FDTD Modelling and Experimental Verification of FWM in Semiconductor Micro-Resonators

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Overview

- **FWM in micro-resonator**
  - wavelength conversion, optical switches, etc.
  - in a nonlinear resonator: $\chi^{(3)}$ enhanced by ~(FE)^8
  - challenging task, numerically & theoretically
  - require accurate information on resonance field in complicated stru.
  - strong material dispersion of semiconductors
- **Full wave FDTD analysis**
  - accurate, but numerically expensive
  - efficient parallel code
  - detailed modeling capability
    - Kerr nonlinearity
    - material dispersion
    - Effective Dielectric Const. (EDC) tech. for modeling curved surfaces
  - first attempt to apply full wave analysis to FWM in actual devices

FDTD analysis of nonlinear and dispersive media

Optical Kerr effect

polarization: $P_K(t) = \varepsilon_0 \ \chi_K^{(3)} \ E^3(t)$ ⇨ finite difference eq.

⇧ const. nonlinear susceptibility

Lorentz dispersion

polarization: $\tilde{P}_L(\omega) = \tilde{\chi}_L(\omega)\tilde{E}(\omega) = \dfrac{\varepsilon_0 \Delta\varepsilon_L \, \omega_L^2}{\omega_L^2 + 2j\delta_L\omega - \omega^2}\tilde{E}(\omega)$

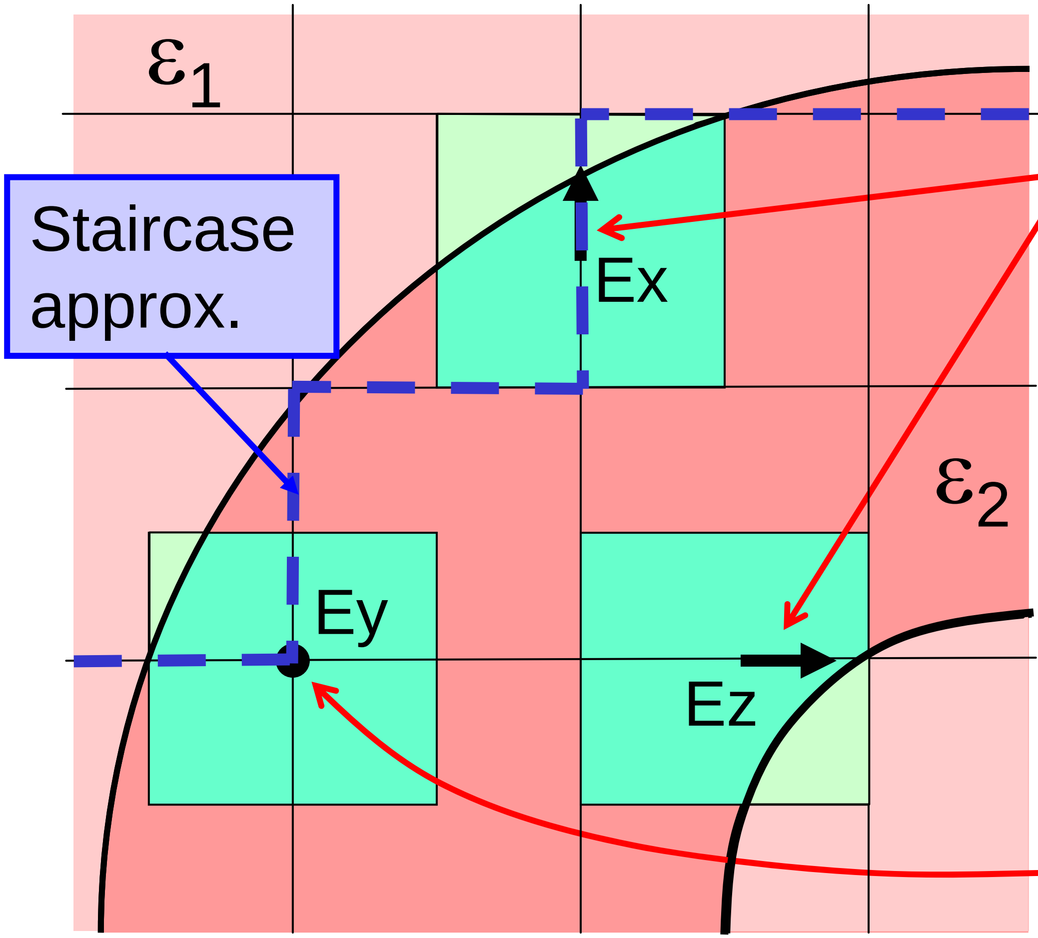
⇩ Φ^{-1}

$$\omega_L^2 P_L(t) + 2\delta_L \frac{dP_L(t)}{dt} + \frac{d^2 P_L(t)}{dt^2} = \varepsilon_0 \Delta\varepsilon_L \, \omega_L^2 E(t)$$

⇨ finite difference eq.

