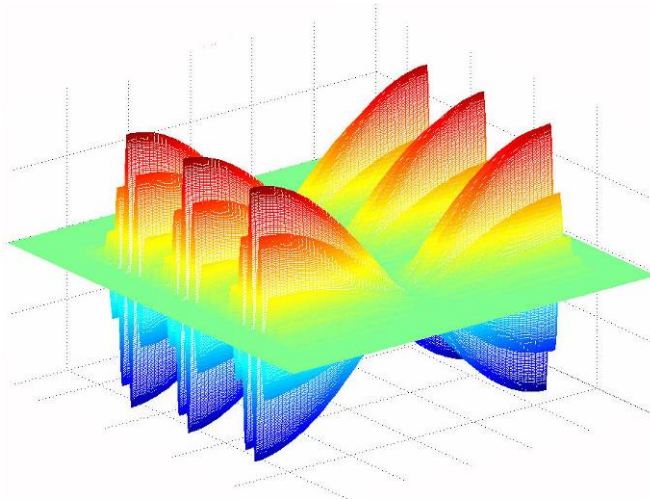


All-optical Coherent Control of Spin Dynamics in Semiconductor Quantum Dots

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Electron spin in a single Quantum Dot

attractive qubit candidate for physical implementation of quantum computation

- **Severe limitation for quantum computing with excitons/biexcitons in QDs : coupling to lattice phonons and non-scalability**

**Long spin coherence and spin relaxation times
ensuring long-lived quantum state:**

- **Long spin relaxation lifetimes $T_1 \sim \text{ms}$ range (Elzerman et al. Nature, 430,431 (2004); Kroutvar et al., Nature 432,81 (2004))**
- **Long electron spin coherence times T_2 measured in bulk SCs exceeding 100 ns (Kikkawa and Awschalom, Science 287, 473 (2000))**
- **Long electron spin coherence times $T_2 \sim 2T_1$ expected in QDs**
- **Extended spin coherence time T_2 in the high-intensity excitation regime when Rabi oscillations occur (Petta et al. Science 309, 2180 (2005))**
- **Optical spin orientation: conversion of quantum information from light to spin and vice versa**
- **The ability to faithfully transmit 'flying qubits' between distant locations**
- **Scalable model (to ≥ 100 coupled qubits)**

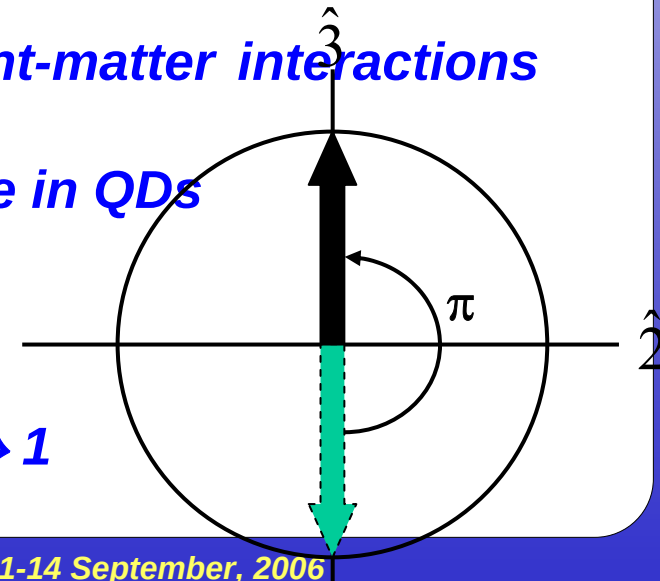


Quantum coherent optical control of electron spin in nanostructures

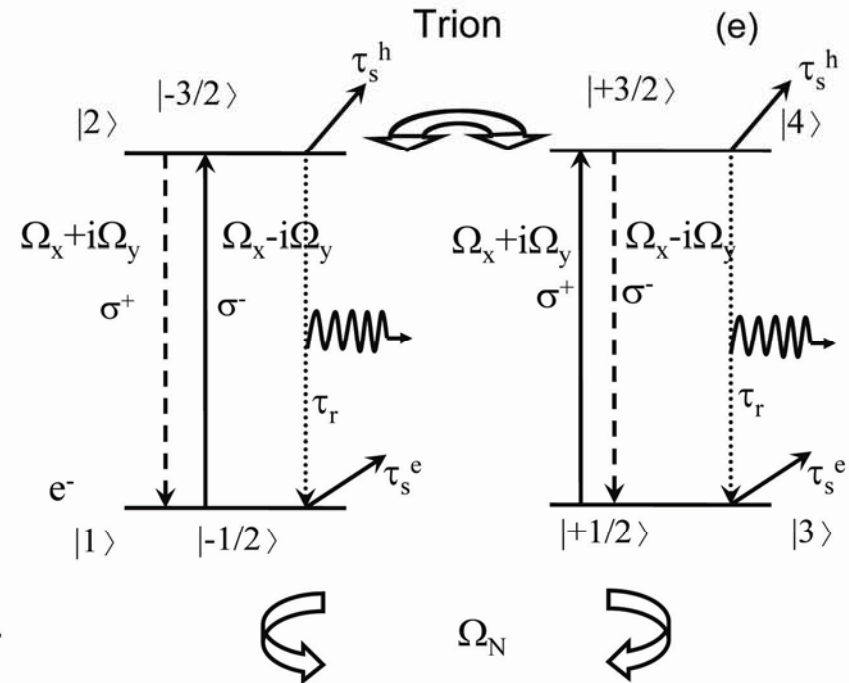
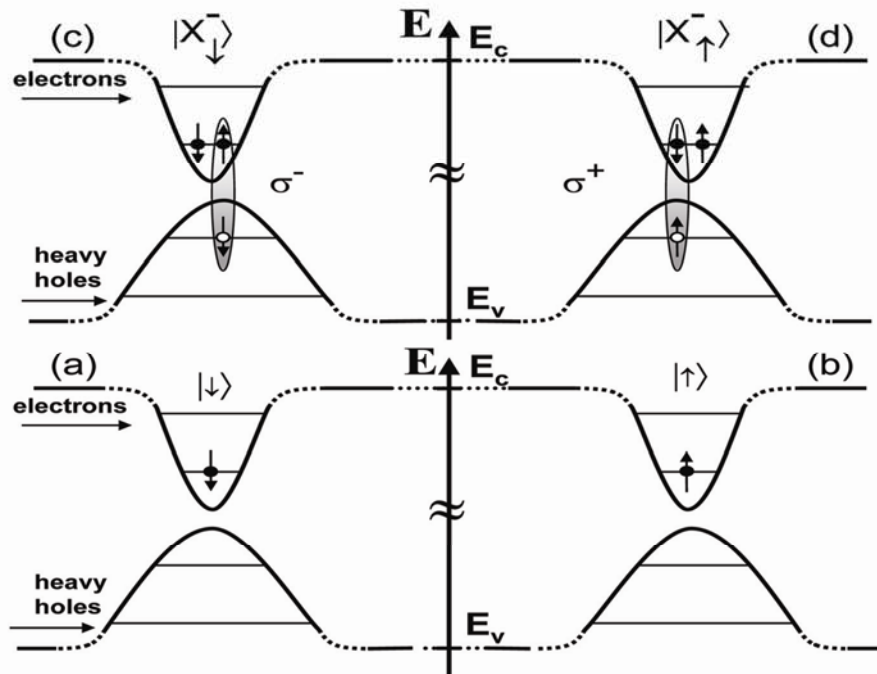
- Predictable manipulation of the quantum dynamics of the system on a time scale shorter than typical dephasing times
- Necessary condition for quantum coherence: use of ultrashort optical pulses
- Recent progress in coherent preparation and detection of single electron spin states (Hartmann et al. PRL 84, 5648 (2000); Warburton et al. Nature 405, 926 (2000); Tischler et al. PRB 66, 081310 (2002); Rugar et al. Nature 430, 329 (2004); Xiao et al. ibid. 430, 435 (2004))

