

Time Dependent Transport Through Molecular Devices

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Collaborators

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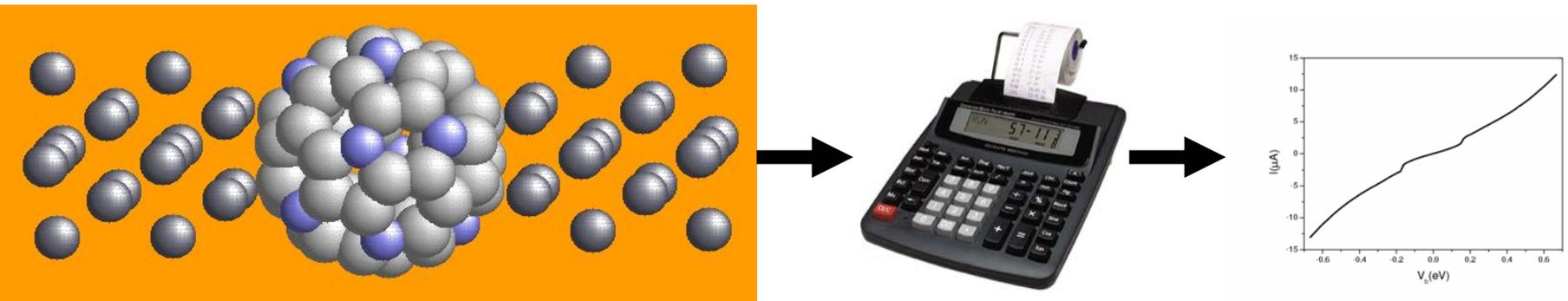
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Nano-device Transport Simulation

First principles Device Simulator



Our method



NEGF + DFT

Taylor, Guo, and Wang,
PRB **63**, 245407 (2001)



Non-equilibrium and non-linear
quantum transport properties



Chemical information
of molecular devices

Equivalent “KS” equation for an open system:

$$\left\{ \begin{array}{l} -\nabla^2 U(x) = 4\pi\rho(x) = 4\pi i \int dE [G^<(E, U)]_{xx} \\ G^<(E, U) = iG^r(E, U)[\Gamma_L f_L + \Gamma_R f_R]G^a(E, U) \\ G^r(E, U) = \frac{1}{E - H - U - V_{ex}[\rho] - \Sigma^r} \end{array} \right.$$

G^r expanded using s, p, d real space LCAO basis set

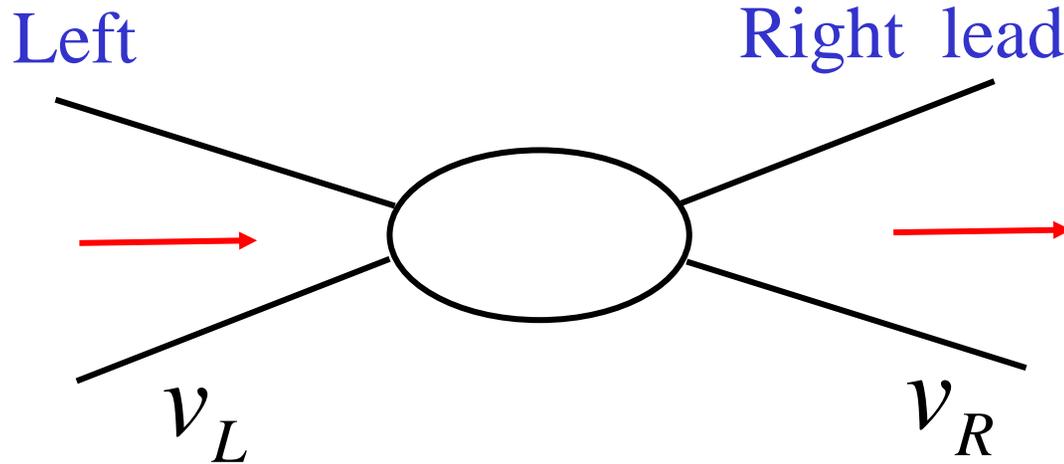
NEGF+DFT formalism

- When NEGF+DFT iteration converges, we obtain the potential landscape V on the Hartree level.
- This potential includes effect of charge transfer.
- Once the scattering potential V is known, it then becomes a scattering problem. We can use the NEGF formalism to calculate all kinds of quantum transport properties.

