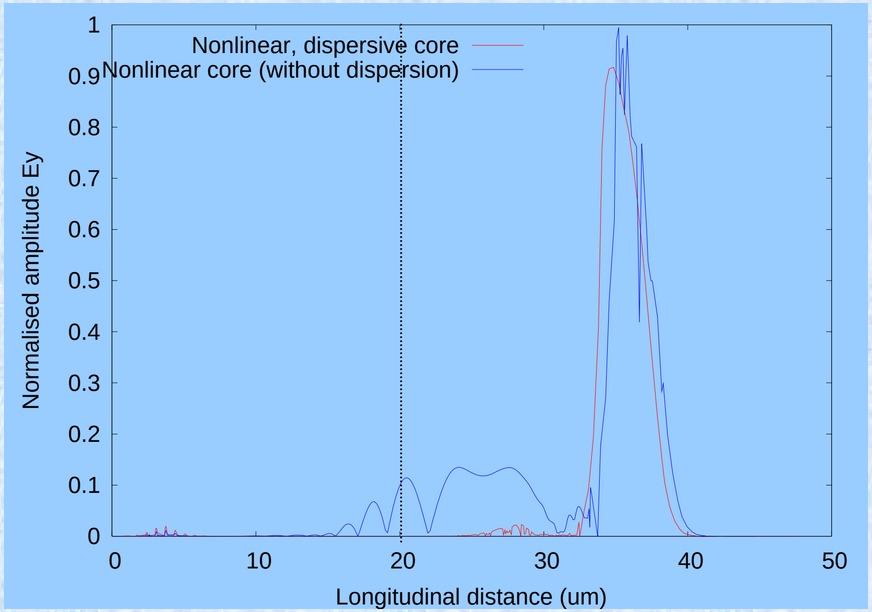


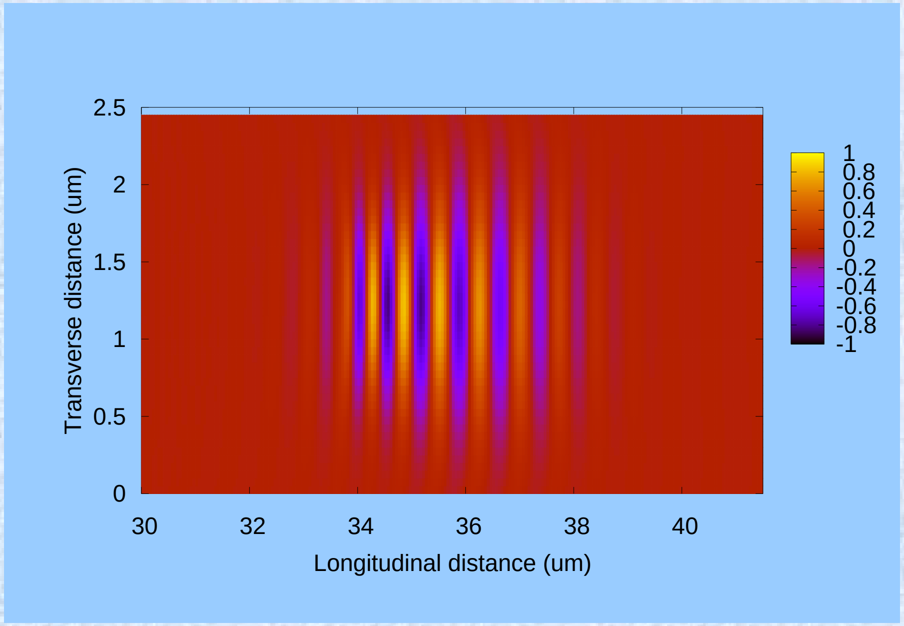
The signal profile inside the nonlinear core with positive nonlinearity