Exploiting the resonant optical nonlinearities of the QDs in the high-intensity excitation regime:

- Optical Rabi oscillations: coherent nonlinear light-matter interactions in discrete-level systems
- Population flopping over many periods is possible in QDs
 - systems with long coherence lifetimes
 - large dipole moments
- A Rabi flop corresponds to one-qubit rotation where the bit is rotated through π (i.e. from state 0 $\rightarrow$ 1 and vice versa)



Energy-level diagram of a negatively charged exciton (trion)



Similarities between QD and atomic systems:

- Discrete-level electronic structure (3D carrier quantum confinement)
- Fulfil DiVincenzo criteria: a few-energy level system, well isolated spectrally and insensitive to external perturbations (due to localised eigenstates)

Model of the coherent spin dynamics Theoretical fundamentals

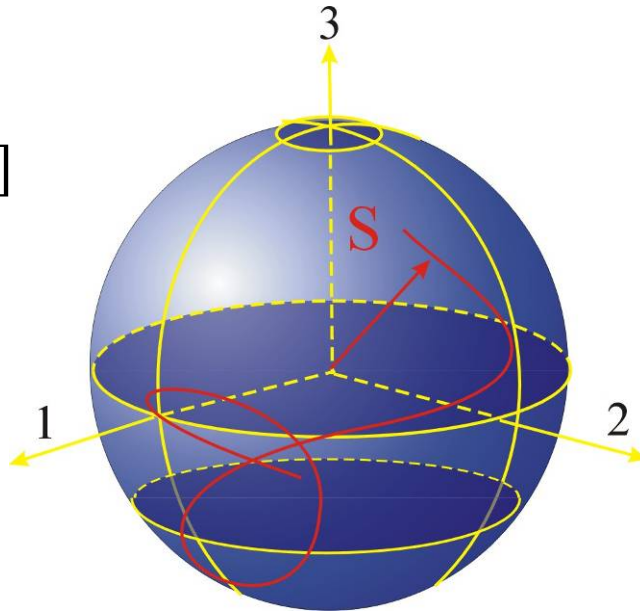
Dynamical evolution of an N-level quantum system:

**Liouville equation
(Schrödinger picture):**

$$i\hbar \frac{\partial \hat{\rho}}{\partial t} = [\hat{H}, \hat{\rho}]$$

**Pseudospin equation for the real state
vector $S = (S_1, S_2, \dots, S_{N^2-1})$ (Heisenberg picture)
(Hioe and Eberly, PRL, 47, 838, 1981)**

$$\frac{\partial S_i}{\partial t} = f_{ijk} \gamma_j S_k \quad i, j, k = 1, \dots, N^2 - 1$$



Using Gell-Mann's λ -generators of the $SU(N)$ Lie algebra:

Pseudospin (coherence) vector

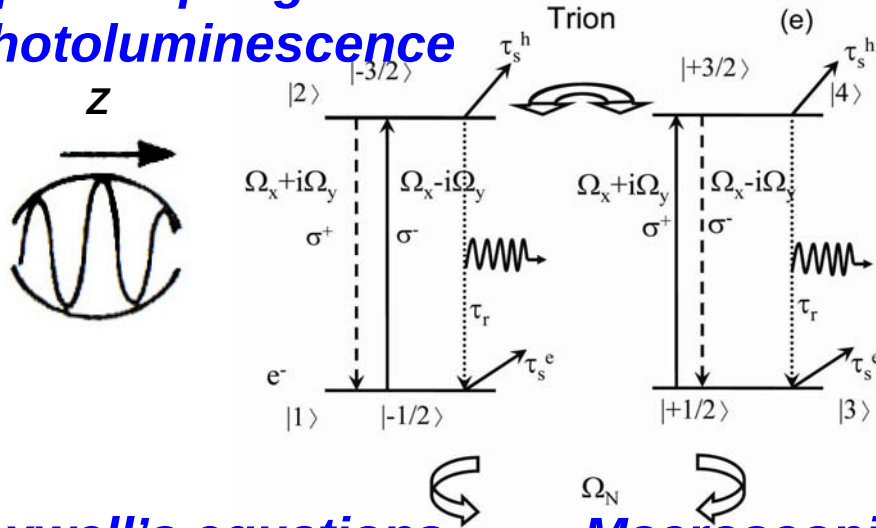
$$S_j(t) = \text{Tr}(\hat{\rho}(t) \hat{\lambda}_j)$$

