Carrier Recombination in Semiconductor Lasers: Beyond the ABC

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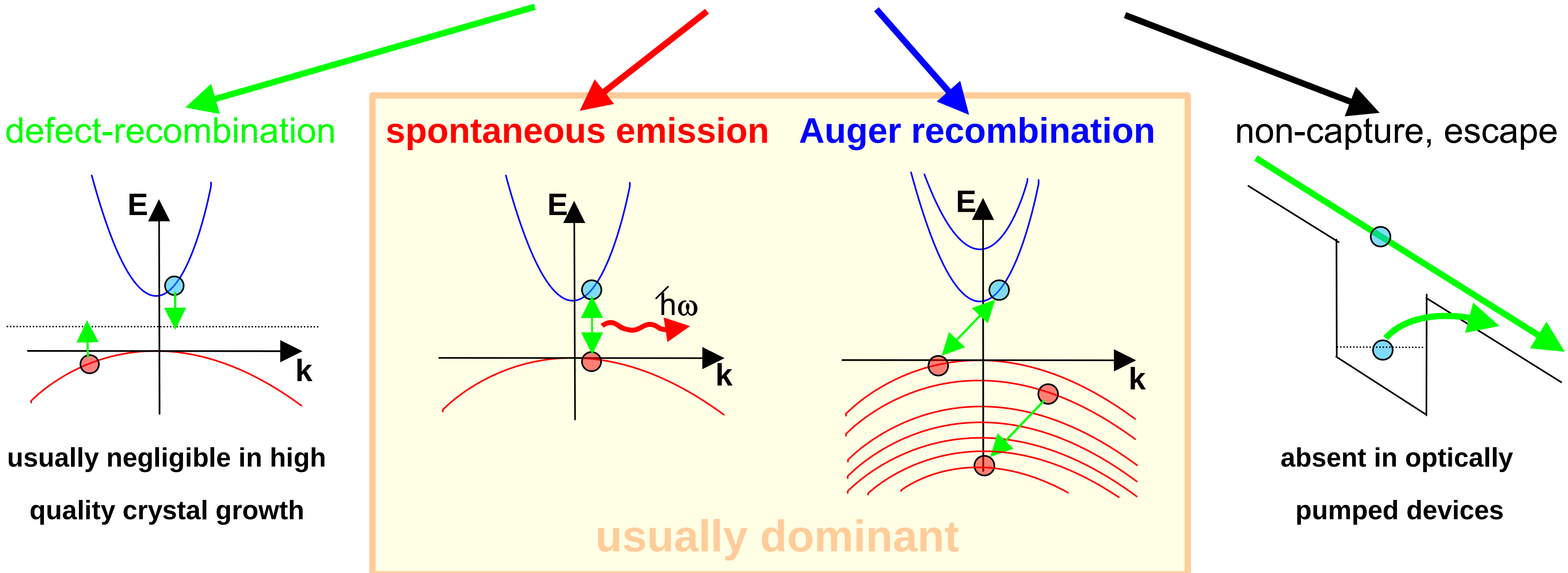
ABC's of Semiconductor Lasers

Classical Parametrization of Loss Current J_{loss}:

$$J_{loss} = A\,N + B\,N^{2} + C\,N^{3} + J_{rest}$$



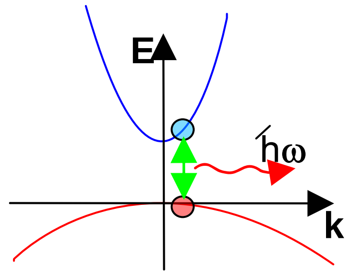
Problems with A, B, C - Parametrization:

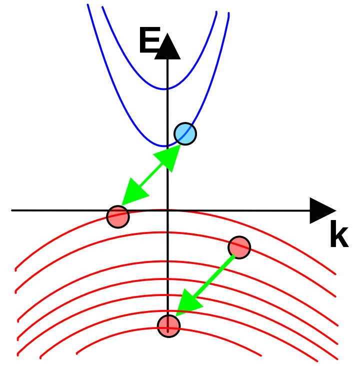
. **parameters only very roughly known and only for special cases;**
depend on well- and barrier-materials, layer widths, temperatures, densities...

. **simple density-dependence far from reality**

ABC's of Semiconductor Lasers

Problems with B, C: $J_{spont} = B\,N^2$? $J_{aug} = C\,N^3$?



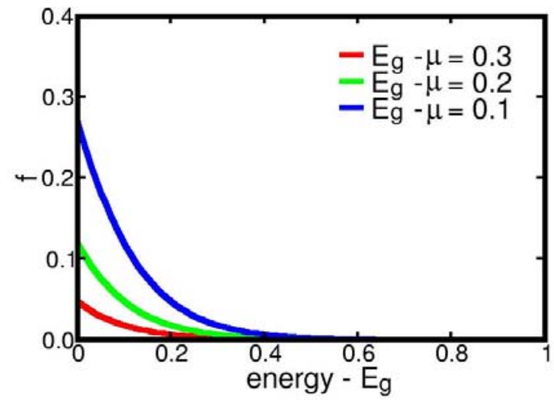


Low Density: $E_g - \mu \gg 1/\beta = k_B T$

Maxwell-Boltzmann Distributions: $\Longrightarrow$

$$J_{spont}(low\ density) \approx BN^2$$

$$J_{aug}(low\ density) \approx CN^3$$

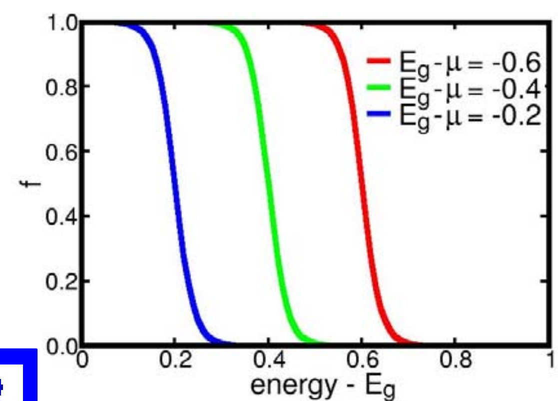


High Density: $\mu - E_g \gg k_B T$

Fermi Distributions: $f \approx \begin{cases} 0 \text{ if } k > k_F \\ 1 \text{ if } k < k_F \end{cases}$

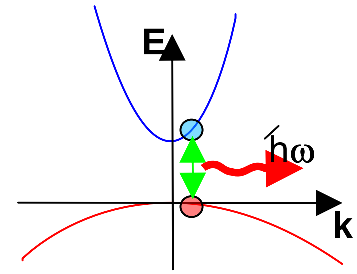
$$J_{spont}(high\ density) \approx BN,$$

$$J_{aug}(high\ density) \approx CN^x,\ x < 3$$



Theory

Gain:



Semiconductor Bloch equations (**SBE**):

$$\frac{d}{dt}P^{ji}_{\mathbf{k}} = \frac{1}{i\hbar}\left\{ \sum_{i',j'} \left[\mathcal{E}^{h}_{jj',\mathbf{k}}\delta_{ii'} + \mathcal{E}^{e}_{ii',\mathbf{k}}\delta_{jj'}\right] P^{j'i'}_{\mathbf{k}} + \left[1 - f^{e,i}_{\mathbf{k}} - f^{h,j}_{\mathbf{k}}\right] \mathcal{U}_{i,j,\mathbf{k}} \right\} + \left.\frac{d}{dt}P^{ji}_{\mathbf{k}}\right|_{corr}$$

$$\mathcal{E}^{e}_{ii',\mathbf{k}} = \varepsilon^{e,i}_{\mathbf{k}}\delta_{ii'} - \sum_{i'',\mathbf{q}} V^{ii''i'i''}_{\mathbf{k}-\mathbf{q}} f^{e,i''}_{\mathbf{q}} \qquad \mathcal{U}_{ij,\mathbf{k}} = -\mu_{ij,\mathbf{k}}E(t) - \sum_{i',j',\mathbf{q}} V^{ij'ji'}_{\mathbf{k}-\mathbf{q}} P^{j'i'}_{\mathbf{q}}$$

Quantum-Boltzmann scattering in 2. Born-Markov approximation to determine dephasing of *P*, lineshape of $\alpha(\omega)$:

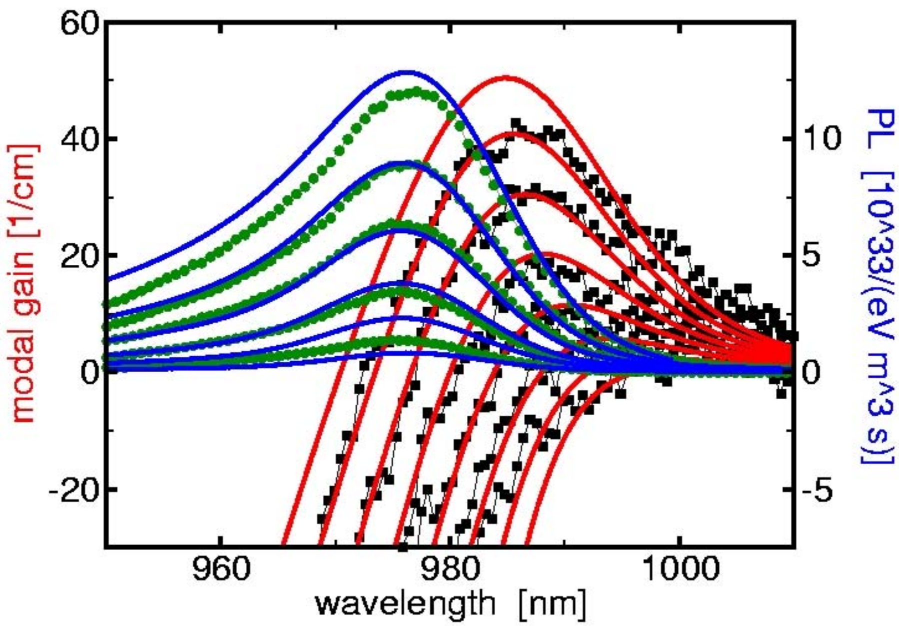
$$\left.\frac{\hbar}{\pi}\frac{d}{dt}P^{ji}_{\mathbf{k}}\right|_{ee} = \sum_{n,\mathbf{k}',\mathbf{q}} 2\left|\tilde{V}^{inni}_{\mathbf{q}}\right|^2 \mathcal{D}\left(\varepsilon^{e,i}_{\mathbf{k}'+\mathbf{q}} - \varepsilon^{e,i}_{\mathbf{k}} - \varepsilon^{n}_{\mathbf{k}} + \varepsilon^{n}_{\mathbf{k}'-\mathbf{q}}\right) \times$$

$$\left[f^{e,i}_{\mathbf{k}} f^{n}_{\mathbf{k}'}\left(1 - f^{n}_{\mathbf{k}'-\mathbf{q}}\right) + \left(1 - f^{e,i}_{\mathbf{k}}\right)\left(1 - f^{n}_{\mathbf{k}'}\right) f^{n}_{\mathbf{k}'-\mathbf{q}}\right] P^{ji}_{\mathbf{k}+\mathbf{q}} + \ldots$$

Theory

Gain:

With explicit treatment of scattering :



(From J. Hader, et al., IEEE J. Sel. Topics Quantum Electron. 9, 688 (2003))

. correct amplitudes, spectral positions, shifts

. no unphysical absorption

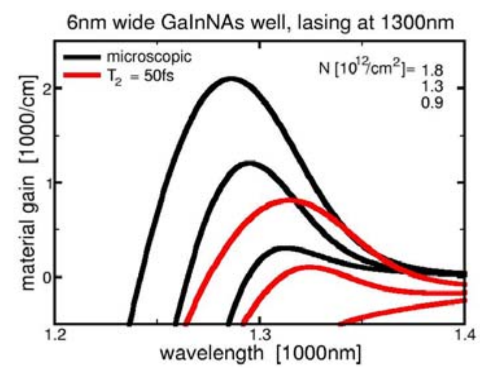
. correct density dependence for SE and gain

Without explicit treatment of scattering but lineshape functions :

. wrong amplitudes, spectral positions, shifts

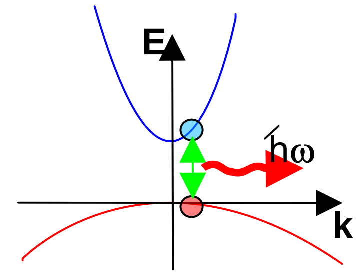
. unphysical absorption

. drastically wrong density dependence for gain and SE



Theory

Spontaneous Emission; KMS vs. SLE:



$$J_{SE} = eR_{SE} = e\int d\omega S(\omega)$$

Kubo Martin Schwinger Relation (**KMS**) between absorption/gain, $\alpha(\omega)$, and SE, $S(\omega)$:

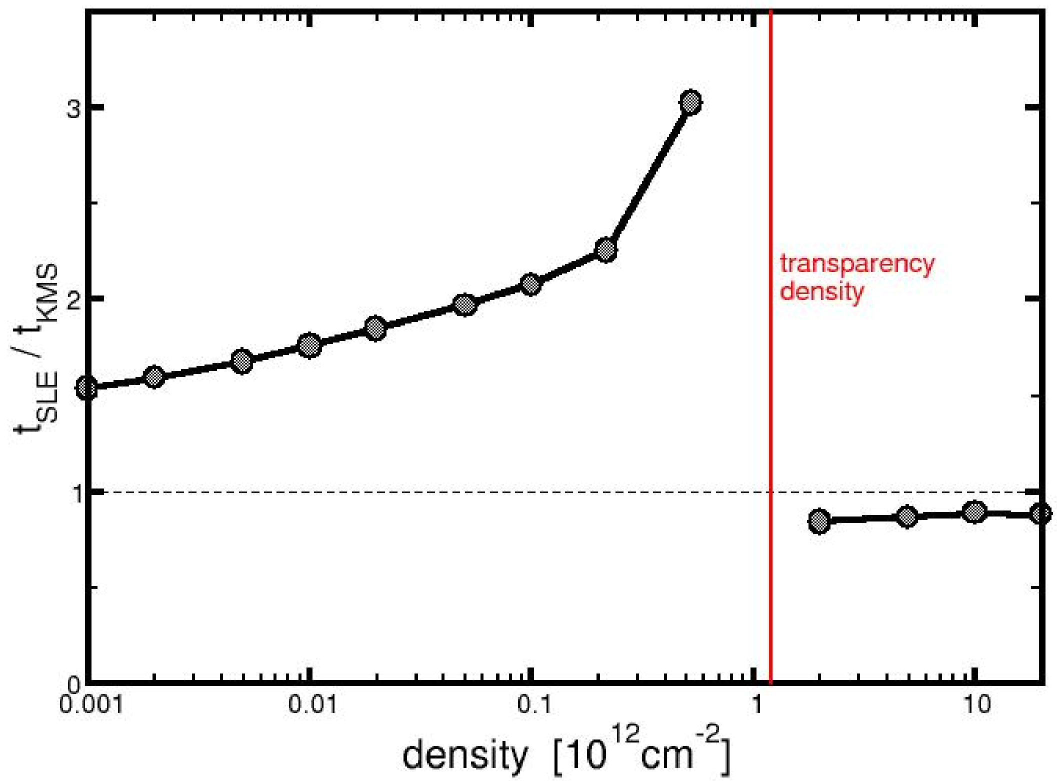
$$S(\omega) = -\frac{1}{\hbar}\left(\frac{\epsilon_b\omega}{\pi c}\right)^2 \alpha(\omega)\left[e^{\frac{\hbar\omega-\mu}{k_B T}} - 1\right]^{-1}$$

Semiconductor Luminescence Equations (**SLE**):

- Equations of motion for photon assisted polarizations: <b^+v^+c>
- Similar to SBE, I.e. equations of motion for polarizations: <v^+c>, <c^+v>
- Scattering in 2. Born-Markov approximation

Theory

Spontaneous Emission; KMS vs. SLE:



KMS:

. numerically very simple

. ok for low density lineshapes

. fails close to transparency

. overestimates low density SE

. some tens of percents wrong in
the gain regime

Theory

Auger Recombination:

Quantum-Boltzmann scattering in 2. Born-Markov approximation to determine Auger transitions

$$\frac{d f_k^{i,s}}{dt} = \frac{2\pi}{\hbar}\sum_{\mathbf{k}',\mathbf{q},s'} \mathcal{R}e\Big\{\sum_{j_1,j_2,j_3}\left(\left|\tilde{V}_q{}^{\substack{i\\s}\ \substack{j_3\\s'}\ \substack{j_1\\s'}\ \substack{j_2\\-s}}\right|^2 - \tilde{V}_q{}^{\substack{i\\s}\ \substack{j_3\\s'}\ \substack{j_1\\s'}\ \substack{j_2\\-s}}\ \bar{V}_{|\mathbf{k}'-\mathbf{q}+\mathbf{k}|}{}^{\substack{j_1\\s'}\ \substack{j_2\\-s}\ \substack{j_3\\s'}\ \substack{i\\s}}\right)\times$$

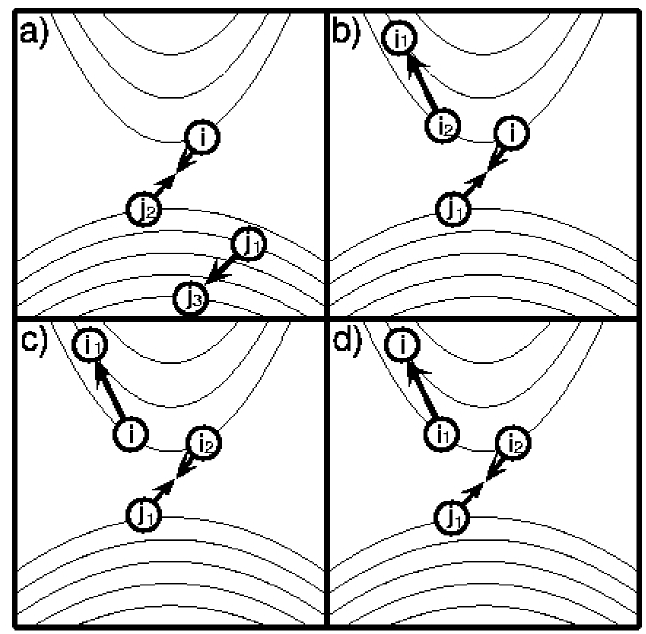
$$\mathcal{D}\left(-\varepsilon_k^{i,s} - \varepsilon_{|\mathbf{k}'-\mathbf{q}|}^{j_1,s'} - \varepsilon_{|\mathbf{q}-\mathbf{k}|}^{j_2,-s} + \varepsilon_{k'}^{j_3,s'}\right)\times$$

Impact Ionization →

$$\left[f_{k'}^{j_3,s'}\left(1-f_{|\mathbf{q}-\mathbf{k}|}^{j_2,-s}\right)\left(1-f_{|\mathbf{k}'-\mathbf{q}|}^{j_1,s'}\right)\left(1-f_k^{i,s}\right) -\right.$$

Auger Recombination →

$$\left.\left(1-f_{k'}^{j_3,s'}\right) f_{|\mathbf{q}-\mathbf{k}|}^{j_2,-s} f_{|\mathbf{k}'-\mathbf{q}|}^{j_1,s'} f_k^{i,s}\right] + \ldots$$