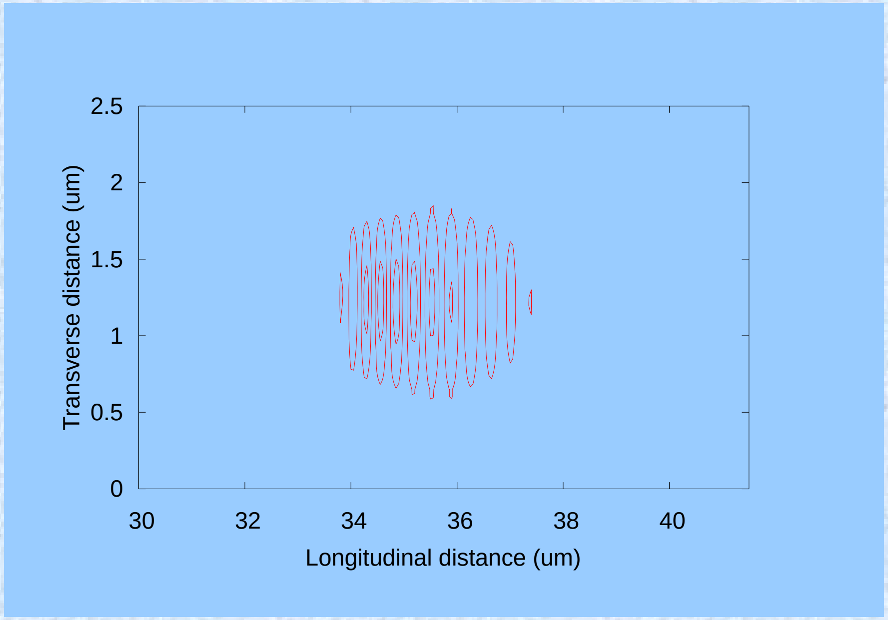


The profile when the core is represented by a dispersive nonlinear model

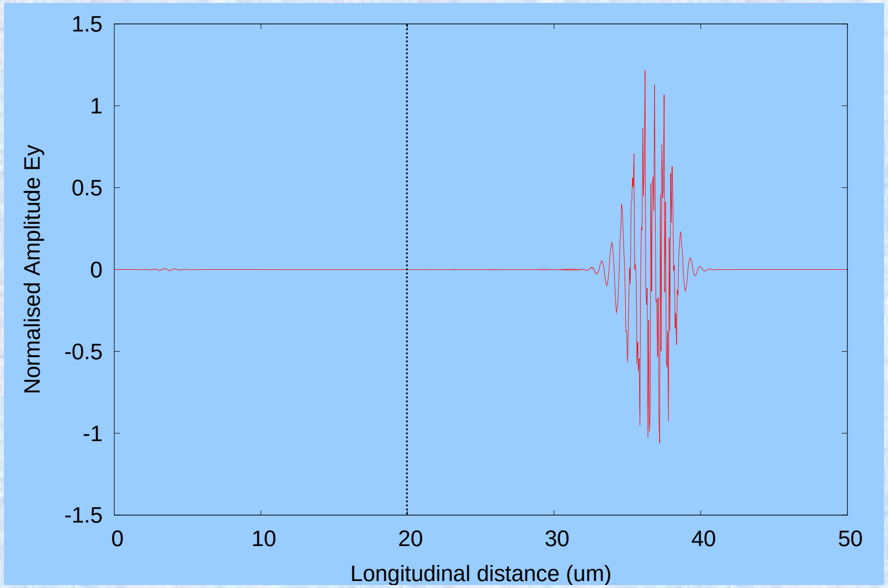


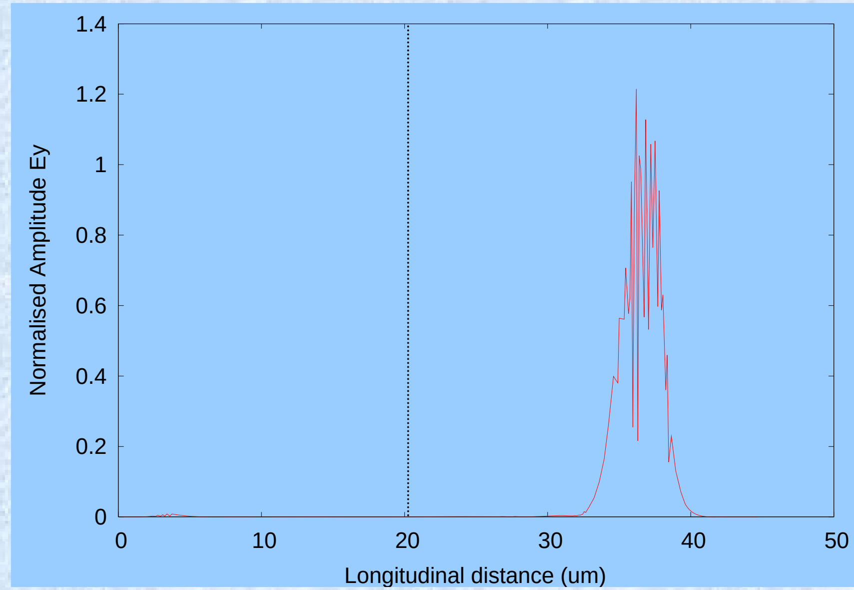


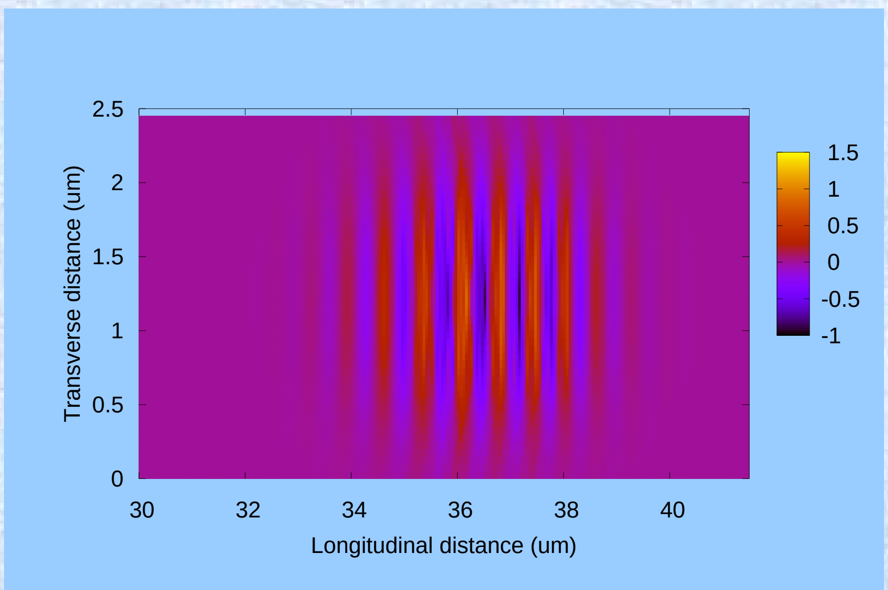