torque vector

$$\gamma_j(t) = \frac{1}{\hbar} \text{Tr}(\hat{H}(t) \hat{\lambda}_j)$$

Coherent Maxwell-pseudospin equations for a four-level system

Optical spin generation and readout of the electron spin state via polarised photoluminescence



$$\hat{H} = \hbar \begin{pmatrix} 0 & -\frac{1}{2}(\Omega_x - i\Omega_y) & 0 & 0 \\ -\frac{1}{2}(\Omega_x + i\Omega_y) & \omega_0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2}(\Omega_x + i\Omega_y) \\ 0 & 0 & -\frac{1}{2}(\Omega_x - i\Omega_y) & \omega_0 \end{pmatrix}$$

$$\Omega_x = \frac{\wp}{\hbar} E_x; \quad \Omega_y = \frac{\wp}{\hbar} E_y$$

Maxwell's equations

Macroscopic polarisation

Pseudospin equations

$$\begin{cases} \frac{\partial \mathbf{H}}{\partial t} = -\frac{1}{\mu} \nabla \times \mathbf{E} \\ \frac{\partial \mathbf{E}}{\partial t} = \frac{1}{\varepsilon} \nabla \times \mathbf{H} - \frac{1}{\varepsilon} \frac{\partial \mathbf{P}}{\partial t} \end{cases}$$

$$\begin{aligned} P_x &= -\wp N_a (S_1 + S_6) \\ P_y &= -\wp N_a (S_7 - S_{12}) \end{aligned}$$

$$\sigma = \text{diag}(1/T_1, 1/T_2, \dots, 1/T_{15})$$

$$\frac{\partial \mathbf{S}}{\partial t} = \mathbf{M} \mathbf{S} - \sigma (\mathbf{S} - \mathbf{S}_E)$$

$$\mathbf{M} = \begin{pmatrix} \underline{P}_{6 \times 6} & \underline{Q}_{6 \times 6} & \underline{R}_{6 \times 3} \\ -\underline{Q}_{6 \times 6}^T & \underline{P}_{6 \times 6} & \underline{S}_{6 \times 3} \\ -\underline{R}_{3 \times 6}^T & -\underline{S}_{3 \times 6}^T & \underline{0}_{3 \times 3} \end{pmatrix}$$

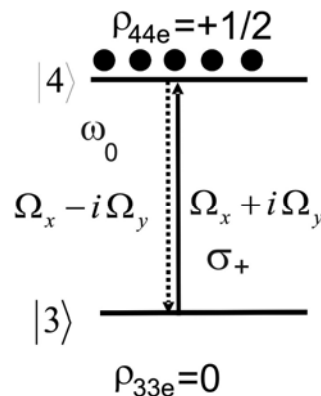
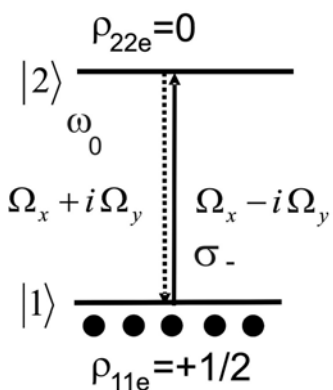
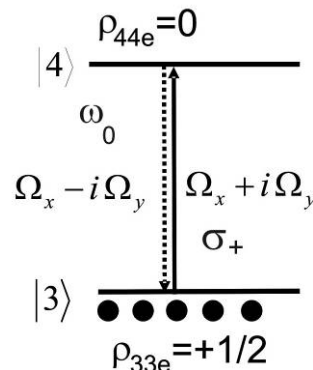
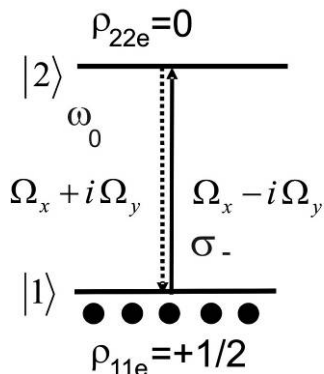
Source optical field

$$\sigma^- \begin{cases} E_x(z=0, t) = E_0 \text{sech}(10\Gamma) \cos(\omega_0 t) \\ E_y(z=0, t) = -E_0 \text{sech}(10\Gamma) \sin(\omega_0 t) \end{cases}$$

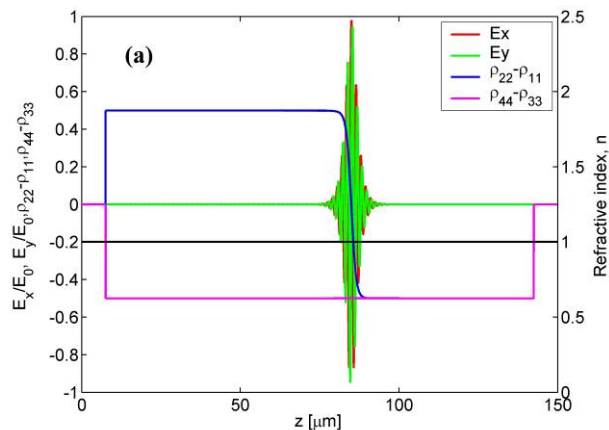
$$\sigma^+ \begin{cases} E_x(z=0, t) = E_0 \text{sech}(10\Gamma) \cos(\omega_0 t) \\ E_y(z=0, t) = E_0 \text{sech}(10\Gamma) \sin(\omega_0 t) \end{cases} \quad \Gamma = [t - (T_p/2)] / (T_p/2)$$