Theory-Experiment Comparison

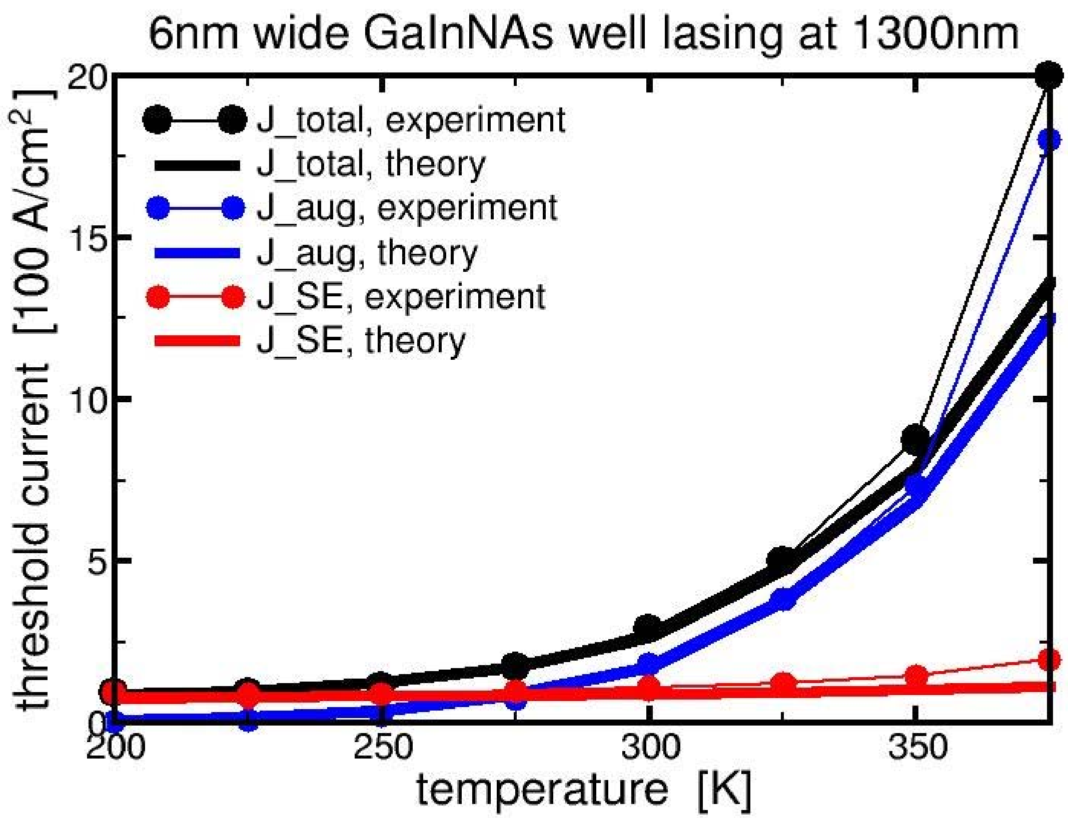
Theoretical Procedure:

. calculate gain for various densities

. search for density that overcomes intrinsic losses (mirror losses) = threshold density

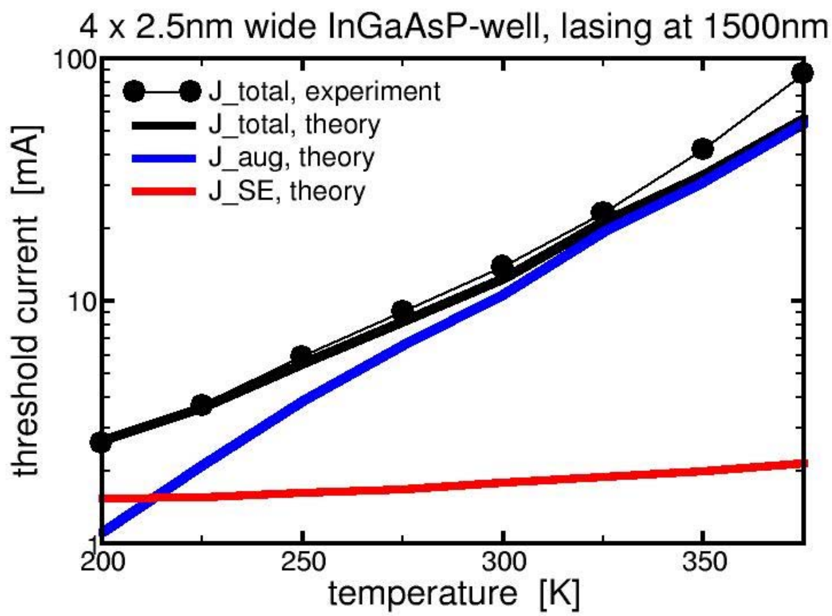
. calculate spontaneous emission and Auger recombination for this density

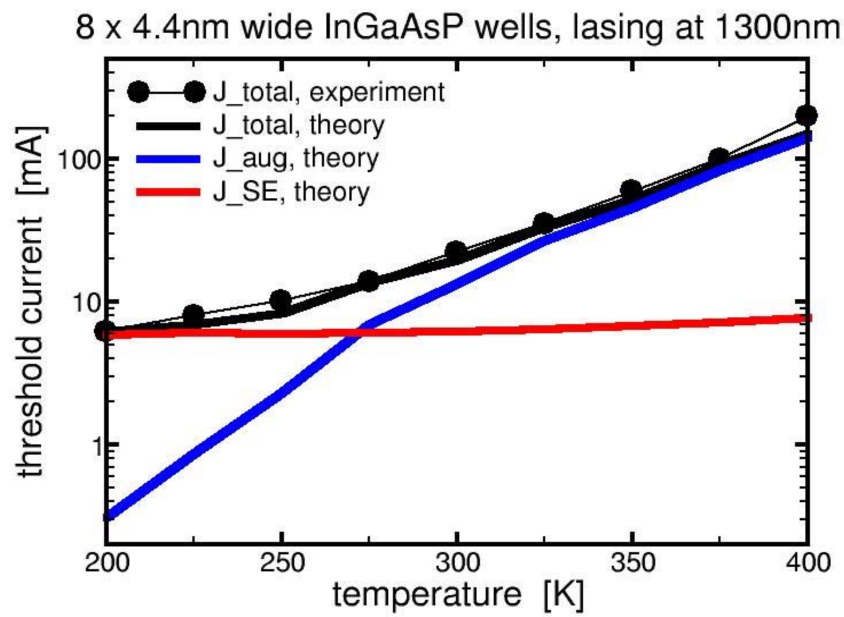
. **put on top of experimental result without adjustment**



(experimental data from R.Fehse, et al., IEEE J. Sel. Topics Quantum Electron. 8, 801 (2002))