Issues in time dependent quantum transport

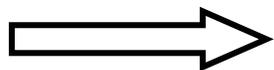
- *Steady state ac transport*
- Charge relaxation time
- Transient transport (response time)

ac transport theory



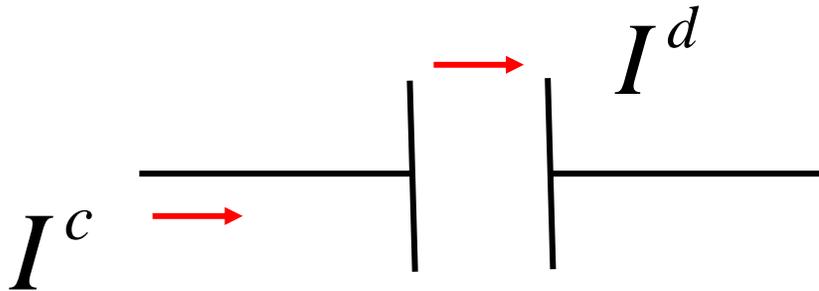
Conduction current : $I^c = \frac{dq}{dt}$

Continuity equation : $I_L^c + I_R^c + \frac{dQ}{dt} = 0$



Current is not conserved !

What is $\frac{dQ}{dt}$?



Parallel plate capacitor

Displacement current $J^d = \frac{1}{4\pi} \frac{\partial D}{\partial t}$ from Jackson's book

$$\frac{dQ}{dt} = \frac{\partial}{\partial t} \int \rho dV = \int \frac{1}{4\pi} \nabla \cdot \frac{\partial D}{\partial t} dV = \nabla \cdot \int J^d dV = I_L^d + I_R^d$$

Poisson Eq. $\nabla \cdot D = 4\pi\rho \longrightarrow \frac{\partial \rho}{\partial t} = \frac{1}{4\pi} \nabla \cdot \frac{\partial D}{\partial t}$

Total displacement current: $I^d = \frac{dQ}{dt}$

Q: charge accumulation in the device.

$$I_L^c + I_R^c + \frac{dQ}{dt} = 0$$

Conclusion

Current consists of two contributions at ac bias: the usual conduction current and the displacement current due to Coulomb interaction.

* Define total current

$$I^t = I^c + I^d$$

So that $I_L^t + I_R^t = 0$

Current conservation

- * One could partition the total displacement current into each lead

$$I_L^d + I_R^d = I^d$$

This has been done by Buttiker when external bias is small.

Buttiker et al, PRL **70**, 4114 (1993) and Wang et al, PRL **82**, 398 (1999).

- * Dynamic conductance $I^t(\omega) = G(\omega)(v_L - v_R)$

a). $\omega = 0$ $G(0) \longrightarrow$ DC conductance, a real quantity.