Selective spin state excitation

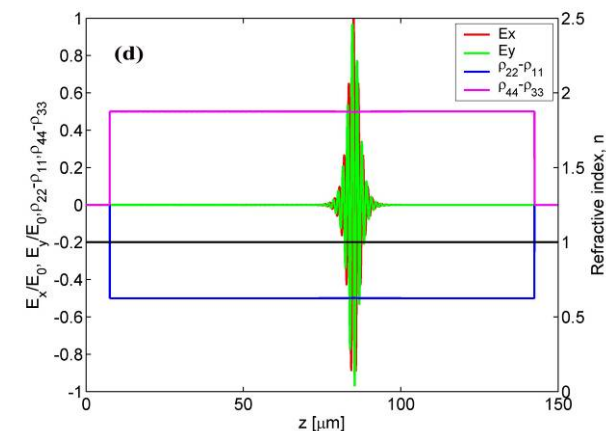
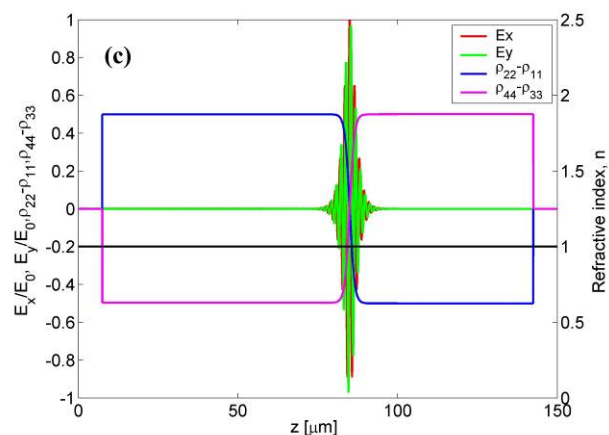
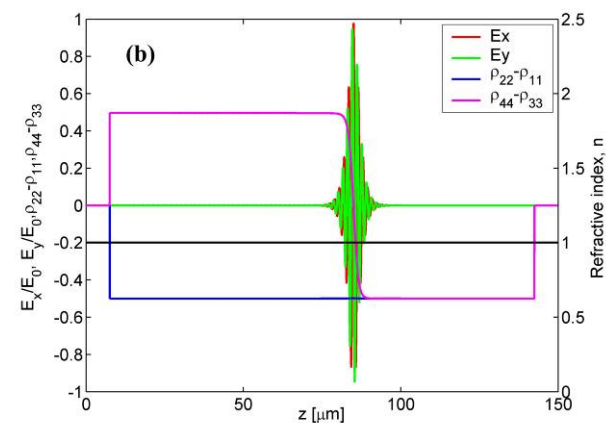
π -pulse $E_0=2.1093 \times 10^9 \text{ V.m}^{-1}$,
 $T_p=100 \text{ fs}$, $\lambda=1.5 \mu\text{m}$



$\sigma_- (\pi\text{-pulse})$



$\sigma_+ (\pi\text{-pulse})$



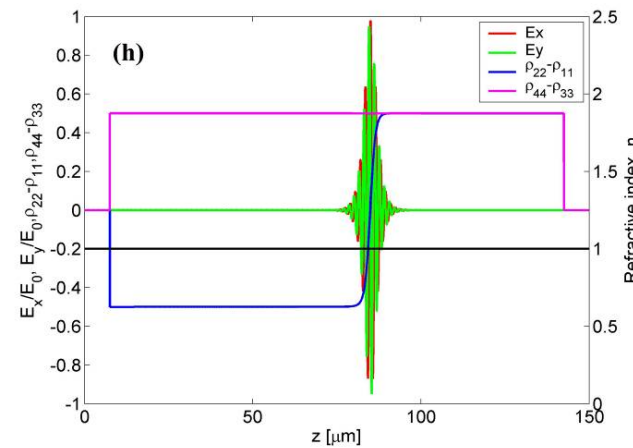
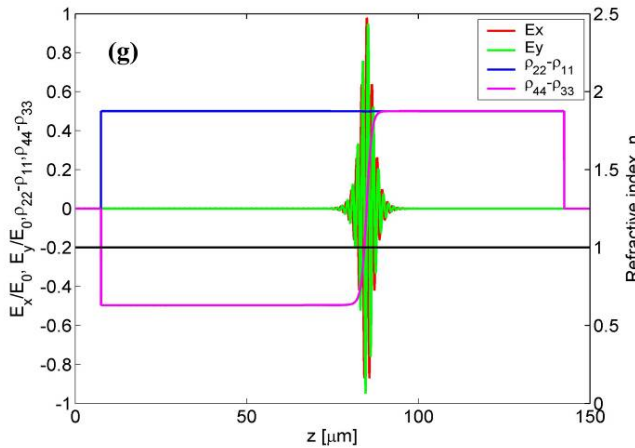
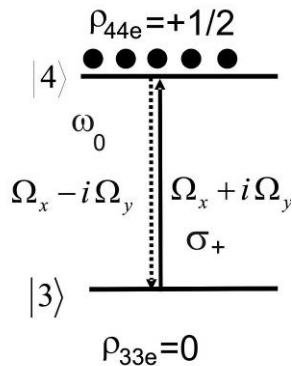
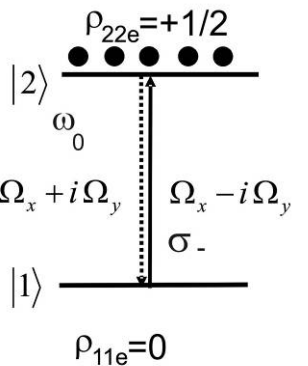
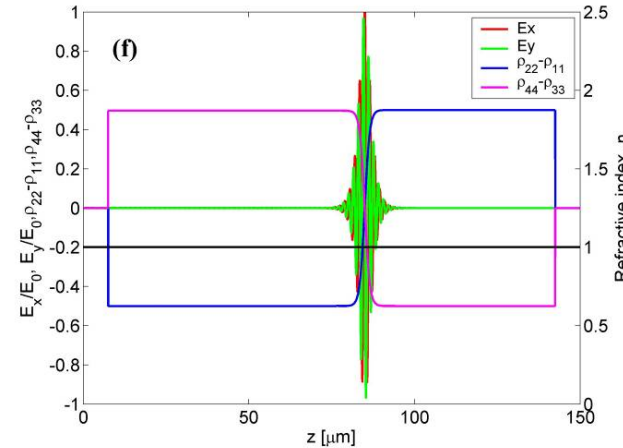
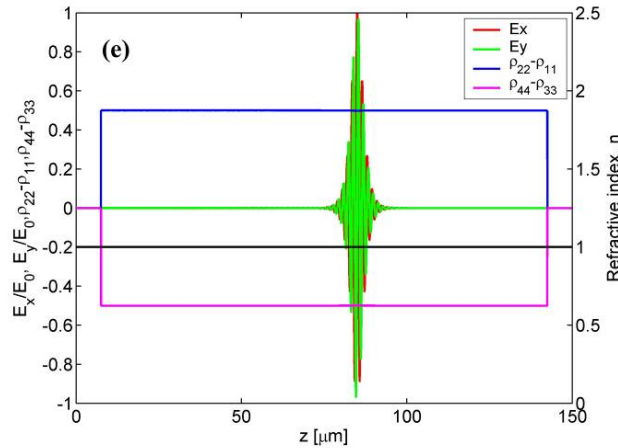
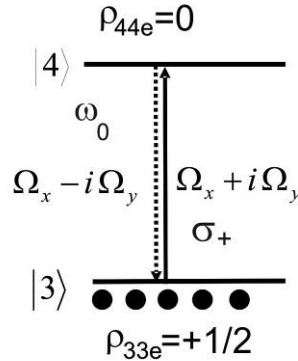
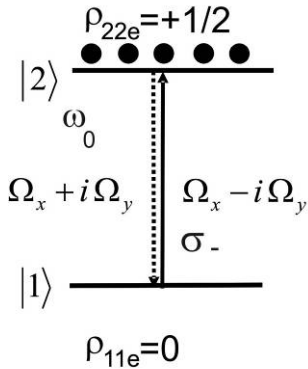
$T_1=T_6=T_7=T_{12}=100 \text{ ps}$,
 $T_{13}=T_{14}=T_{15}=100 \text{ ps}$