Theory-Experiment Comparison





(experimental data from

A.F. Phillips, et al., IEEE

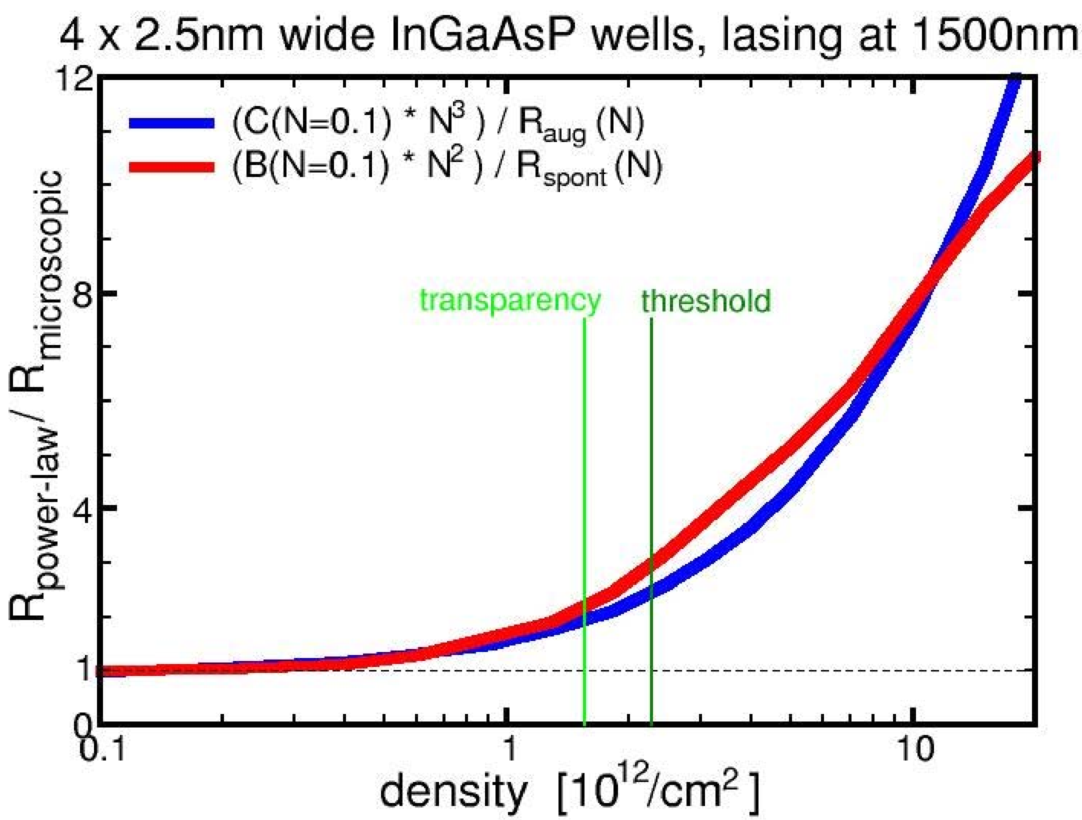
J. Sel. Topics Quantum

Electron. 5, 401 (1999))

Results

How good are the ABC's?:

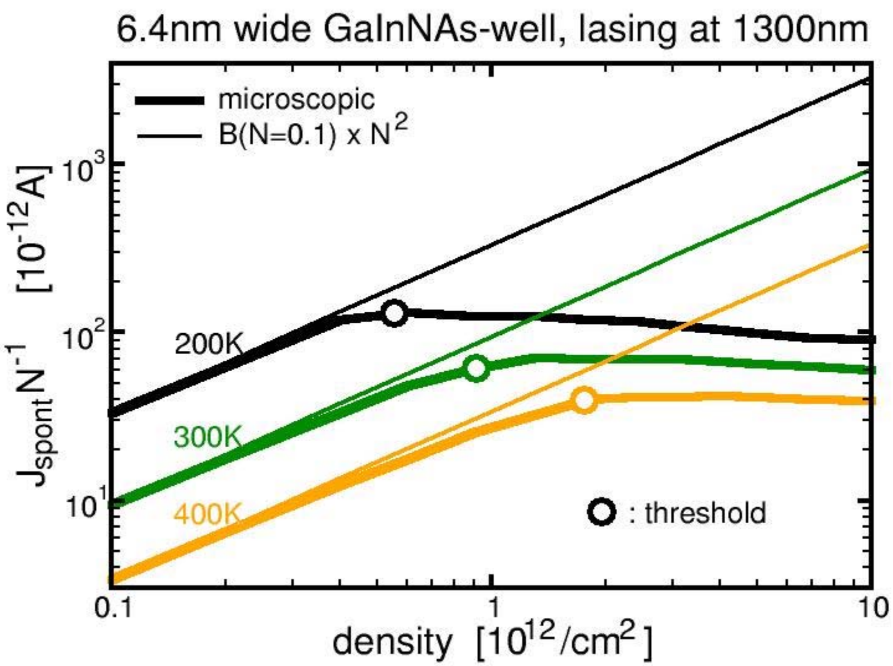
. Error of more than two already at transparency for B- and C-laws.



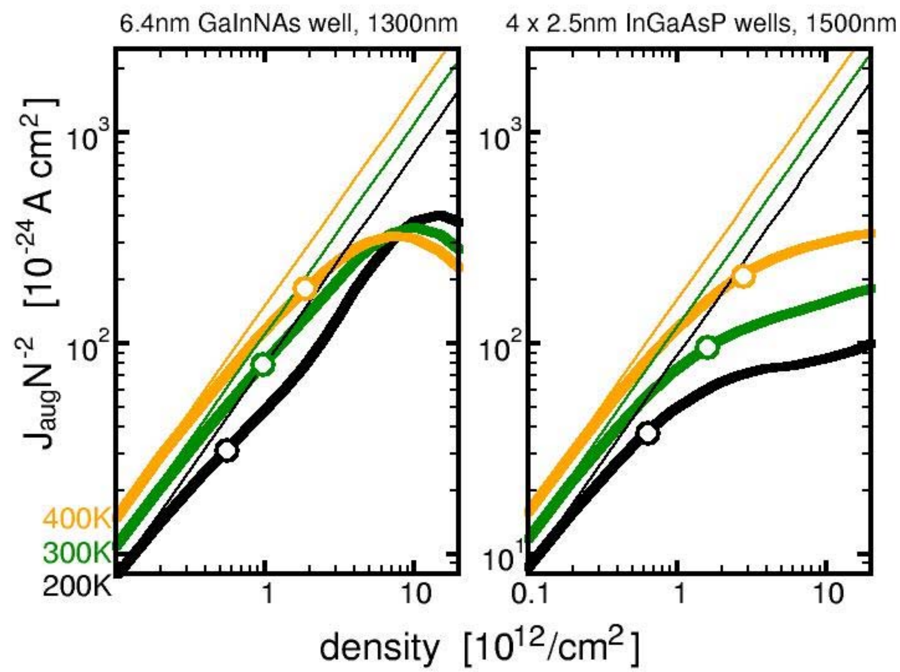
Results

How good are the ABC's?:

- J_{spont} increases only linear with N at high densities



- J_{aug} increases far less than cubic with N; sometimes even less than quadratic



Closed-Loop Laser Design

Predicting Input-Output Characteristics Using Basic Structural Information

J. Hader, et al. Optics Letters, in print.

Experimental Input:

- nominal structural parameters (layer widths, material compositions, device length, L, mirror reflectivities, R_1, R_2 → outcoupling loss, $\alpha_{out} = 1/(2L)\ \ln[1/(R_1 R_2)]$
- internal loss α_{int} → threshold loss, $\alpha_{thr} = \alpha_{int} + \alpha_{out}$
- low excitation PL

Step 1:

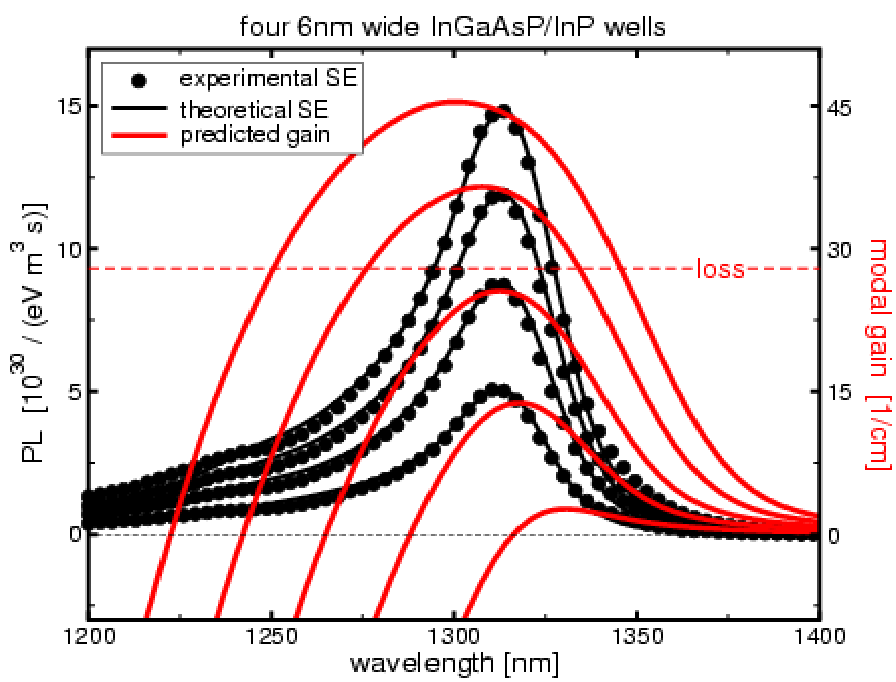
- calculate PL using fit parameter free SLE's; compare to measured PL

→ inhomogenous broadening and actual structural compositions

Step 2:

- calculate gain using fit parameter free SBE's and apply inhomogeneous broadening; look up density for which gain compensates α_{thr}

→ threshold density, N_{thr}



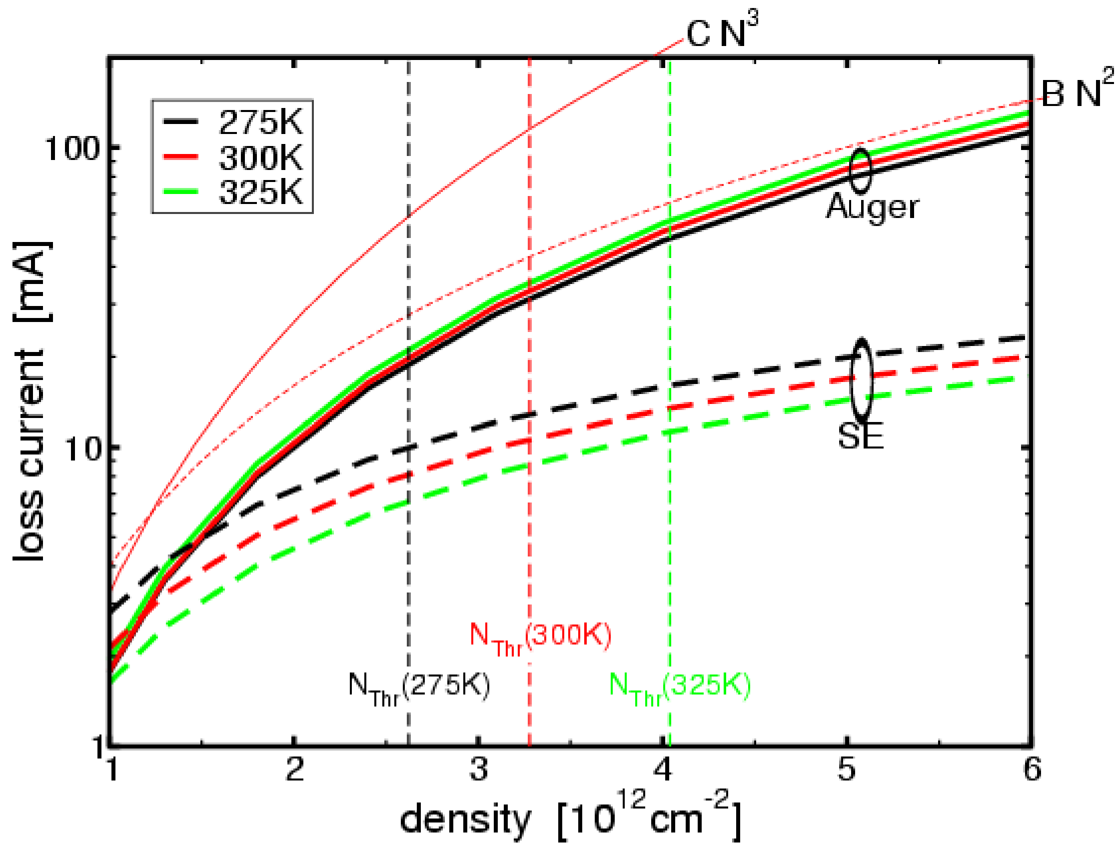
Closed-Loop Laser Design

Predicting Input-Output Characteristics Using Basic Structural Information

Step 3:

- use fit parameter free SLE's and Auger model to calculate spontaneous emission- and Auger-losses at threshold, $J_{se}(N_{thr})$, $J_{aug}(N_{thr})$,

→ threshold current, $J_{SE}(N_{thr})+J_{aug}(N_{thr})$



Closed-Loop Laser Design

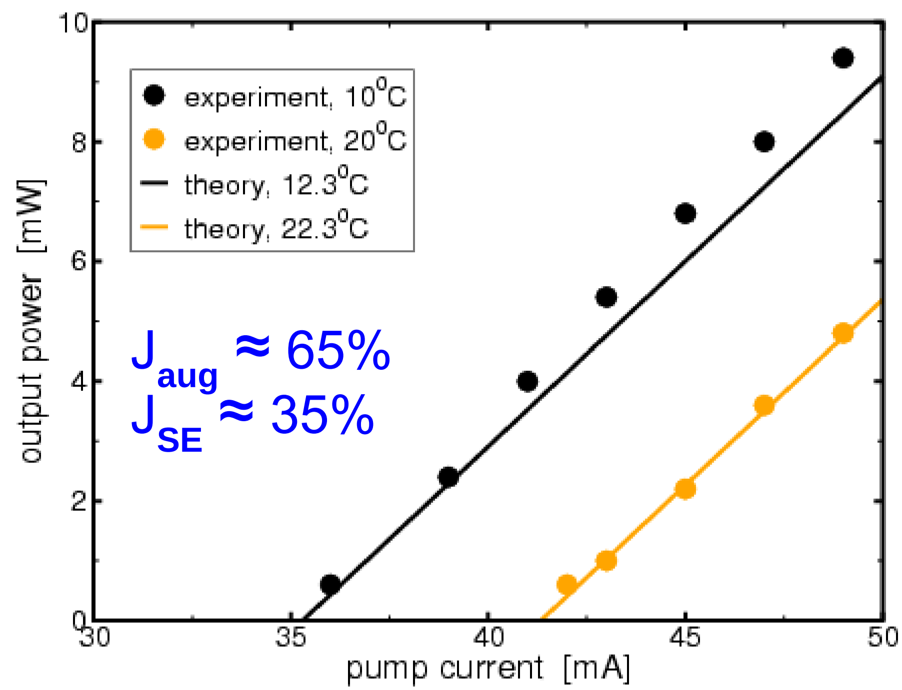
Predicting Input-Output Characteristics Using Basic Structural Information