⇨ solved in Yee's leapfrog algorithm

FDTD implementation of Effective Dielectric Const. (EDC) Tech.



For E fields not being tangential to the interface:
ε_{eff}: solve electrostatic Laplace equation

$$\nabla \cdot (\varepsilon \nabla \varphi) = 0$$

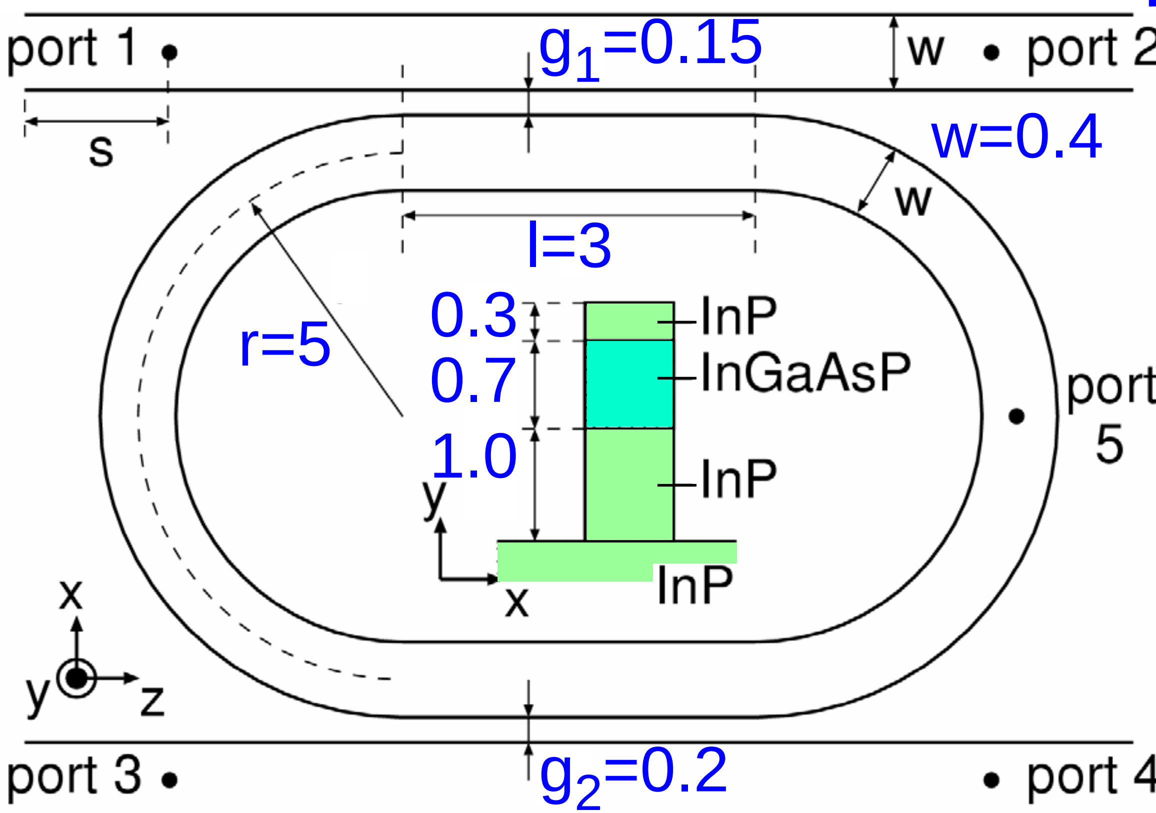
by FD method

For E fields tangential to the interface:
Tangential B.C.
(E is continuous)
ε_{eff}: arithmetic average

2nd-order accuracy is maintained!