b). $\omega \neq 0$ $G(\omega) = G_R(\omega) + iG_I(\omega)$



expressed in terms of
non-equilibrium
Green's function



dissipation



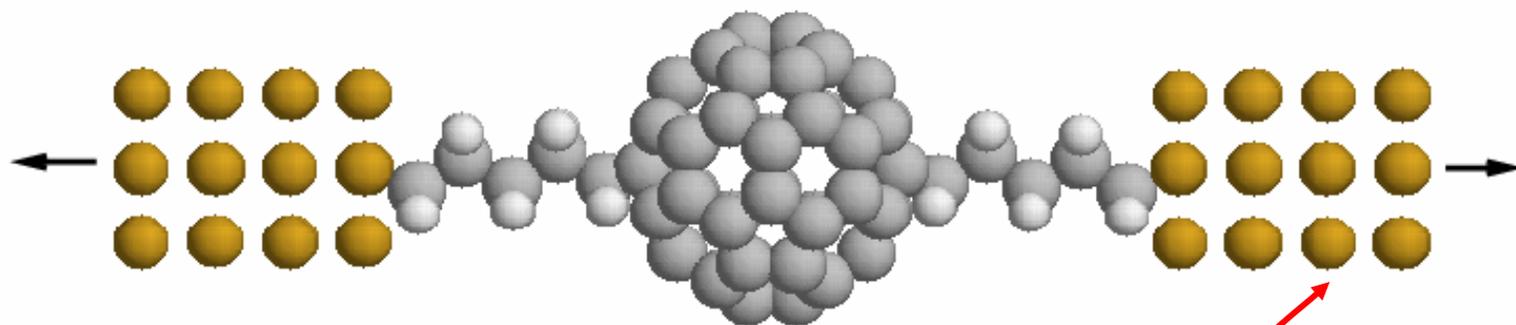
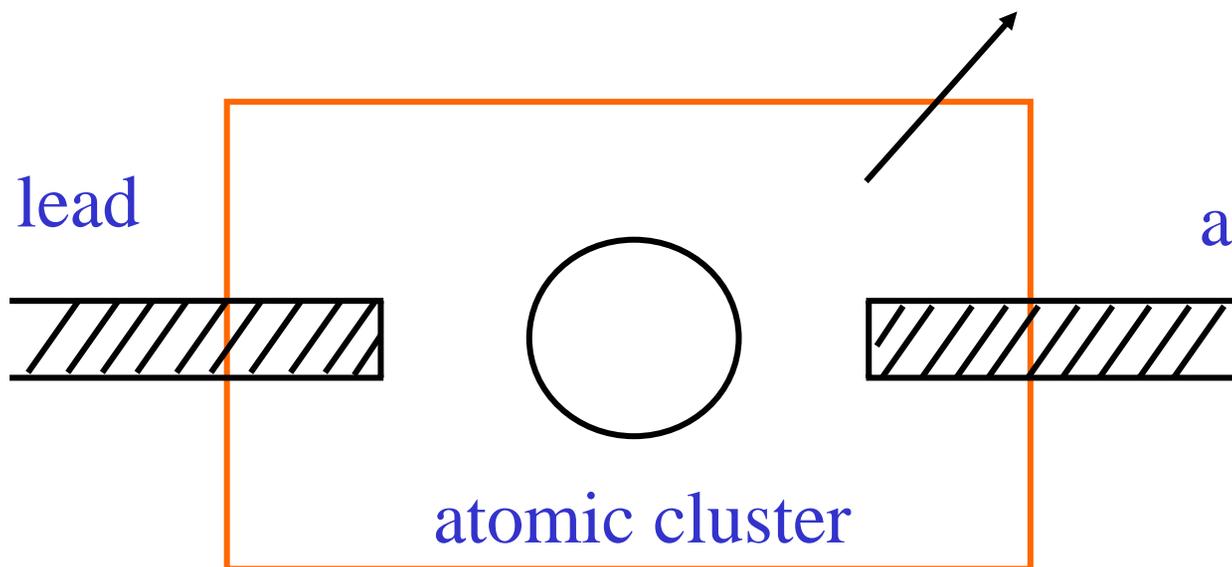
phase difference
between I and v

Typical molecular device

Simulation box

atomic lead

atomic lead



Al lead (100)