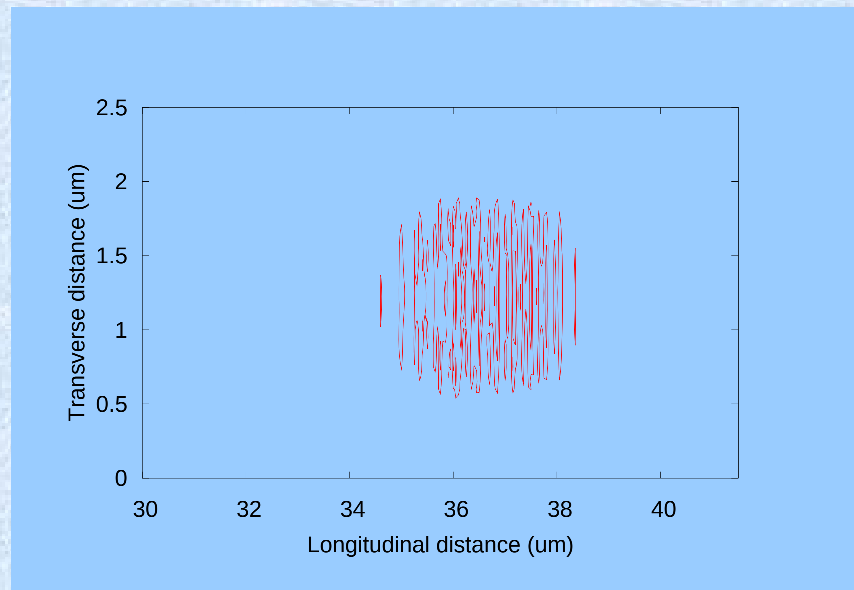


The pulse profile when core material has a negative nonlinearity coefficient

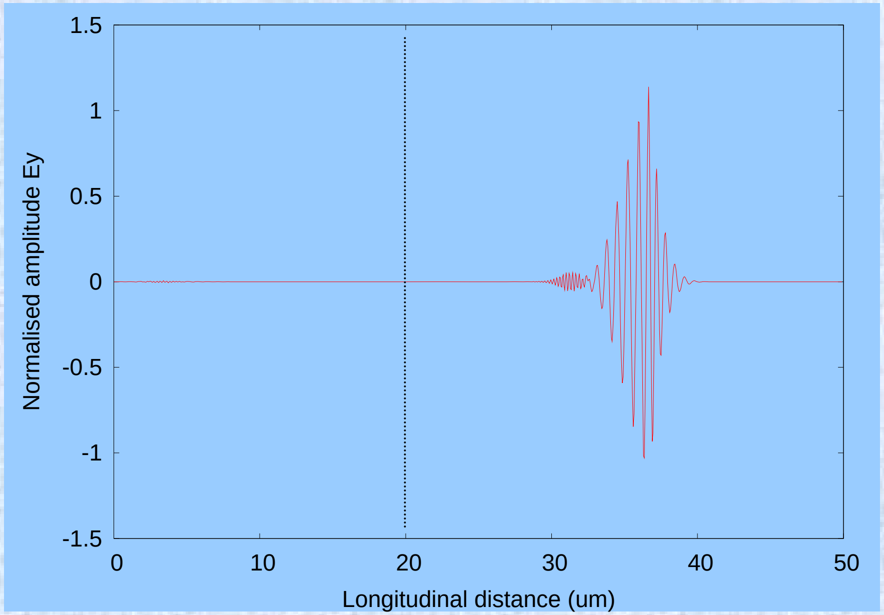


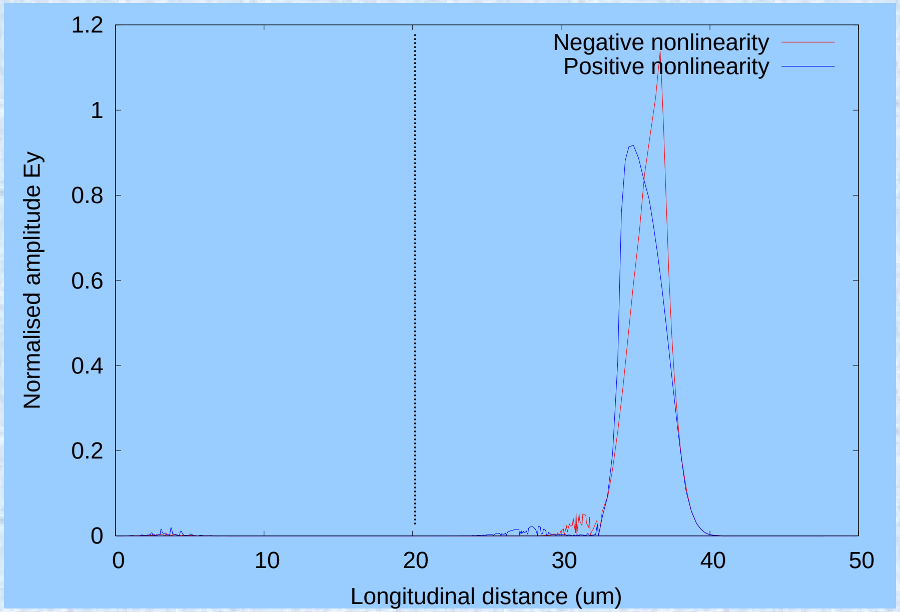


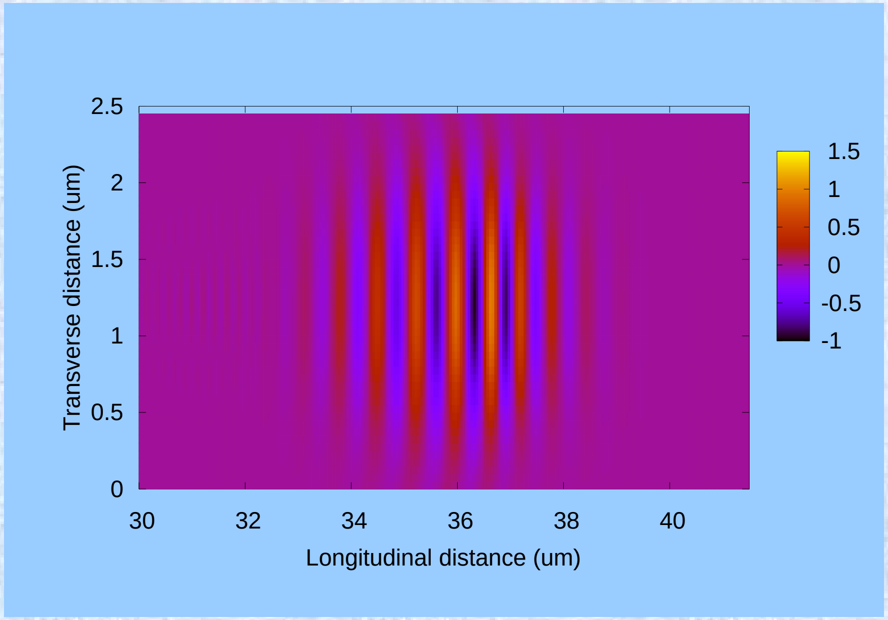