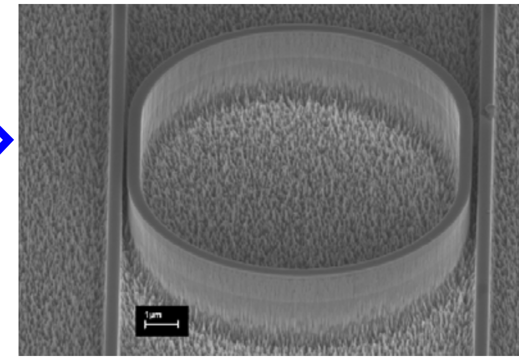
Modeling of InP/InGaAsP micro-resonator

Resonator structure (2D and 3D)



Δx=Δz=0.025, Δy=0.1μm

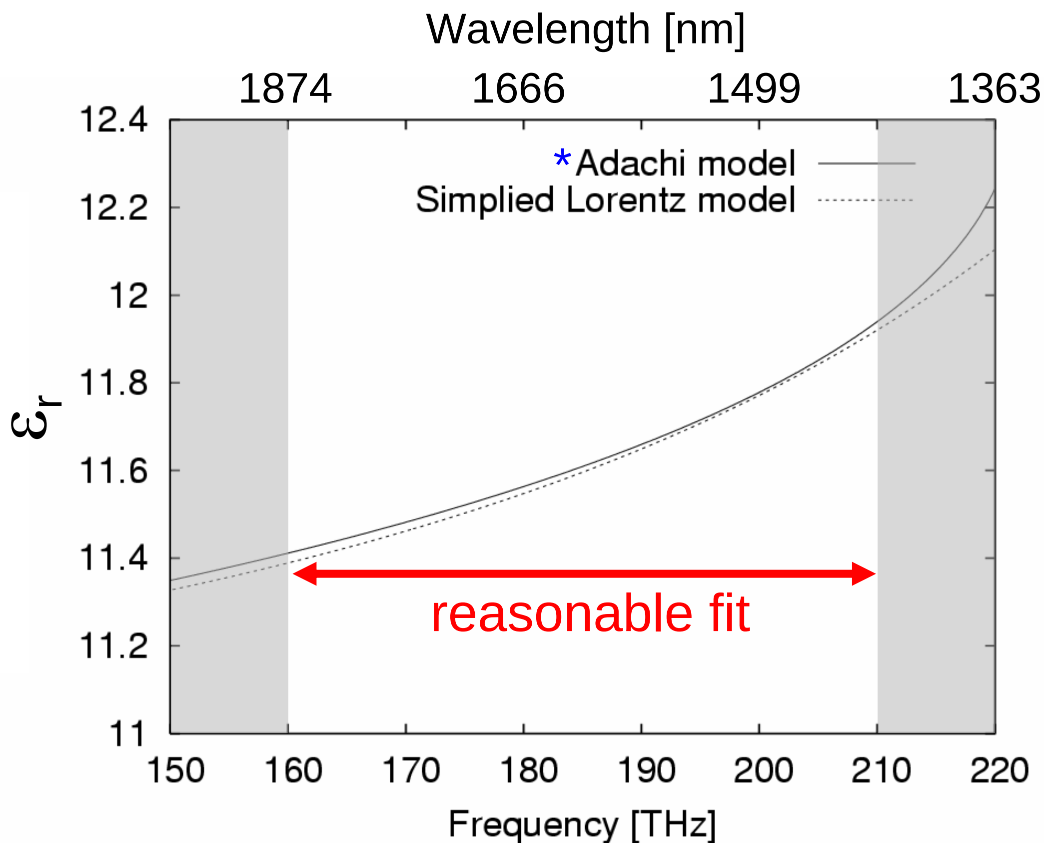
(unit: μm)



- substrate: InP
- computation region: 13.5x17.4 μm^2, for 3D; hight = 2.7 μm
- **InGaAsP**: n=3.42, $\chi^{(3)}$ = 3.8x10^{18} m^2/V^2
- **InP**: n=3.17(1.55 μm)
- for 2D analysis; n=3.34 (slab wg.)
- quasi-TM pol.