Selective spin state excitation

π -pulse $E_0 = 2.1093 \times 10^9 \text{ V.m}^{-1}$,
 $T_p = 100 \text{ fs}$, $\lambda = 1.5 \mu\text{m}$

σ^- (π -pulse)

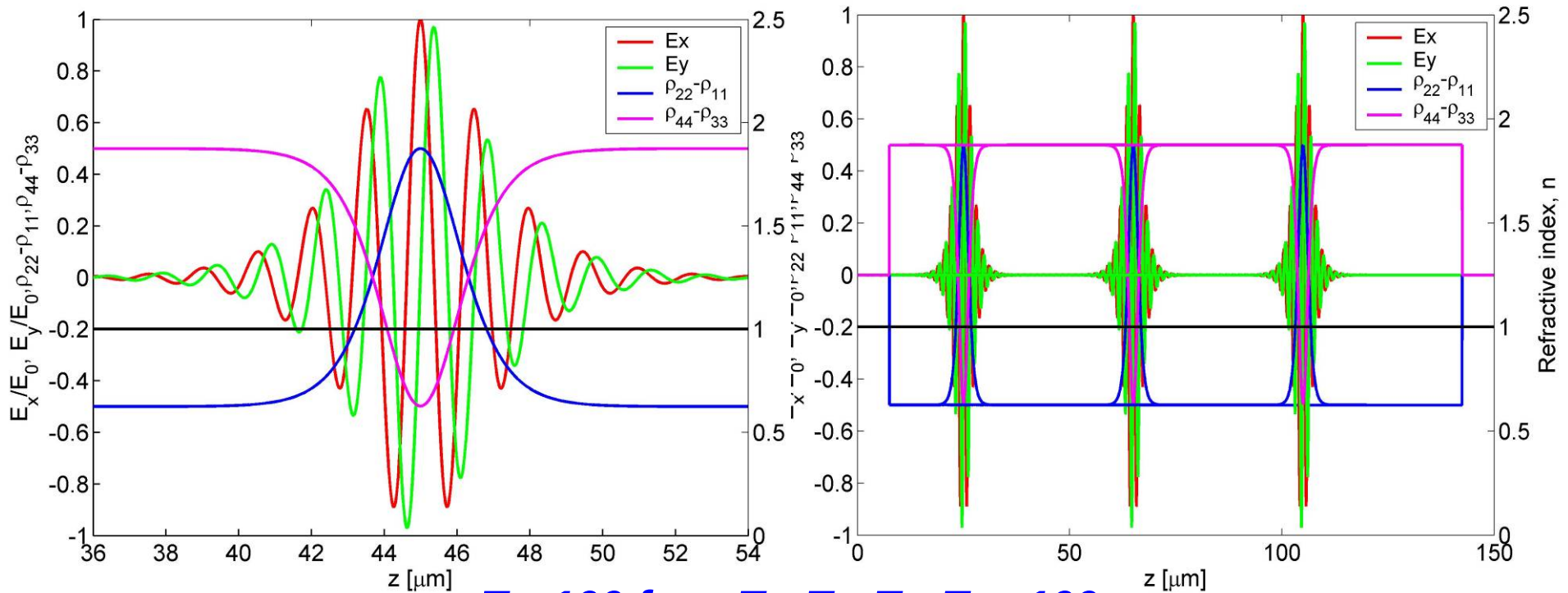
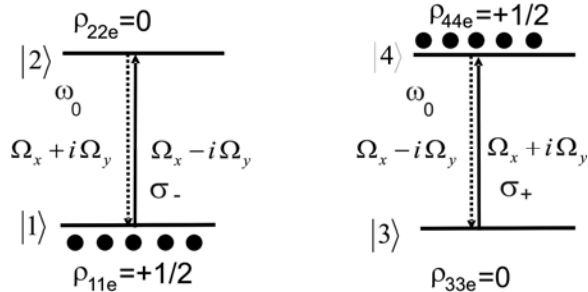
σ^+ (π -pulse)



$T_1 = T_6 = T_7 = T_{12} = 100 \text{ ps}$,
 $T_{13} = T_{14} = T_{15} = 100 \text{ ps}$