CH₂

C₆₀

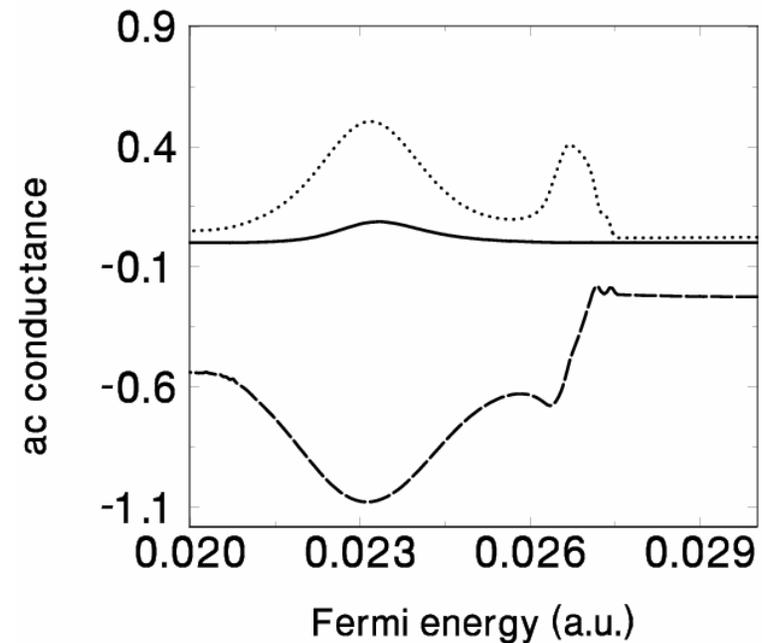
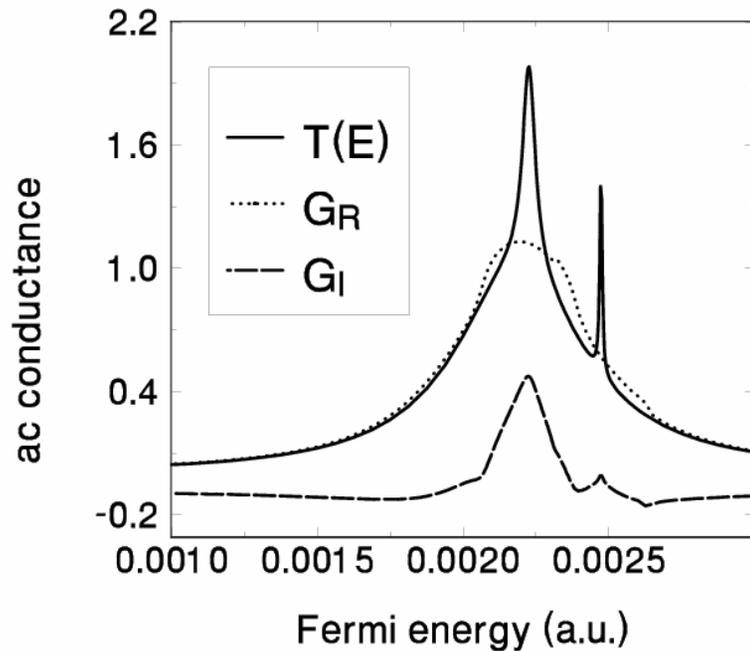
Laser

Due to highly resistive CH₂ chain the system is in the tunneling regime.

Results

J.L.Wu, B.G. Wang, J. Wang, and
H. Guo, Phys. Rev. B 72, 195324 (2005)

Frequency: 100GHz



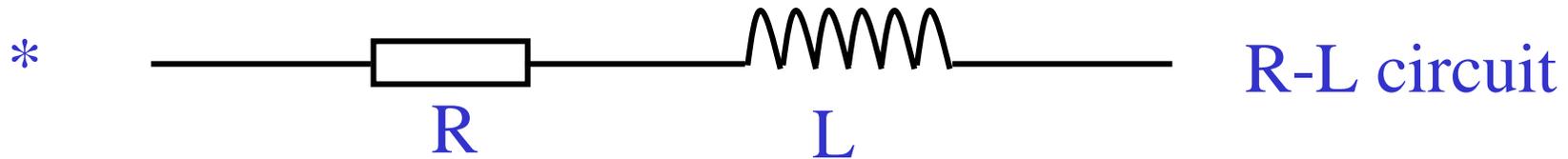
Fermi energy can be shifted by gate voltage
See Cronin et al, APL 84, 2052 (2004).

* two kinds of resonant behaviors

a). $T = 2$, near the resonance, $G_R(\omega) < G_R(0)$

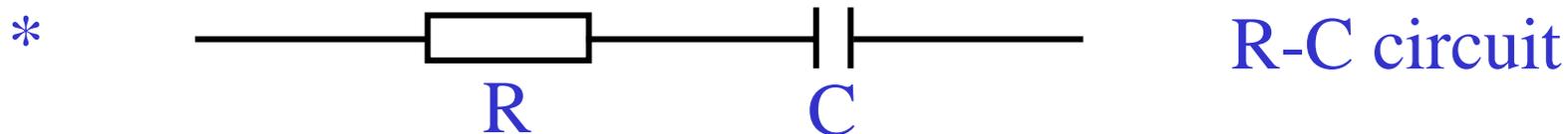
b). $T \ll 1$, $G_R(\omega) > G_R(0)$

Classical circuit (qualitatively understanding)