Modeling of InP/InGaAsP micro-resonator

Material dispersion of **InGaAsP** core



dielectric const. for Q(1.35)-InGaAsP:

$$\varepsilon_r = \varepsilon_\infty + \chi^{(1)} + \chi^{(3)} E^2$$

$$\chi^{(1)} = \frac{\Delta\varepsilon\,\omega_p^2}{\omega_p^2 - \omega^2}$$

$\chi^{(3)} = 3.8 \times 10^{18}\ \mathrm{m^2/V^2}$

with $\varepsilon_{\mathrm{inf}} = 10.05$

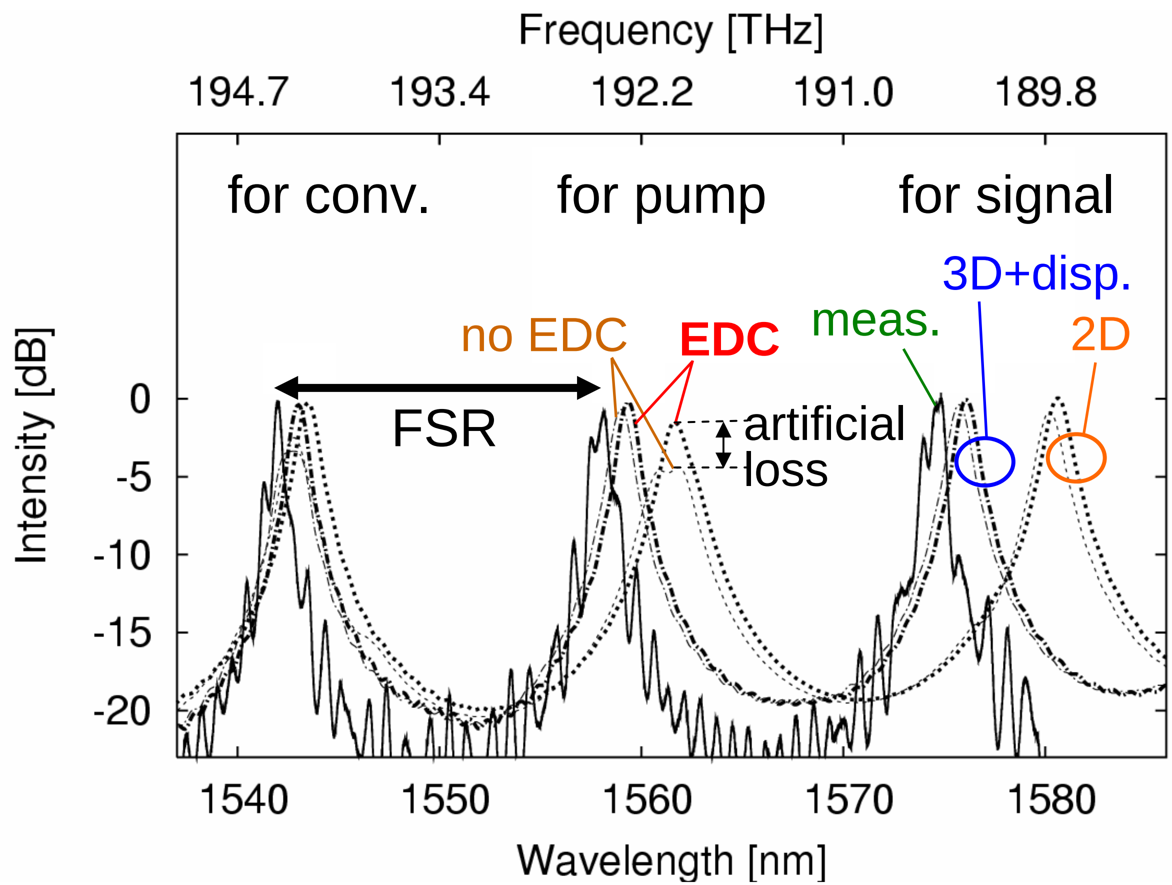
$\Delta\varepsilon = 1.0$

$\omega_p = 1.948 \times 10^{15}$ rad/s

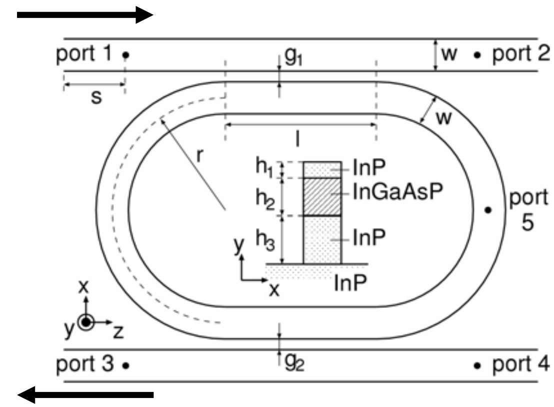
*S. Adachi, "Optical properties of $In_{1-x}Ga_xAs_yP_{1-y}$ alloys", *Phys. Rev. B*, vol. 39, no. 17, pp. 12612–12621, 1989.