Step 4, Comparison to Experiment:

Assumptions:

- slope efficiency = $\alpha_{out} / \alpha_{thr}$
- internal efficiency = 100%
- homogeneous mode under pumped area

- **No adjustments of any parameters.**
- **No free parameters.**
- **True predictions for threshold and temperature dependence.**



NOTE:

When using adjustable parameters like an Auger-constant, C, and its temperature dependence, a reasonable **FIT** to the threshold and its temperature dependence can always be obtained.

Summary

Spontaneous Emission:

- B N ^{2}-assumption leads to an error of several orders of magnitude even if low-density B is known
- above threshold N ^{2}-assumption completely breaks down
- here, only linear increase with density due to phase space filling
- Numerically expensive SLE's have to be used especially for densities near transparency

Auger Recombination:

- C N ^{3}-assumption leads to an error of up to one order of magnitude even if low-density C is known
- measured and/or calculated literature values for C vary by 1-2 orders of magnitude for similar systems
- C strongly temperature- and density dependent
- N_{thr} 25% wrong ⟷ Auger-current wrong by factor 2

J. Hader, et al., IEEE J. Quantum Electron. **41**, 1217 (2005)

J. Hader, et al., Appl. Phys. Lett. **87**, 201112 (2005)

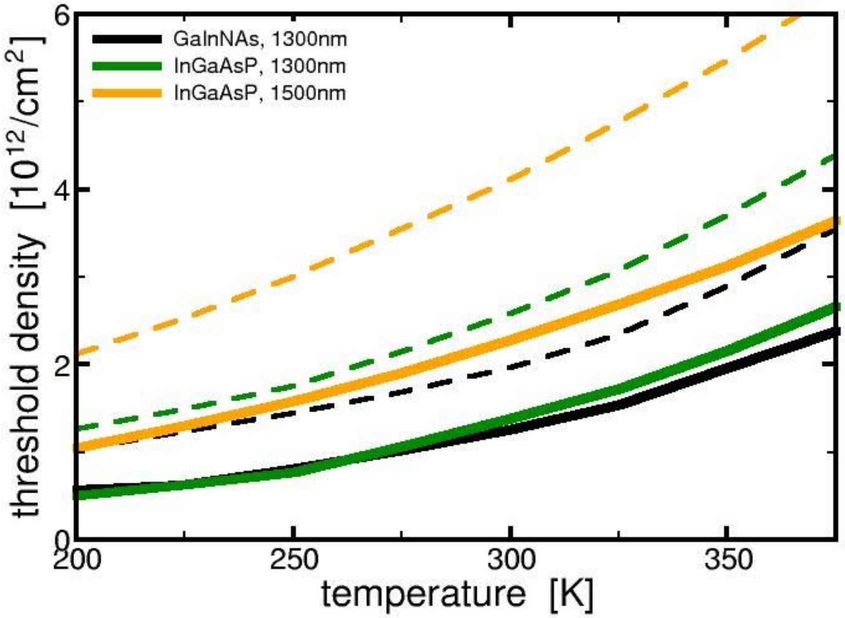
J. Hader, et al., Optics Lett., in print.

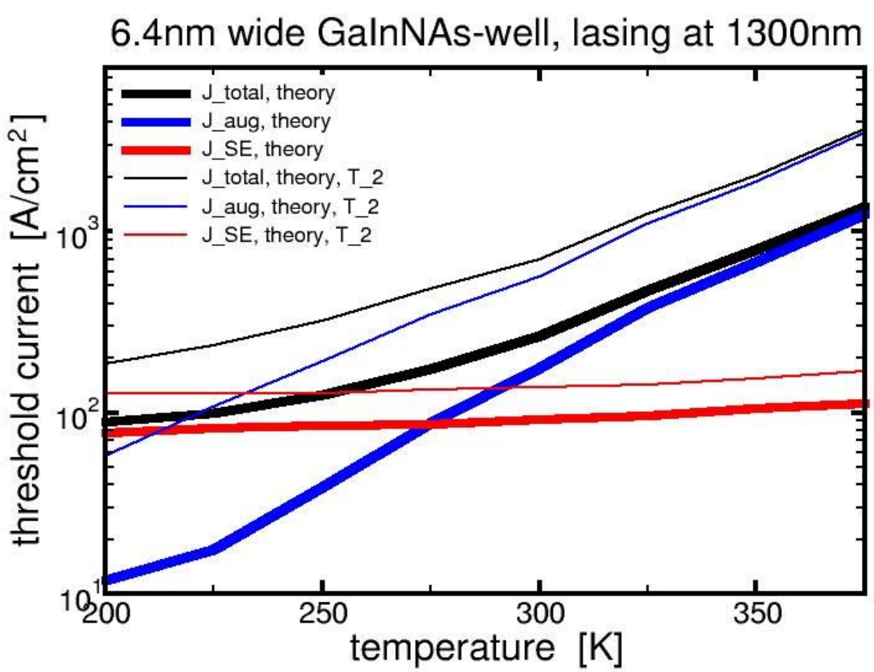
Shortcomings of Simpler Approaches

Dephasing Time Approximation:

threshold density overestimated by about factor of 2

$\Longrightarrow$ **up to one order of magnitude error in loss-currents**





Shortcomings of Simpler Approaches

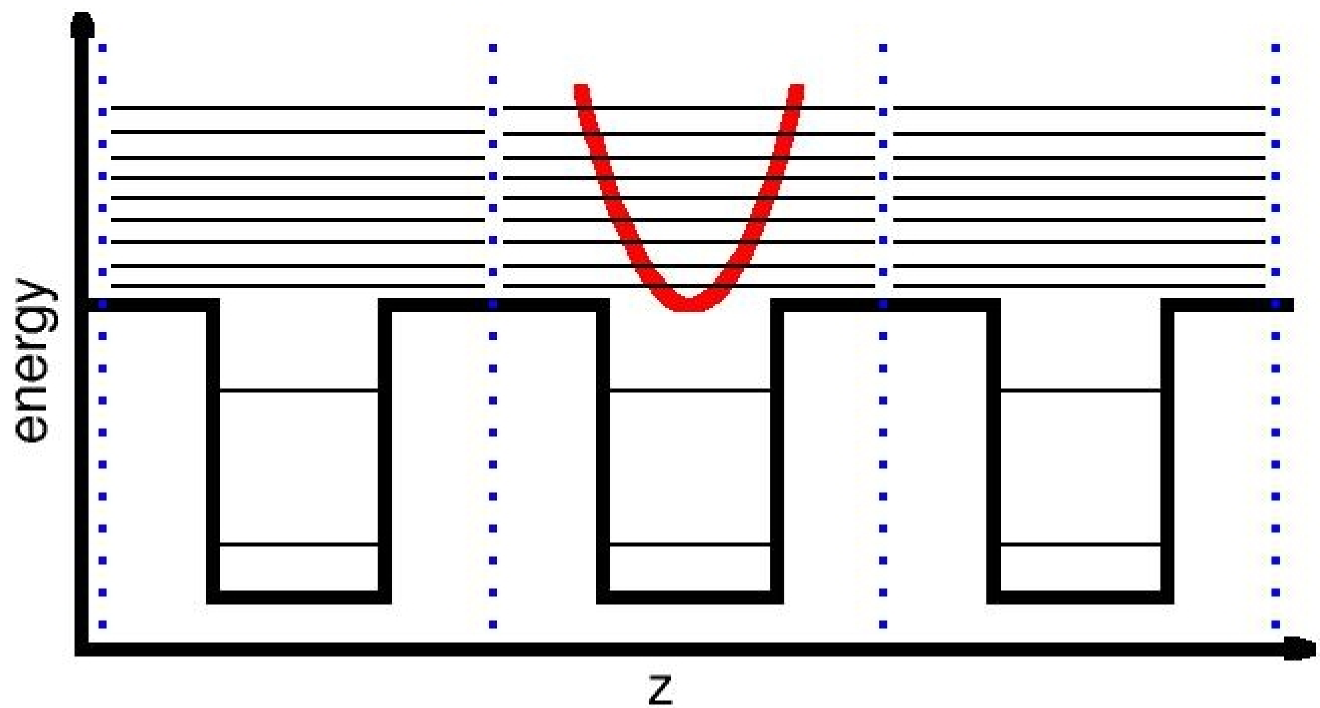
Bulk Approximation for Barrier States:

subband approximation:

- similar density of states as bulk
- *seems* to be good for periodic MQW systems
- negelcts coupling between well-unit-cells
- neglects formation of subbands and mixing of wavefunction-character

bulk approximation:

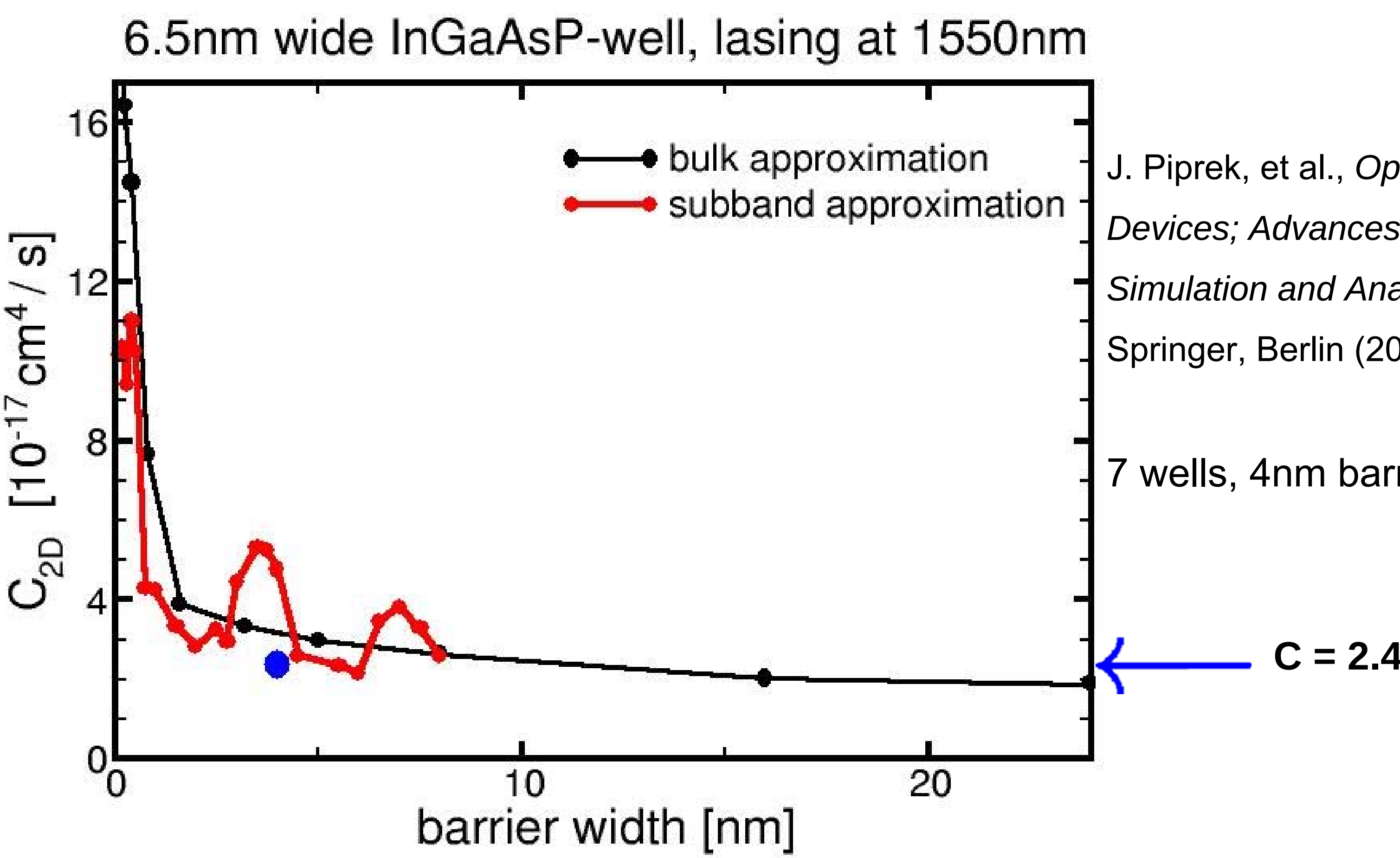
- good for total barrier widths of more than about ten excitonic Bohr radii



Shortcomings of Simpler Approaches

Bulk Approximation for Barrier States:

- unphysical resonances in width dependence
- wrong by factor of about 2



J. Piprek, et al., *Optoelectronic Devices; Advances in Simulation and Analysis*, Springer, Berlin (2005):

7 wells, 4nm barriers