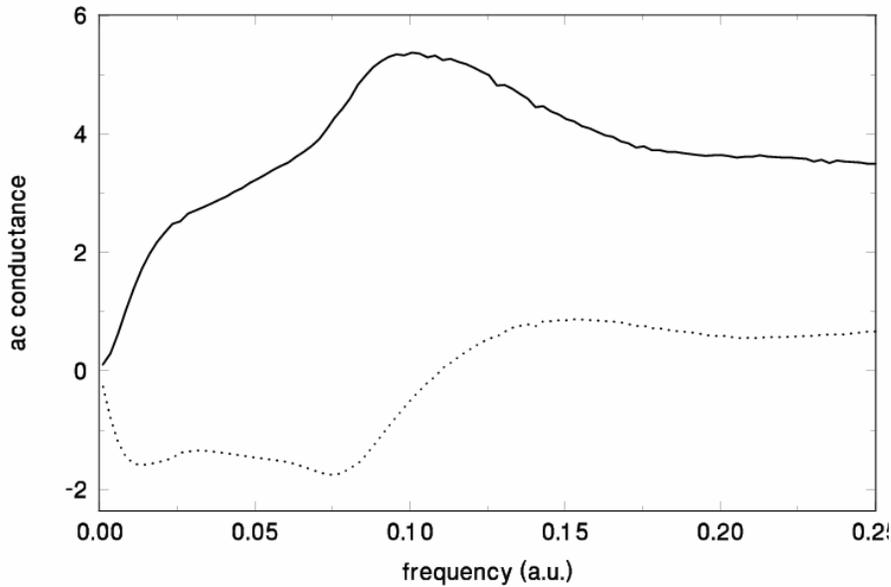
$$G(\omega) = \frac{1}{R - i\omega L} \quad (Z = R - i\omega L)$$

$$\approx \frac{1}{R} + \frac{i\omega L}{R^2} - \frac{\omega^2 L^2}{R^3} + \dots$$

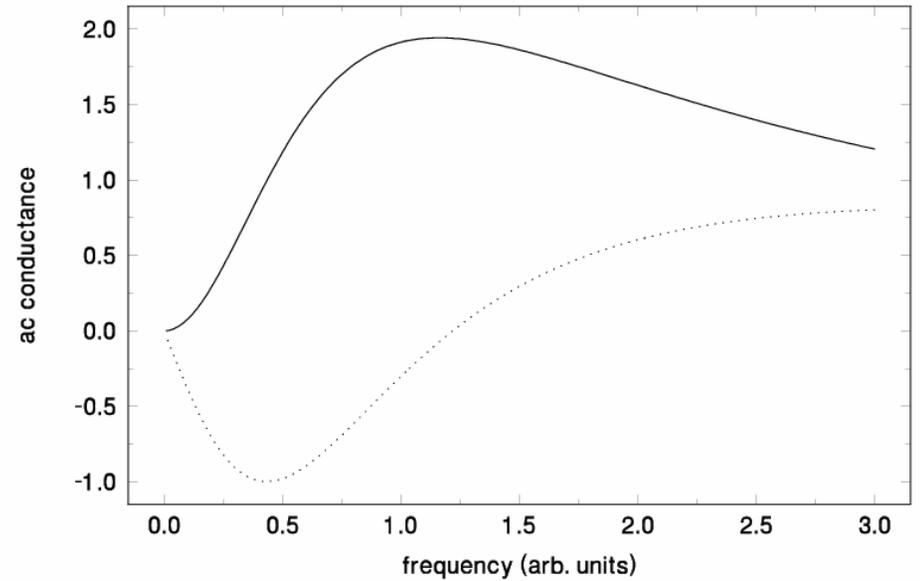


$$G(\omega) = \frac{1}{R - \frac{1}{i\omega C}} = \frac{-i\omega C}{1 - i\omega C R} = -i\omega C + \omega^2 C^2 R$$

Comparison (at fixed Fermi energy)