Modeling of InP/InGaAsP micro-resonator

Linear pulse response analysis



pulse input (quasi-TM)



output

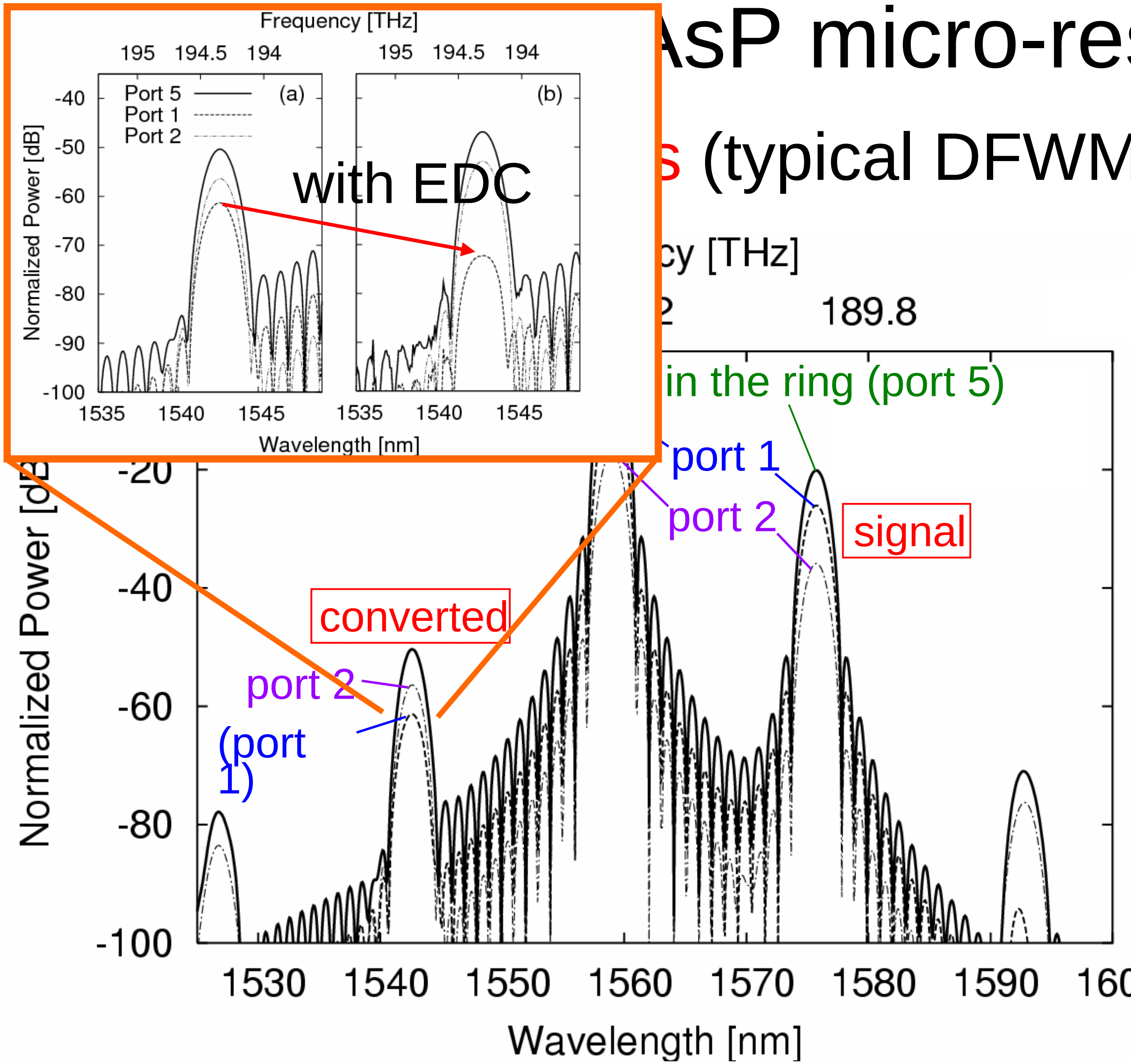
FSR error (FDTD vs. meas.)