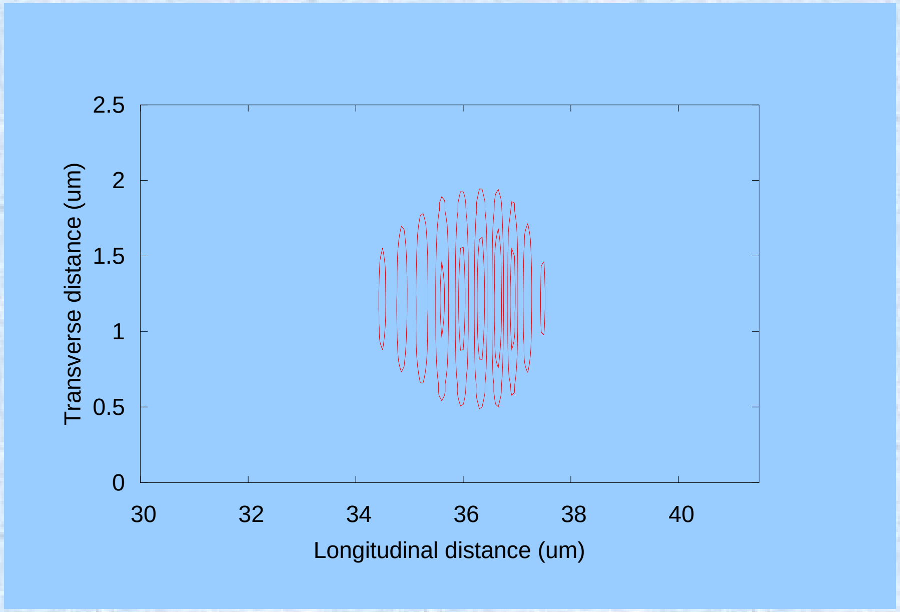


Dispersive material having a negative nonlinearity coefficient

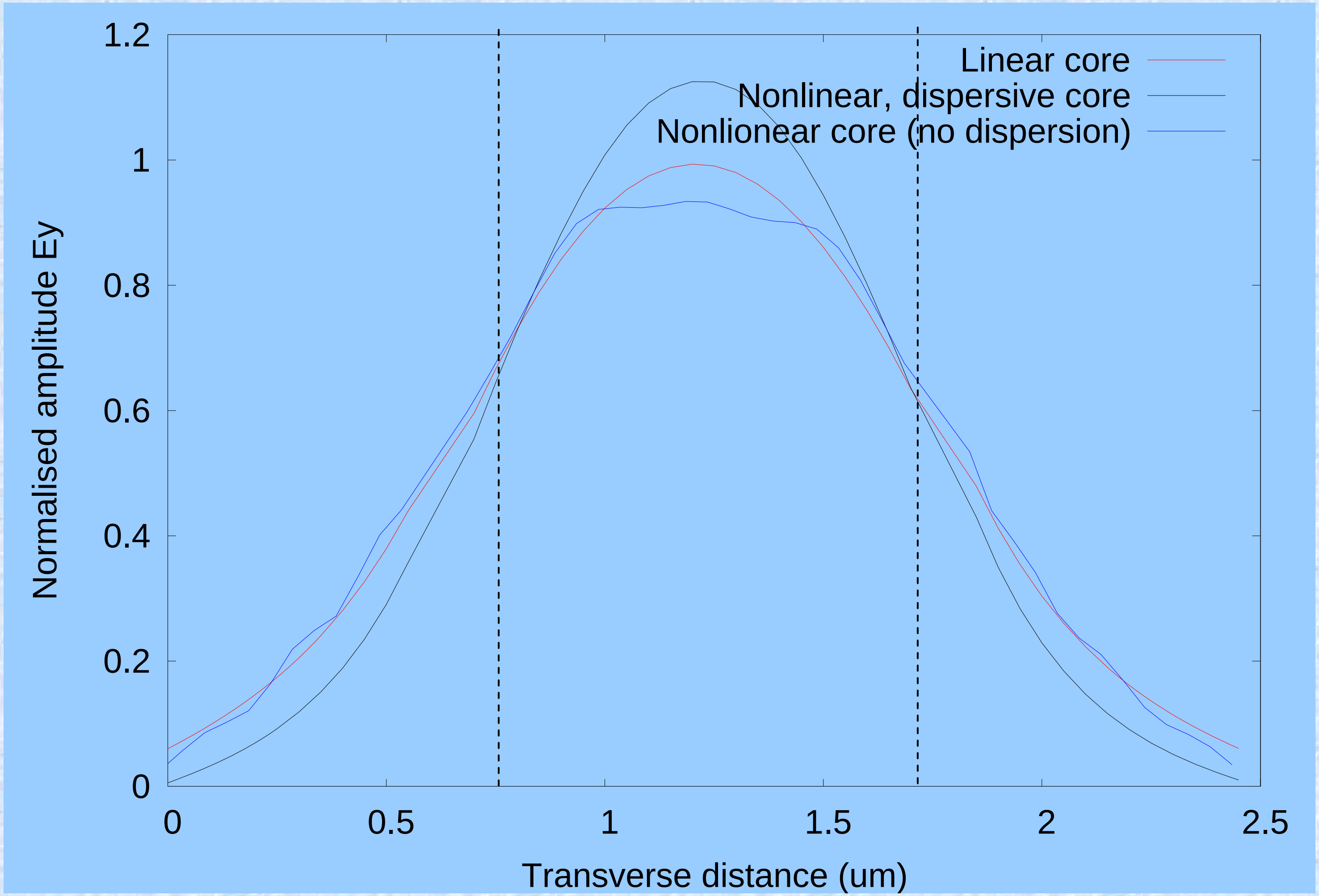








Transverse mode profile



Conclusions

- A TLM methodology for modeling nonlinear, dispersive materials is presented.
- The TLM method is not based on inherent assumptions like SVEA and gradual refractive-index change.
- A Duffing model of dispersive nonlinearity is used which is based on the Lorentz model.
- A TM-wave formulation is developed by employing a Jacobian method to solve the system of coupled nonlinear equations.
- The method gives vital spatio-temporal information about the waveguide in optical waveguides.
- More complex structures and phenomena can be simulated using the algorithm.

Thank you !