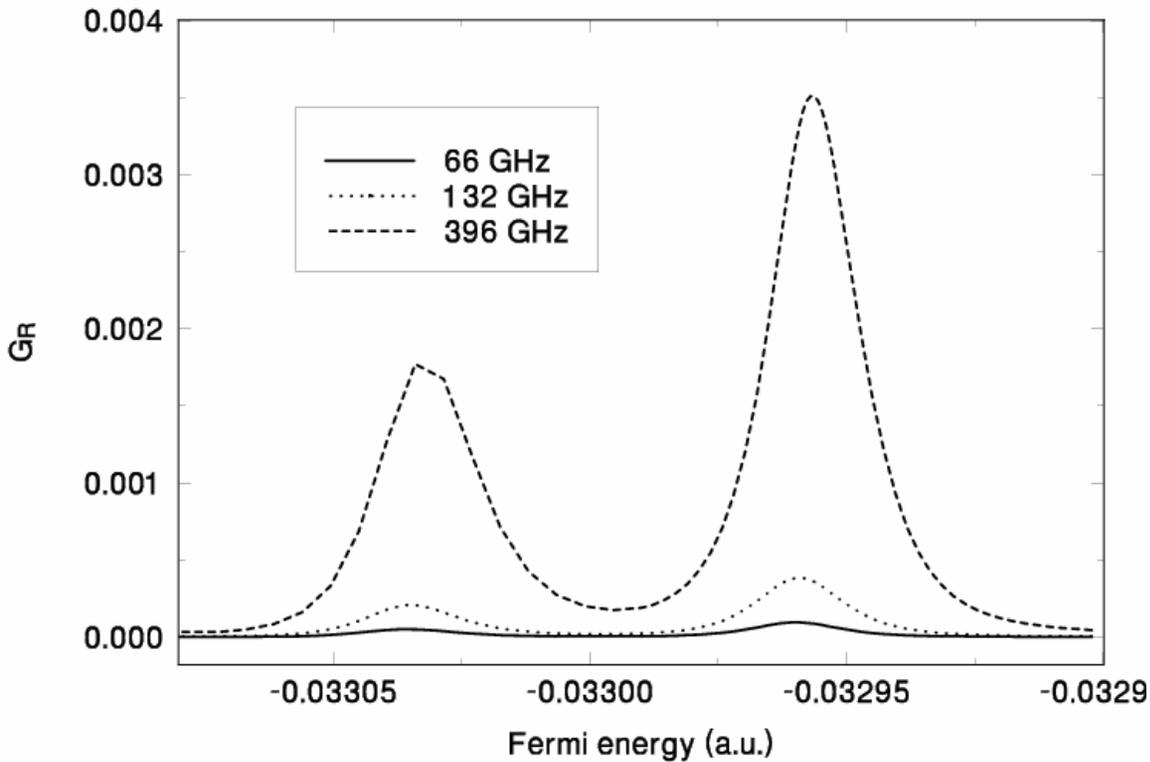


Ab initio calculation

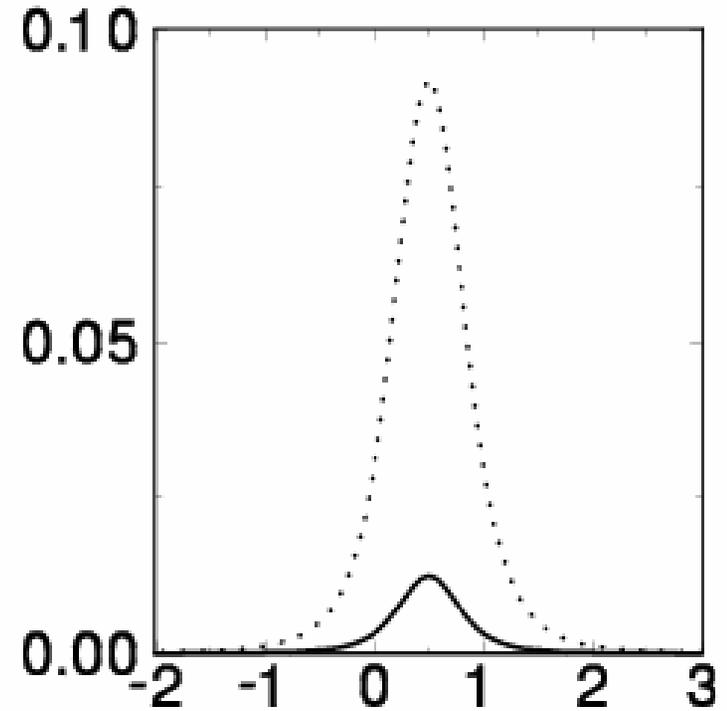


Model calculation

Large magnification



Ab initio calculation



Model calculation

On experimental side

- Oscillation up to 0.7 THz has been achieved in InAs/AlSb resonant tunneling diodes.
(Brown et al APL **58**, 2291 (1991).)
- SWNT transistor operated at 50GHz has been achieved. (Rosenblatt et al APL **87**, 153111 (2005).)
- It would be a challenge to measure the large magnification of ac conductance at 100GHz.