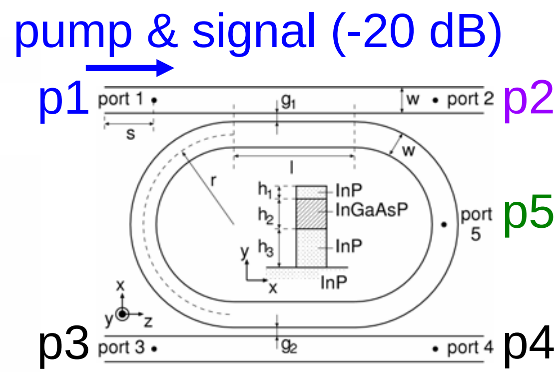
2D	2D +disp.	3D	3D +disp.
13 %	4%	10%	1%

- **EDC** reduces stair-case error (artificial loss)

Modeling of
AsP micro-resonator

s (typical DFWM spectrum)





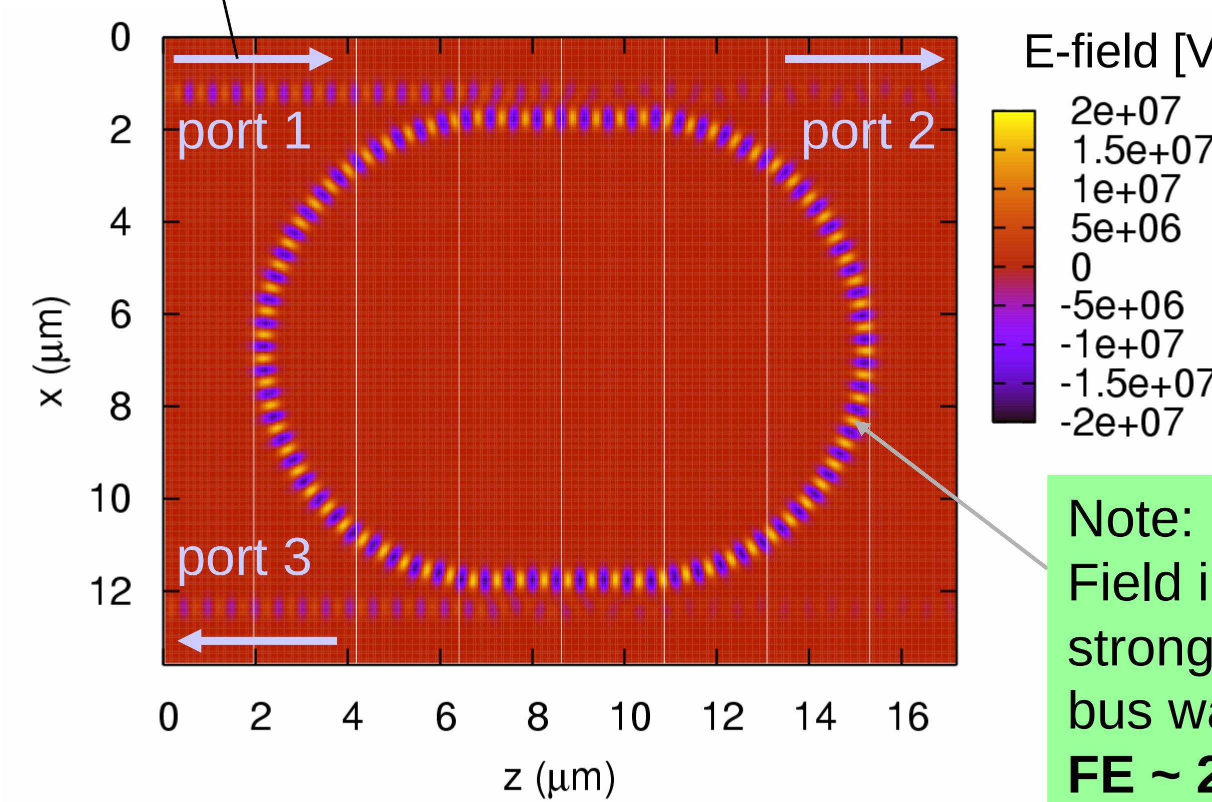
calculation conditions:

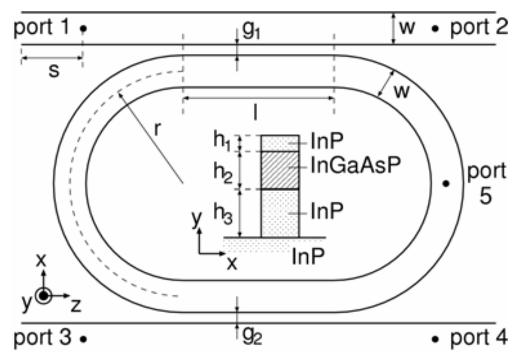
- pump & signal input at p1
- **3D FDTD (max 18 ps)**
- max pump field at center of waveguide: ***Ep* = 5.65x10⁶ V/m (=pump power 13 dBm)**
- material dispersion
- no EDC