Self-Induced Transparency in a degenerate four-level system

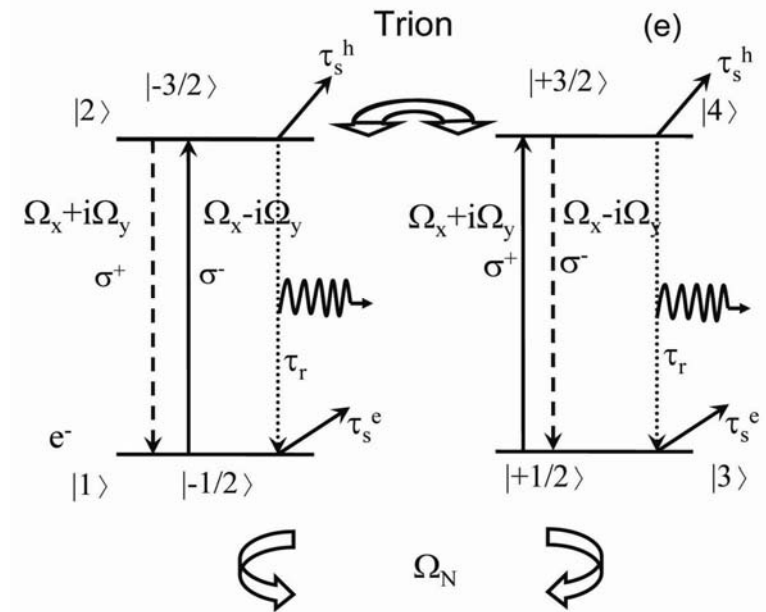
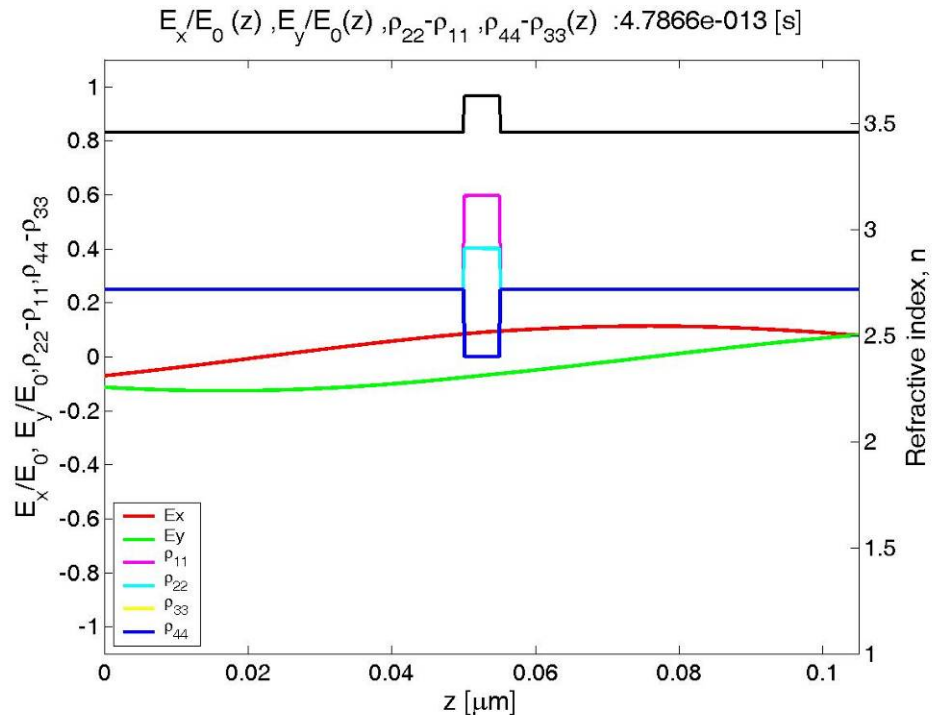
2π -pulse $E_0 = 4.2186 \times 10^9 \text{ V.m}^{-1}$



$T_p = 100 \text{ fs} \ll T_1 = T_6 = T_7 = T_{12} = 100 \text{ ps};$
 $T_{13} = T_{14} = T_{15} = 100 \text{ ps}$

Trion optically-induced coherent spin dynamics

GaAs/Al_{0.3}Ga_{0.7}As self-assembled modulation-doped QD



$\lambda = 787 \text{ nm}$, $T_p = 1 \text{ ps}$, $T_2 = T_4 = T_8 = T_{10} = \infty$

$T_3 = T_9 = \tau_s^e = 0.5 \text{ ns}$ hyperfine interaction (Shabaev et al., PRB 68, 201305, 2003; Merkulov et al., PRB 65, 205309, 2002)

$T_5 = T_{11} = \tau_s^h \sim 1 \mu\text{s}$ phonon-assisted process (Takagahara et al., PRB 62, 16840, 2000)

$T_1 = T_6 = T_7 = T_{12} = 0.5 \text{ ns}$ trion spin dephasing time (Economou et al., PRB 71, 195327, 2005)