Issues in time dependent quantum transport

- Steady state ac transport
- Charge relaxation time
- Transient transport (response time)

For a nano-capacitor

Frequency dependent conductance

$$G(\omega) = -i\omega C_{\mu} + \omega^2 C_{\mu}^2 R_q$$

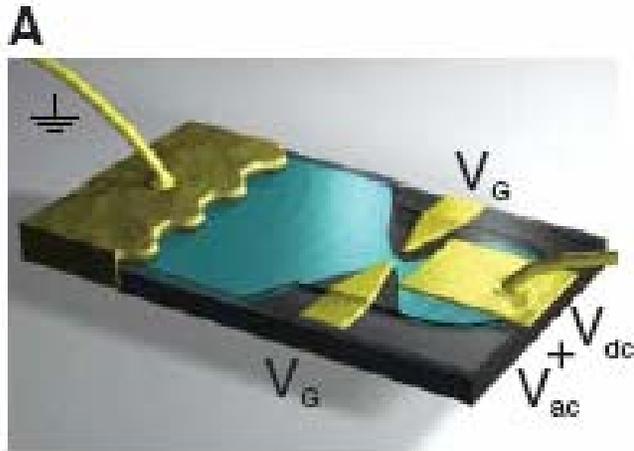
C_{μ} Electrochemical capacitance

For nano-capacitor $\frac{1}{C_{\mu}} = \frac{1}{C} + \frac{1}{C_{DOS}}$ T.P. Smith et al,
Phys. Rev. B32, 2695 (1985)

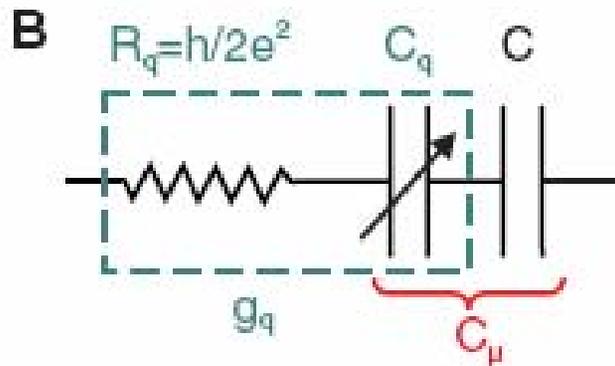
R_q Charge relaxation resistance, $h/2e^*e$ for a single channel

$\tau = R_q C_{\mu}$ Charge relaxation time

Recent experiment on mesoscopic systems,
Gabelli et al, Science 313, 499 (2006)