Modeling of InP/InGaAsP micro-resonator

Field enhancement in resonator (q-TM E-field)

pump & signal input

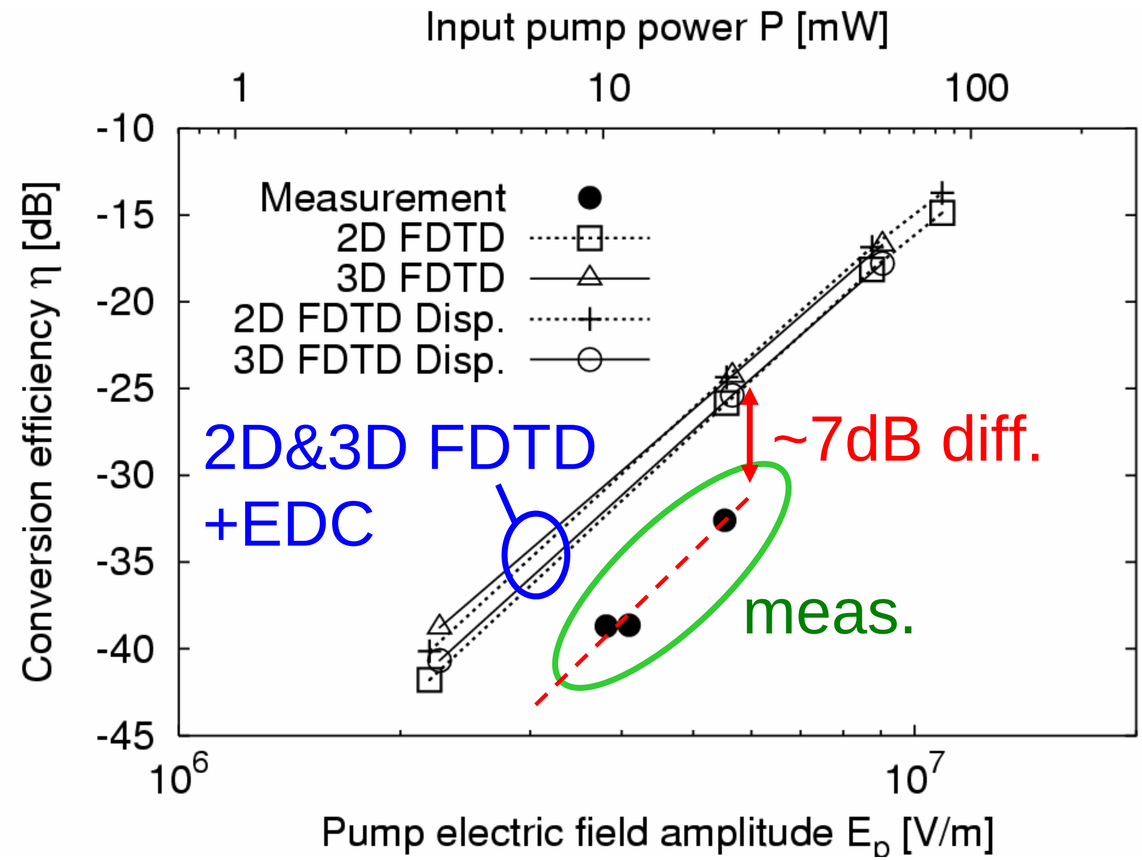




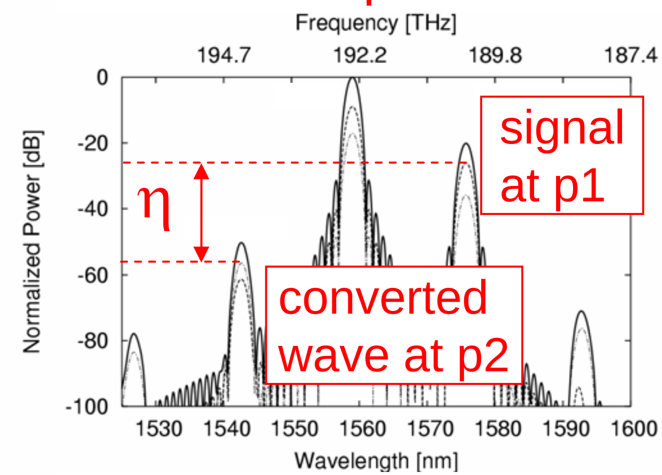
Note:
Field in the ring is much stronger than that in the bus waveguides.
FE ~ 2.4