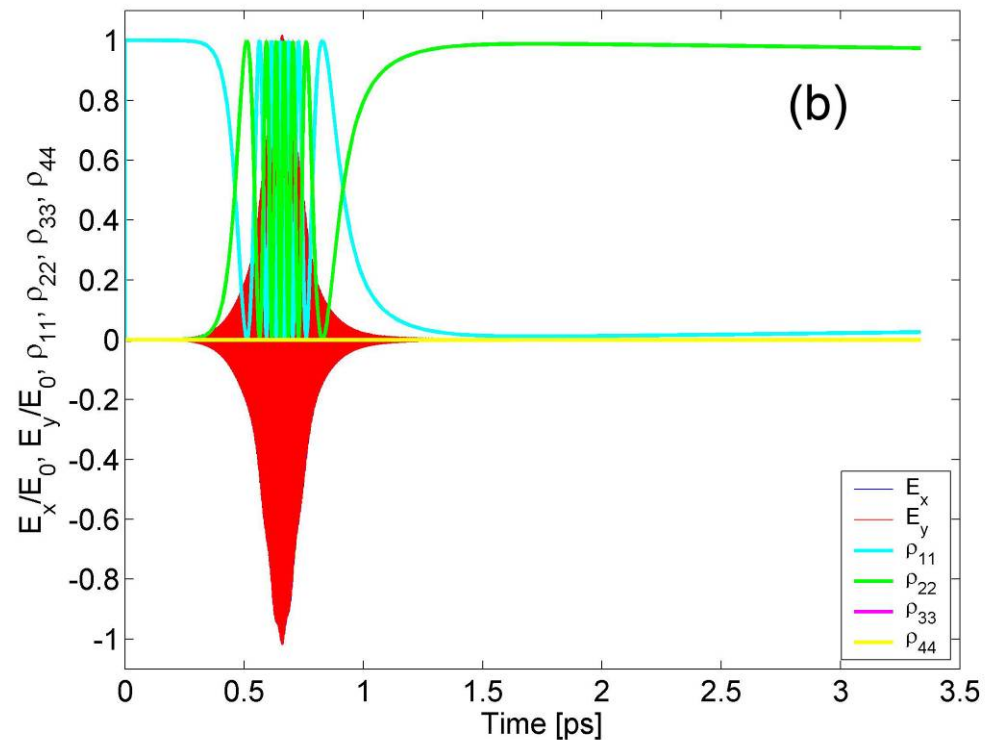
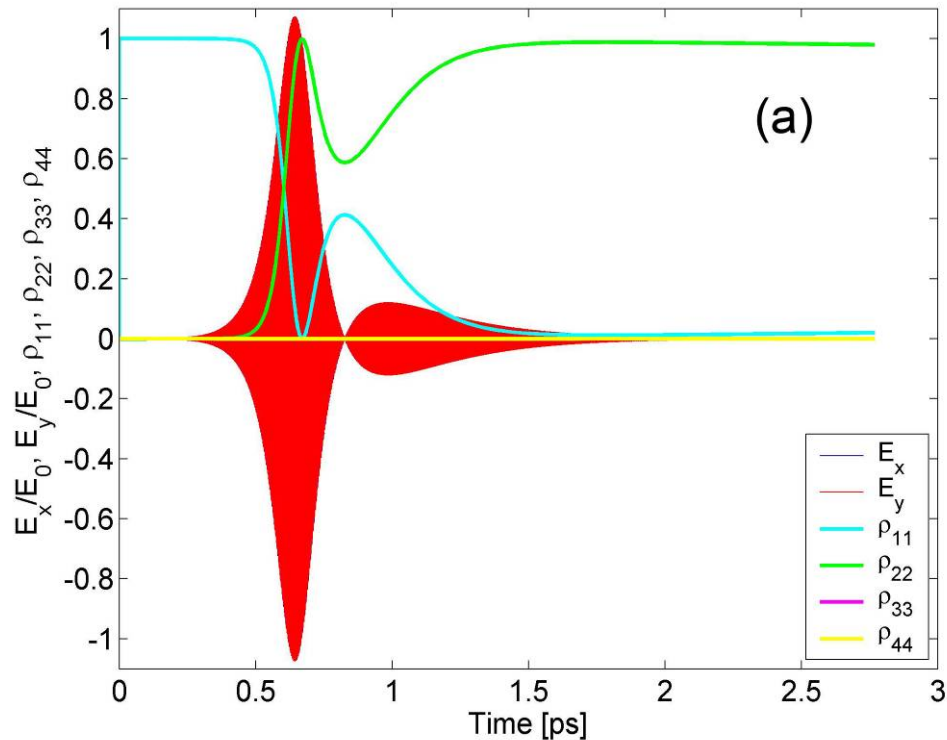
$T_{13} = T_{14} = T_{15} = \tau_r = 100 \text{ ps}$ trion radiative decay time (recombination time) (Shabaev et al. 2003, Economou et al. 2005, Greilich et al., PRL 96, 227401, 2006)

Trion optically-induced coherent spin dynamics

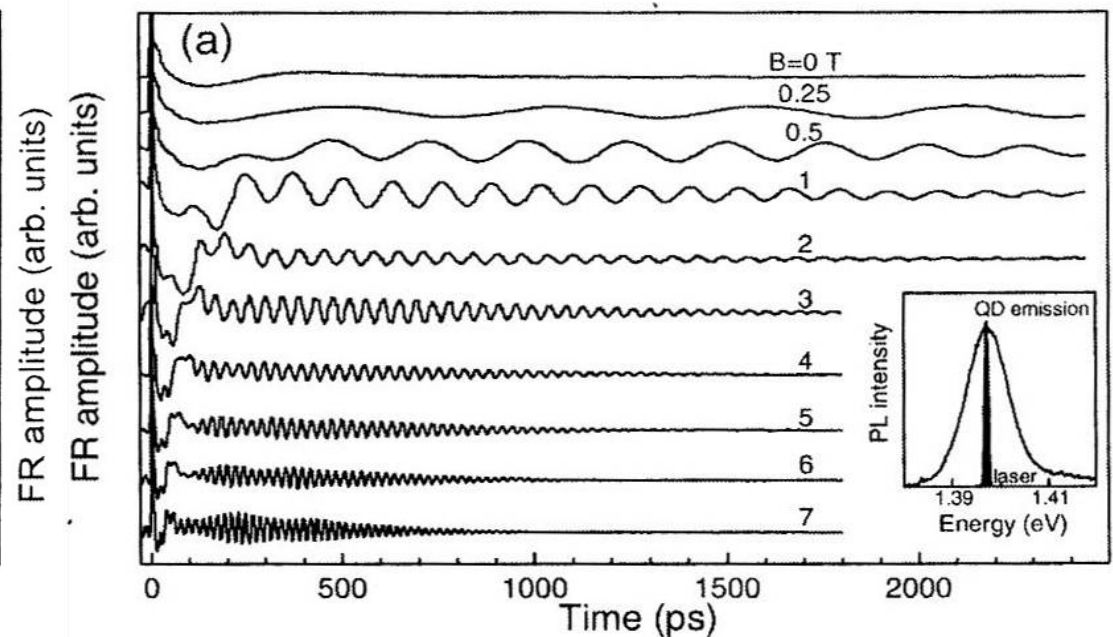
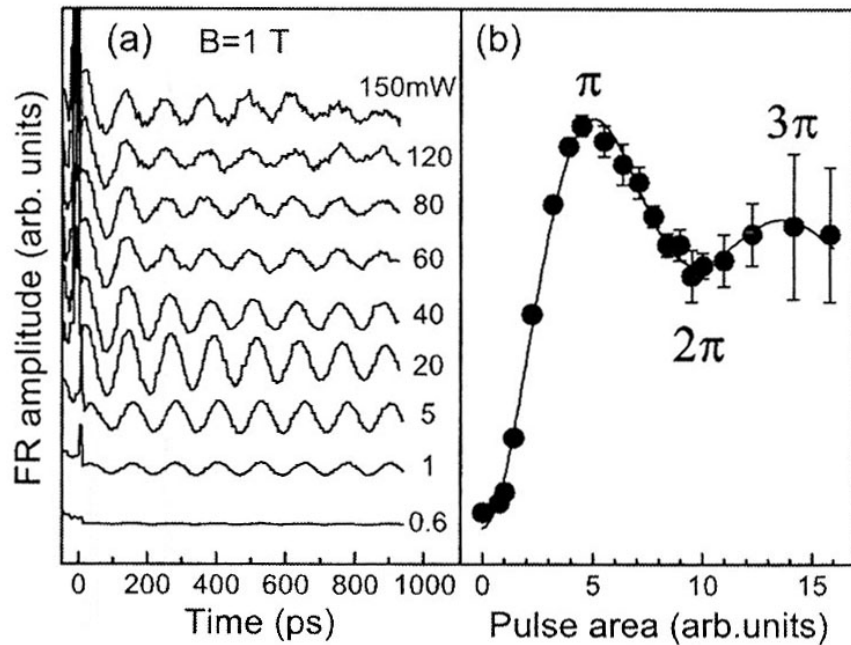
σ^- -pulse $T_p=1$ ps, $\lambda=787$ nm,
 $\omega_0=2.39 \times 10^{15}$ rad.s⁻¹