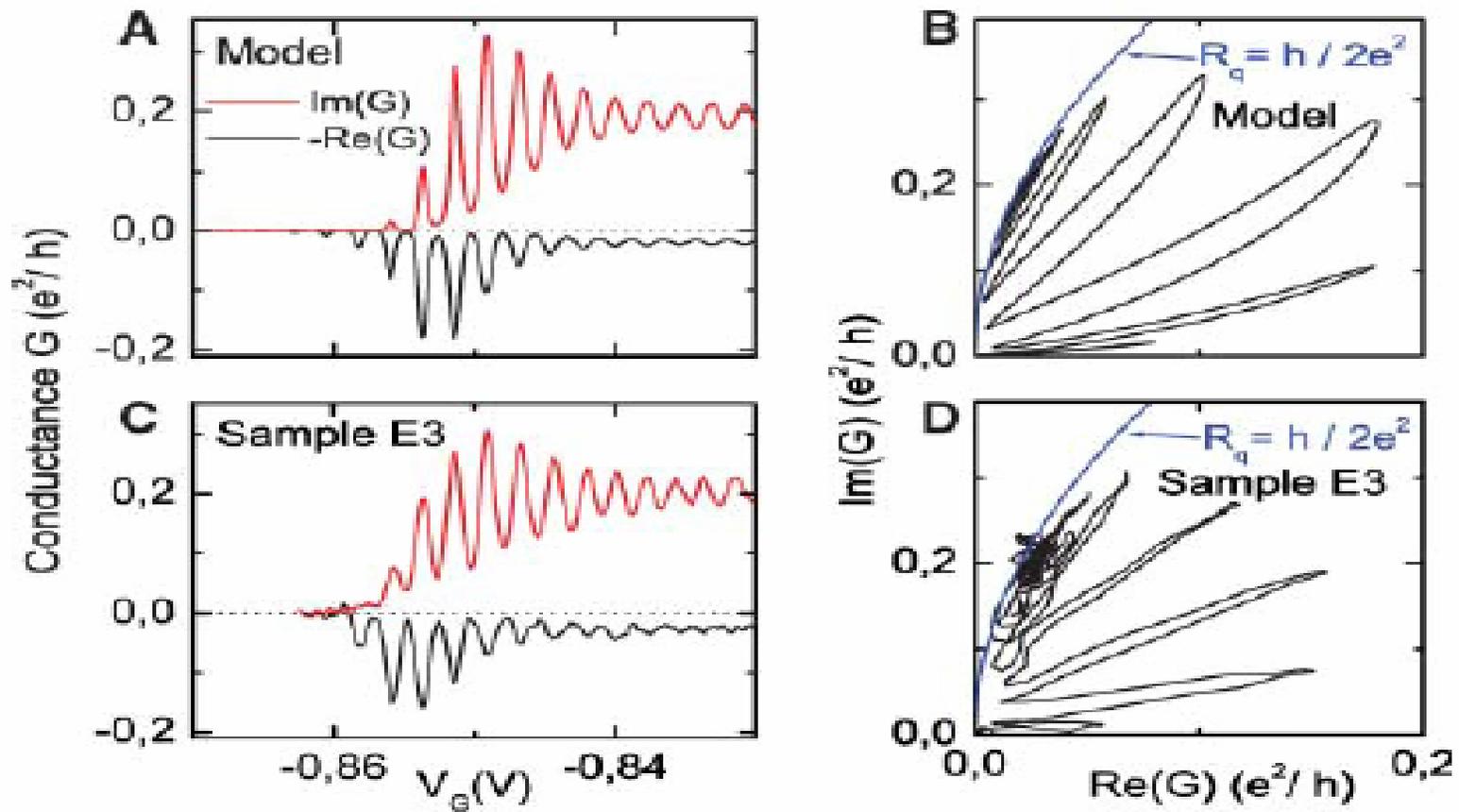


$$\frac{1}{g_q(\omega)} = \frac{h}{2e^2} + \frac{1}{-i\omega C_q}$$



Frequency=1.2GHz

$$G = \frac{-i\omega C g_q(\omega)}{-i\omega C + g_q(\omega)} = \frac{-i\omega C_\mu (2e^2/h)}{-i\omega C_\mu + (2e^2/h)}$$



What is the nature of charge relaxation for atomic devices?

In addition,

- Non-equilibrium relaxation resistance?
- What happens for a leaky nano-capacitor?
- Frequency dependent capacitance and charge relaxation resistance?

Issues in time dependent quantum transport

- Steady state ac transport
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How fast a nano-electronic device can turn on/off a current ?

- For nano-devices, a quantum mechanical first principle calculation is needed.
- Problem of atomic leads: must go beyond the wideband limit.
- Our theoretical formalisms:
 - (1). Time domain, [Zhu et al, Phys. Rev. B 71, 075317 \(2005\)](#)
 - (2). Exact solution in k-space for a pulse, [Maciejko, Wang, and Guo, Phys. Rev. B 74, 085324 \(2006\)](#).
- Our theoretical results on model calculation show that there is a big difference between wideband limit and exact solution.

Difficulty with ab initio calculation

- Numerical problem with a triple integral.
- An ansatz can be used so that the triple integral reduces to a double integral.
- Very calculational demanding.
- Alternatively solution: find out all the poles of retarded Green's function.

Thank You!