Comparison with measurements

Wavelength conversion efficiencies of FWM



FWM spectrum



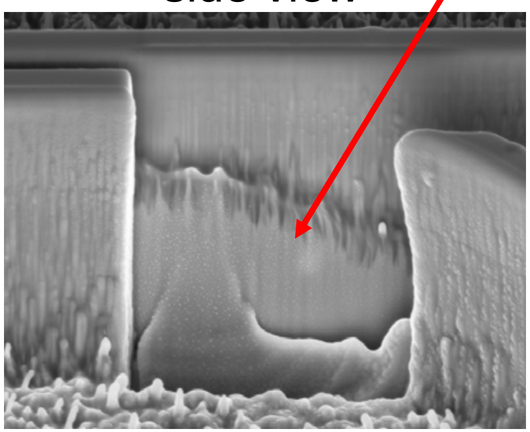
Results:

- 2D, 3D FDTD w/wo disp. agree well.
- FDTD vs. meas. ~7dB difference

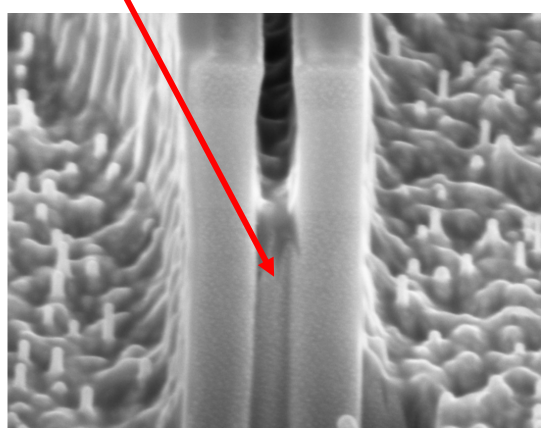
Discussions

- Wavelength conversion efficiencies η of FWM
  - 7dB difference of η between FDTD and measurement
  - fabrication tolerance for coupling gap (20% wider)
    g_1~0.185 μm (nominal 0.15), g_2~0.240 μm (nom.0.20)
  - Incompletely etched residual in gaps

side view



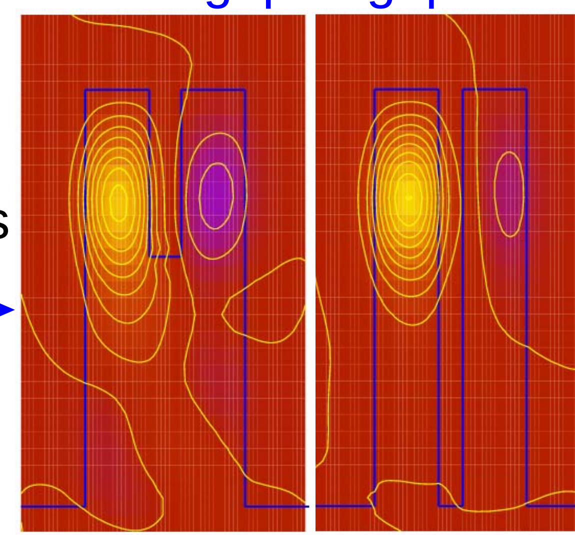
front view



ring bus

3D analysis

incomplete etch in gaps | normal gap



⇒ **significant reduction of η !**

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Discussions

- Computational resource
  - 3D FDTD with 64 CPUs (U.Karlsruhe, HP XC-6000)
    run time: 28 hours.
    total memory: up to approx. 4GB
  - 3D FDTD with 12 CPUs (Dell Poweredge cluster)
    run time: 98 hours.
  - 2D FDTD with 32 CPUs (with 3D code, XC-6000)
    run time: approx. 150 min
    total memory: up to approx. 500MB

Conclusions

- 2D & 3D FDTD anlaysis of DFWM in micro-resonator
  - optical Kerr nonlinearity
  - material dispersion: Lorentz model incl.
  - efficient stair-case roughness compensation by EDC (Effective Dielectric Constant) Technique
  - parallel computation allows efficient 3D analysis
  - Results:

    7dB difference in conversion efficiency, mainly due to fabrication tolerance and incomplete etching in gap region

With full-wave FDTD analysis, it will be possible to show what happens in an actual device, and then what to do next!