$E_0=5 \times 10^6$ V.m⁻¹

$E_0=4 \times 10^7$ V.m⁻¹ ($\pi/2$ - pulse)
Rabi oscillations

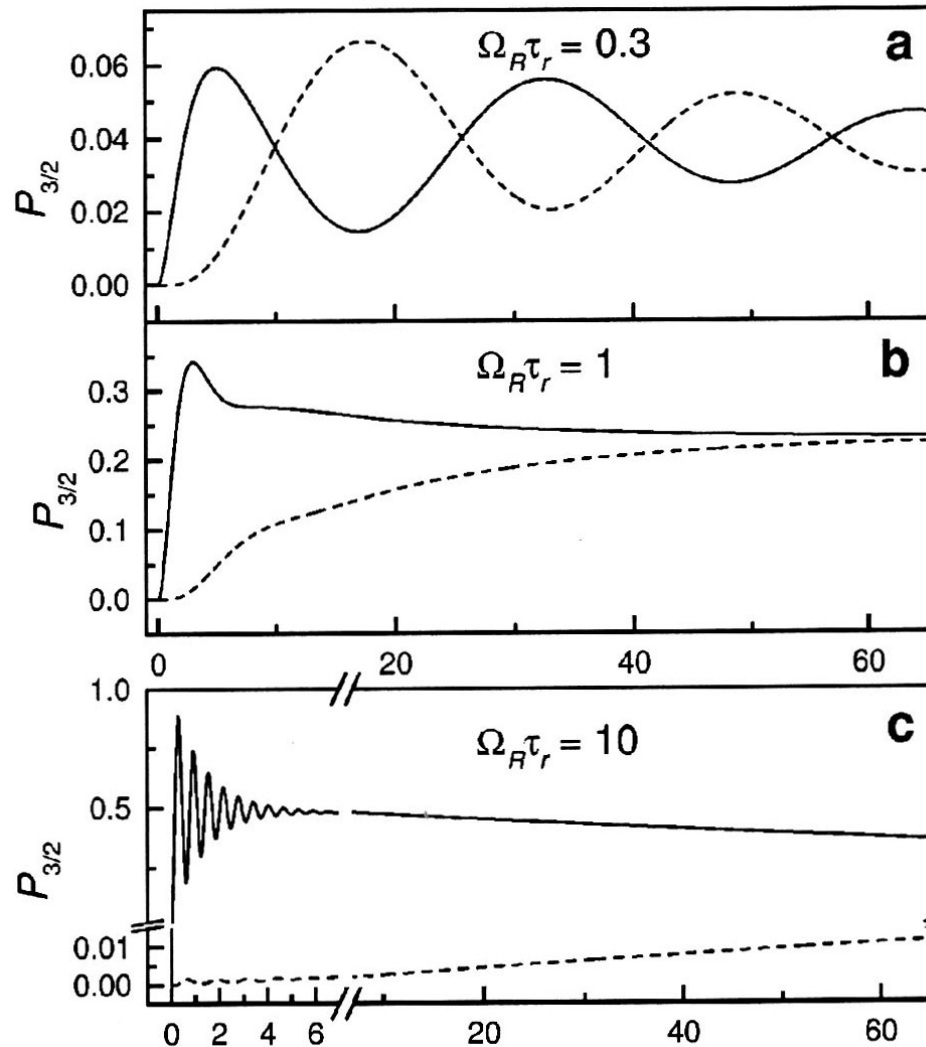


Trion optically-induced coherent spin dynamics (time-resolved Faraday rotation experiments)



Greilich et al. , PRL 96, 227401 (2006)

Time dependence of the $|3/2\rangle$ trion spin population (theory)



Shabaev et al. , PRB 68, 201305 (2003)



Summary/Outlook

A novel model of the optically-induced coherent spin dynamics in QDs:

- *Selective spin excitation with predefined helicity of the optical pulse demonstrated*
- *Rabi flopping of the population inversion and polarised Self-induced Transparency solitons numerically demonstrated*
- *Model for the non-equilibrium coherent optically-induced spin dynamics in QDs embedded in optical waveguides and semiconductor microcavities*
- *Beyond SVEA, RWA and rate equations approximations: valid for few optical cycle pulses*